University of Technology
Building and Construction Engineering Department
Final Exam 2012-2013

Subject: Engineering Statistics
Division: All branches
Examiner: committee

Year: 2^{nd}
Time: 3 hours
Date: 19 / 6 / 2013

[Answer 4 questions only]- [أجب عن اربعة اسئلة فقط]

Q1: The frequency table below shows the yield strength of steel bars results.

a) Construct a histogram, frequency table, and frequency polygon?
b) Calculate mean and median?
c) Calculate the variance and the Standard Deviation?
d) Calculate the percentage of the compressive strength results < 422 N/mm^2?

Class interval (N/mm^2)	400-409	410-419	420-429	430-439	440-449
Frequency	20	30	35	30	18

Q2: Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples have a binomial distribution and are independent with regard to the presence of the pollutant.

a) Find the probability that in the next 18 samples, exactly 2 contain the pollutant.
b) Determine the probability that at least four samples contain the pollutant?
c) Determine the probability that $3 \leq X < 7$ (X the number of samples that contain the pollutant)

Q3: An engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength normally distributed with standard deviation $\sigma = 60$ psi. A random sample of 12 specimens has a mean tensile strength of $\bar{x} = 3250$ psi

a) Test the hypothesis Ho: $\mu = 3500$ versus H1: $\mu < 3500$ using $\alpha = 0.01$
a) Test the hypothesis Ho: $\mu = 3500$ versus H1: $\mu \neq 3500$ using $\alpha = 0.05$

4: **The number of cracks in a section of interstate highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.**

a) **What is the probability that there are no cracks that require repair in 5 miles of highway?**
b) **What is the probability that at least one crack requires repair in 0.5 mile of highway?**

Q5: **A random sample of size n_1= 16 is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size n_2= 9 is taken from another normal population with mean 70 and standard deviation 12. Let $\bar{x}_1$ and $\bar{x}_2$ be the two sample means. Find**

a) **The probability that $\bar{x}_1 - \bar{x}_2$ exceeds 4**
b) **The probability that $3.5 \le \bar{x}_1 - \bar{x}_2 \le 5.5$**

بعض القوانين المفيدة

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{or} \quad Z = \frac{\bar{x} - \mu}{\sigma} \quad \text{or}$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2 f_i}{n} \quad ; \quad \mu = \bar{x} \mp Z_c \frac{\sigma}{\sqrt{n}}$$

$$P(X = x_i) = \binom{n}{x} p^x q^{n-x} \quad ; \quad P(X = x_i) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P = 2\left[1 - \Phi_{|z|}\right] \qquad n = \left[\frac{Z_c \, \sigma}{e}\right]^2$$

$$P = \left[1 - \Phi_z\right]$$

$$P = \Phi_z$$

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

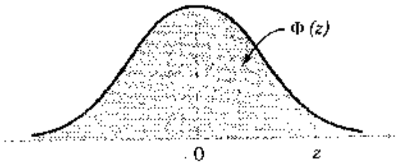


Table II Cumulative Standard Normal Distribution *(continued)*

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

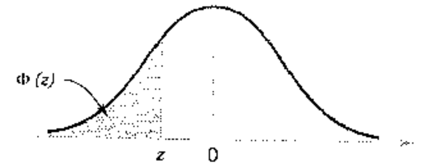


Table II Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

1.

Q1:

class interval	f_i	X_i	$X_i f_i$	$(X_i-\bar{X})^2 f_i$
400 – 409	20	404.5	8090	7761.912
410 – 419	30	414.5	12435	2822.118
420 – 429	35	424.5	14857.5	3.171
430 – 439	30	434.5	13035	3183.32
440 – 449	18	444.5	8001	7418.35
	$\Sigma f_i = 133$		$\Sigma =$	$\Sigma = 21187.97$

$$\text{Mean} = \bar{X} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{56418.5}{133} = 424.1992$$

$$\text{median} = a + \frac{\frac{n}{2} - n_1}{f_m} \Delta$$

$$\text{median} = 419.5 + \frac{\frac{133}{2} - 50}{35} (10) = 424.21$$

$$\text{Variance} = S^2 = \frac{\Sigma (x_i - \bar{x})^2 f_i}{n} = \frac{21187.97}{133}$$

$$= 159.3$$

$$S.D = \sqrt{159.3} = 12.62$$

$$P(X < 422)$$

طريقة القانون

$$P_1 = a + \frac{np - n_1}{f} \Delta$$

$$422 = 419.5 + \frac{133P - 50}{35} (10)$$

C2

$$P(X=2)=\binom{18}{2}(0.1)^2(0.9)^{16}$$

Now $\binom{18}{2} = 18!/[2!\ 16!] = 18(17)/2 = 153$. Therefore,

$$P(X=2) = 153(0.1)^2(0.9)^{16} = 0.284$$

Determine the probability that at least four samples contain the pollutant?

The requested probability is

$$P(X \geq 4) = \sum_{x=4}^{18} \binom{18}{x}(0.1)^x(0.9)^{18-x}$$

However, it is easier to use the complementary event,

$$\begin{aligned} P(X \geq 4) &= 1 - P(X<4) = 1 - \sum_{x=0}^{3}\binom{18}{x}(0.1)^x(0.9)^{18-x} \\ &= 1 - [0.150 + 0.300 + 0.284 + 0.168] = 0.098 \end{aligned}$$

Determine the probability that $3 \leq X < 7$. Now

$\binom{18}{0} 0.1^0 * 0.9^{18}$

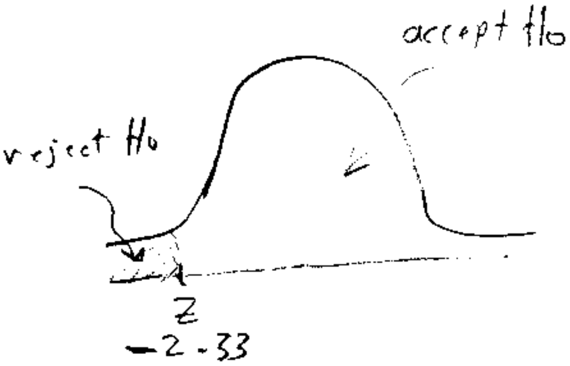
$\frac{18!}{18!\ 0!} = 1 * 1 * 1.5$

$$\begin{aligned} P(3 \leq X < 7) &= \sum_{x=3}^{6}\binom{18}{x}(0.1)^x(0.9)^{18-x} \\ &= 0.168 + 0.070 + 0.022 + 0.005 \\ &= 0.265 \end{aligned}$$

$\underline{Q3:}$

$n = 12$
$\bar{X} = 3250$
$\sigma = 60$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{3250 - 3500}{60/\sqrt{12}} = \frac{-250}{17.3205} = -14.43$$

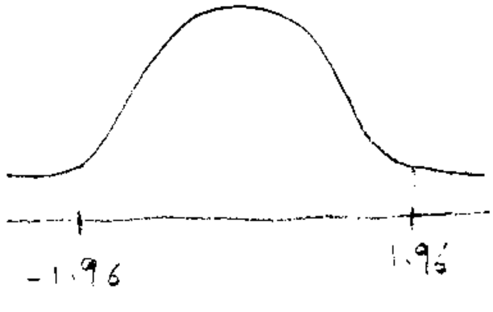


A: $\left.\begin{array}{l} H_0: \mu = 3500 \\ H_1: \mu < 3500 \end{array}\right\}$ $\alpha = 0.01$

$z = -14.43 < -2.33$ ∴ reject Ho
$P = \Phi_z = 0 < 0.01$ ∴ reject Ho

B $\left.\begin{array}{l} H_0: \mu = 3500 \\ H_1: \mu \neq 3500 \end{array}\right\}$ $\alpha = 0.05$

$z = -14.43 << -1.96$ ∴ reject Ho



$$P = 2[1 - \Phi_{|z|}]$$
$$= 2[1 - 1]$$
$$= 0 < 0.05$$ ∴ reject Ho

Q4 C4

$\lambda = 2$ cracks/mile

a) $\lambda = 2 * 5 = 10$ cracks/ 5 miles

$$P(X = x_i) = \frac{e^{-\lambda}\,\lambda^{x}}{X!}$$

$$P(X=0) = \frac{e^{-10}\,\cancel{10^{0}} \to 1}{\cancel{0!} \to 1} = 0.00004539 = 4.54 * 10^{-5}$$

b) $\lambda = 2 * \frac{1}{2} = 1$ crack/ $\frac{1}{2}$ mile

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{e^{-1}\,\cancel{1^{0}} \to 1}{\cancel{0!} \to 1}$$

$$= 1 - 0.3678$$

$$= 0.632$$

Q5:- Sampling theory : C5

$n_1 = 16$, $\mu_1 = 75$ $\sigma_1 = 8$
$n_2 = 9$ $\mu_2 = 70$ $\sigma_2 = 12$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{4 - 5}{\sqrt{\frac{\cancel{64}}{\cancel{16}} + \frac{\cancel{144}}{\cancel{9}}}} = \frac{-1}{\sqrt{20}} = -0.22$$

4

$\Rightarrow$ from table area $= .412$

a) $P(\bar{x}_1 - \bar{x}_2) > 4 = 1 - .412 = \underline{0.588}$

b) $z = \frac{3.5 - 5}{4.472} = \frac{-1.5}{4.472} = -0.335 \Rightarrow P = \underline{.3707}$

$z = \frac{0.5}{4.472} = 0.111 \Rightarrow P = 0.5437$

$P(3.5 \leq \bar{x}_1 - \bar{x}_2 \leq 5.5) = 0.5437 - 0.3707$

$= 0.173$