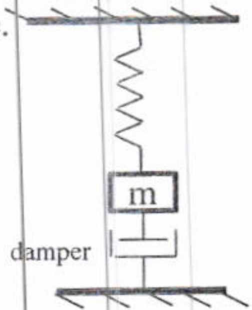
University of Technology
Building and Construction Eng. Dept.
Final Exam – First Attempt – 2013/2014
Subject : Engineering Analysis & Numerical Methods   Class: 3rd year
Division : All Divisions   Time : 3 Hours
Date :01/ 06 / 2014

Part One: **Engineering Analysis – Q1 , Q2 , Q3 and Q4**
Answer three questions only *(48 marks)*

Q1: A mass (m) of (**10** kg) is suspended from a spring shown in the figure.
Given the following data :

1) Damping constant **C= 60 N.sec/m**
2) Damped frequency $\omega_D^* = 30$ rad/sec

Find : 1)Spring stiffness (k) . 2) The complete solution of spring motion , if $y(0) = 0.01\,m$ and $\frac{dy}{dt}\big|_{t=0} = 0.2\,m/sec$



(16 marks)

Q2: Determine the general solution $[\,x(t)$ & $y(t)\,]$ of the following simultaneous differential equations

$$\frac{dx}{dt} = \frac{dy}{dt} + 3x \qquad \text{and} \qquad x + y + \frac{dx}{dt} = 0$$

(16 marks)

Q3: For the following periodic function whose definition in one period is:

$$f(t) = \begin{cases} t & for \quad 0 \le t \le 1 \\ 2 - t & for \quad 1 < t \le 2 \end{cases}$$

1) Draw the function with extension and find weather it is even, odd, or none.
2) Find the Fourier series for that function. **(16 marks)**

Q4: Find the complete solution of the following partial differential equation:

$$2U_x + 3U_y = 0$$

Where $U(0,0) = 1$ *and* $U(2,-3) = e^2$ **(16 marks)**

Part Two: * Numerical Methods – Q5 , Q6 , Q7 , and Q8
* Answer **three** questions including Q8
* *Solve for 4 digits after decimal point.* *(52 marks)*

Q5: Solve the following equations using Gauss elimination method:

$$\begin{aligned} 2X_1 + X_2 + X_3 &= 5 \\ X_1 + 3X_2 + 2X_3 &= 1 \\ 3X_1 - 2X_2 - 4X_3 &= -4 \end{aligned}$$

(16 marks)

Q6: For the data shown in the table below. Find the velocity after **16 sec** using Newton's divided difference method.

Time (sec)	10	15	20	22.5
Velocity (m/sec)	227.04	362.78	512.35	602.97

(16 marks)

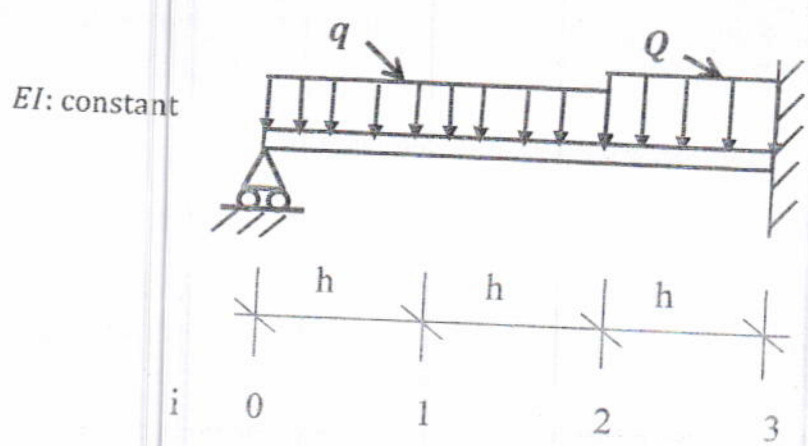
Q7: : Use the 4th Runge – Kutta method to find y at **x=0.9** [y(0.9)] for the following differential equation :

$$\frac{1}{2}y'.y = \cos x \quad if \quad y(0.5) = 2.1 \qquad and \ \ h = 0.4$$

(16 marks)

Q8: Using finite difference method to find the ratio of Q/q when deflection in node 1 equal to that in node 2 $(y_1 = y_2)$, for the loaded beam shown in the figure below :

(20 marks)



Note: 1) $(\frac{df}{dx})_i = \frac{1}{2h}.(y_{i+1} - y_{i-1})$ 2) $(\frac{d^2f}{dx^2})_i = \frac{1}{h^2}.(y_{i+1} - 2\,y_i + y_{i-1})$

3) $(\frac{d^4f}{dx^4})_i = \frac{1}{h^4}.(y_{i+2} - 4y_{i+1} + 6\,y_i - 4\,y_{i-1} + y_{i-2})$

Good luck

$\underline{\underline{Q_1}}$ / $\omega_D^* = \sqrt{\dfrac{4km - c^2}{4m}}$

$30 = \sqrt{\dfrac{4k(10) - 60^2}{2(10)}} \Rightarrow 900 = \dfrac{40k - 3600}{20 * 20}$

$k = \dfrac{360000 + 3600}{40} = 9090 \ N/m$

$m\ddot{y} + c\dot{y} + ky = 0$

$10\ddot{y} + 60\dot{y} + 9090y = 0$

$D_{1,2} = \dfrac{-c}{2m} \mp i . \omega_D^*$

$y_h = e^{\frac{-c}{2m}t}\left[A\cos\omega_D^* t + B . \sin\omega_D^* t\right]$

$y(t) = e^{-3t}\left[A . \cos 30t + B\sin 30t\right]$

$y(0) = 0.01 = e^{0}\left[A . 1 + 0\right] \Rightarrow A = 0.01$

$\underline{\underline{\text{then}}}$ $\dfrac{dy}{dt} = e^{-3t}\left[-30A\sin 30t + 30B\cos 30t\right] +$

$\left[A\cos 30t + B\sin 30t\right] * -3e^{-3t}$

$0.2 = e^{0}\left[0 + 30B\right] + \left[0.01\right] - 3 * 1$

$B = \dfrac{0.2 + 0.03}{30} = 0.0077$

$\therefore y(t) = e^{-3t}\left[0.01\cos 30t + 0.0077\sin 30t\right]$

$\underline{\underline{Q_2}}$ / $\frac{dx}{dt} = \frac{dy}{dt} + 3x \Rightarrow Dx - 3x - Dy = 0$

$x + y + \frac{dx}{dt} = 0 \Rightarrow Dx + x + y = 0$

$$\begin{bmatrix} D-3 & -D \\ D+1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D' = \begin{vmatrix} D-3 & -D \\ D+1 & 1 \end{vmatrix} = D - 3 + D^2 + D = D^2 + 2D - 3$$

$$D'x = \begin{vmatrix} 0 & -D \\ 0 & 1 \end{vmatrix} = 0 \qquad D'y = 0$$

$\therefore x = \frac{D'x}{D'} \Rightarrow x(D^2 + 2D - 3) = 0$

$(D-1)(D+3) = 0$

$D_1 = 1 \qquad D_2 = -3$

$x(t) = C_1 e^{t} + C_2 e^{-3t}$

$y = \frac{D'y}{D'} \Rightarrow y(D^2 + 2D - 3) = 0$

$y(t) = C_3 e^{t} + C_4 e^{-3t}$

$\underline{\text{then}}$ $\frac{dx}{dt} = C_1 e^{t} - 3C_2 e^{-3t}$

sub. in equ. ②:

$\Rrightarrow$ cont.

$$C_1 e^{t} + C_2 e^{-3t} + C_3 e^{t} + C_4 e^{-3t} + C_1 e^{t} - 3C_2 e^{-3t} =$$

e^{t} معاملات : $C_3 = -2C_1$

e^{-3t} معاملات : $C_4 = 2C_2$

$$\therefore\ y(t) = -2C_1 e^{t} + 2C_2 e^{-3t}$$