University of Technology
Building and Construction Eng. Dept.
Final Exam – First Attempt – 2013/2014

Subject : Engineering Analysis & Numerical Methods   Class: 3rd year

Division : All Divisions   Time : 3 Hours

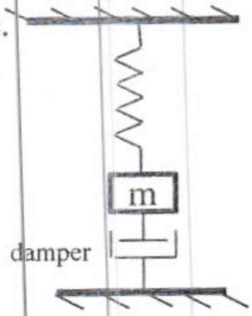
Date :01/ 06 / 2014

Part One: **Engineering Analysis – Q1 , Q2 , Q3 and Q4**
Answer three questions only *(48 marks)*

Q1: A mass (m) of (**10** kg) is suspended from a spring shown in the figure.
Given the following data :

1) Damping constant **C= 60 N.sec/m**
2) Damped frequency $\omega_D^* = 30$ **rad/sec**

Find : 1)Spring stiffness (k) . 2) The complete solution of spring motion , if $y(0) = 0.01\ m$ *and* $\frac{dy}{dt}_{|t=0} = 0.2\ m/sec$



(16 marks)

Q2: Determine the general solution [$x(t)$ & $y(t)$] of the following simultaneous differential equations

$$\frac{dx}{dt} = \frac{dy}{dt} + 3x \qquad \text{and} \qquad x + y + \frac{dx}{dt} = 0$$

(16 marks)

Q3: For the following periodic function whose definition in one period is:

$$f(t) = \begin{cases} t & for \quad 0 \le t \le 1 \\ 2 - t & for \quad 1 < t \le 2 \end{cases}$$

1) Draw the function with extension and find weather it is even, odd, or none.
2) Find the Fourier series for that function. **(16 marks)**

Q4: Find the complete solution of the following partial differential equation:

$$2U_x + 3U_y = 0$$

Where $U(0,0) = 1$ *and* $U(2,-3) = e^2$ **(16 marks)**

Part Two: * Numerical Methods – Q5 , Q6 , Q7 , and Q8
* Answer **three** questions including Q8 *(52 marks)*
* *Solve for 4 digits after decimal point.*

Q5: Solve the following equations using Gauss elimination method:

$$2X_1 + X_2 + X_3 = 5$$
$$X_1 + 3X_2 + 2X_3 = 1$$
$$3X_1 - 2X_2 - 4X_3 = -4$$

(16 marks)

Q6: For the data shown in the table below. Find the velocity after **16 sec** using Newton's divided difference method.

Time (sec)	10	15	20	22.5
Velocity (m/sec)	227.04	362.78	512.35	602.97

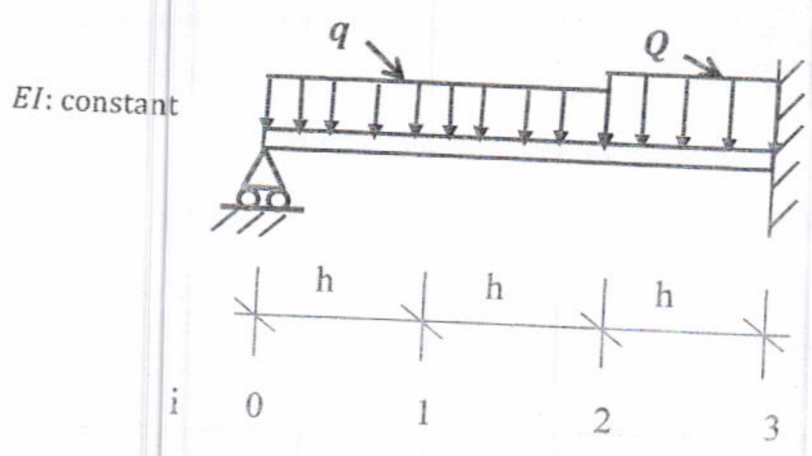
(16 marks)

Q7: : Use the 4[th] Runge – Kutta method to find y at **x=0.9** [y(0.9)] for the following differential equation :

$$\frac{1}{2}y'.y = \cos x \quad if \quad y(0.5) = 2.1 \qquad and \ h = 0.4$$

(16 marks)

Q8: Using finite difference method to find the ratio of Q/q when deflection in node 1 equal to that in node 2 $(y_1 = y_2)$, for the loaded beam shown in the figure below :
(20 marks)



Note: 1) $(\frac{df}{dx})_i = \frac{1}{2h}.(y_{i+1} - y_{i-1})$ 2) $(\frac{d^2f}{dx^2})_i = \frac{1}{h^2}.(y_{i+1} - 2y_i + y_{i-1})$

3) $(\frac{d^4f}{dx^4})_i = \frac{1}{h^4}.(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2})$

Good luck

$\underline{\underline{Q_1}}$ / $\omega_D^* = \sqrt{\frac{4km - c^2}{4m}}$

$30 = \sqrt{\frac{4k(10) - 60^2}{2(10)}} \Rightarrow 900 = \frac{40k - 3600}{20 * 20}$

$k = \frac{360000 + 3600}{40} = 9090 \ N/m$

$m\ddot{y} + c\dot{y} + ky = 0$

$10\ddot{y} + 60\dot{y} + 9090y = 0$

$D_{1,2} = \frac{-c}{2m} \mp i \cdot \omega_D^*$

$y_h = e^{\frac{-c}{2m}t}\left[A\cos\omega_D^* t + B \cdot \sin\omega_D^* t\right]$

$y(t) = e^{-3t}\left[A \cdot \cos 30t + B\sin 30t\right]$

$y(0) = 0.01 = \overset{1}{\cancel{e}}\left[A \cdot 1 + 0\right] \Rightarrow A = 0.01$

$\underline{\underline{\text{then}}}$ $\frac{dy}{dt} = e^{-3t}\left[-30A\sin 30t + 30B\cos 30t\right] +$

$\left[A\cos 30t + B\sin 30t\right] * -3e^{-3t}$

$0.2 = \overset{1}{\cancel{e}}\left[0 + 30B\right] + \left[0.01\right] - 3 * 1$

$B = \frac{0.2 + 0.03}{30} = 0.0077$

$\therefore y(t) = e^{-3t}\left[0.01\cos 30t + 0.0077\sin 30t\right]$

Q2/ $\frac{dx}{dt} = \frac{dy}{dt} + 3x \Rightarrow Dx - 3x - Dy = 0$

$x + y + \frac{dx}{dt} = 0 \Rightarrow Dx + x + y = 0$

$$\begin{bmatrix} D-3 & -D \\ D+1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D' = \begin{vmatrix} D-3 & -D \\ D+1 & 1 \end{vmatrix} = D - 3 + D^2 + D = D^2 + 2D - 3$$

$$D'x = \begin{vmatrix} 0 & -D \\ 0 & 1 \end{vmatrix} = 0 \qquad D'y = 0$$

$\therefore x = \frac{D'x}{D'} \Rightarrow x(D^2 + 2D - 3) = 0$

$(D-1)(D+3) = 0$

$D_1 = 1 \qquad D_2 = -3$

$x(t) = C_1 e^{t} + C_2 e^{-3t}$

$y = \frac{D'y}{D'} \Rightarrow y(D^2 + 2D - 3) = 0$

$y(t) = C_3 e^{t} + C_4 e^{-3t}$

then $\frac{dx}{dt} = C_1 e^{t} - 3C_2 e^{-3t}$

sub. in equ. ②:

$\gg$ cont.

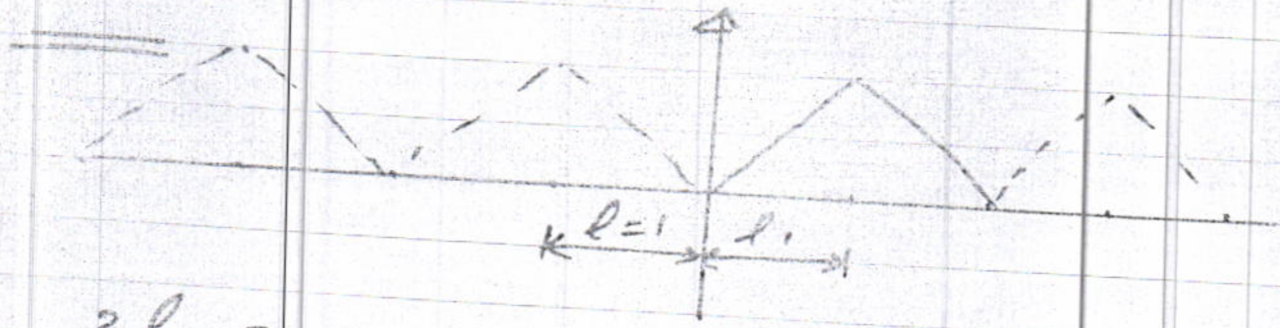
$$C_1 e^{t} + C_2 e^{-3t} + C_3 e^{t} + C_4 e^{-3t} + C_1 e^{t} - 3C_2 e^{-3t} =$$

e^{t} معامل : $C_3 = -2C_1$

e^{-3t} معامل : $C_4 = 2C_2$

$$\therefore y(t) = -2C_1 e^{t} + 2C_2 e^{-3t}$$

Q3 /



$$2l = 2-0 = 2 \Rightarrow l = 1$$

Even function $\Rightarrow b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(t).dt = \frac{2}{1}\int_0^1 t.dt =$$

$$a_n = \frac{2}{l}\int_0^l f(t).\cos\frac{n\pi}{l}t.dt = \frac{2}{1}\int_0^1 t.\cos n\pi t.dt$$

$$= 2\left(\frac{t}{n\pi}\sin n\pi t + \frac{1}{n^2\pi^2}\cos n\pi t\right)_0^1$$

$$= 2\left[\left(\frac{1}{n\pi}\overset{0}{\cancel{\sin n\pi}} + \frac{1}{n^2\pi^2}\cos n\pi\right) - \left(0 + \frac{1}{n^2\pi^2}\right)\right]$$

$$= \frac{2}{n^2\pi^2}\left(\cos n\pi - 1\right) = \frac{2}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$\therefore a_1 = \frac{-4}{\pi^2} \qquad a_2 = a_4 = a_6 = 0$$

$$a_3 = \frac{-4}{9\pi^2}$$

$$\therefore f(t) = \frac{1}{2} - \frac{4}{\pi^2}\left[\cos\pi t + \cos\frac{3\pi}{9}t + ---\right.$$

Q4 / let $u(x,y) = G(x) \cdot F(y)$

$$u_x = G' \cdot F \qquad u_y = G \cdot F'$$

$$2G'F + 3GF' = 0$$

$$\frac{2G'}{G} + \frac{3F'}{F} = 0$$

$$\frac{2G'}{G} = \frac{-3F'}{F} = \lambda$$

$$\therefore 2\frac{dG/dx}{G} = \lambda \Rightarrow 2\frac{dG}{G} = \lambda \cdot dx$$

$$2\ln G = \lambda \cdot x + c_1$$

$$\ln G = \frac{\lambda}{2}x + \frac{c_1}{2} \Rightarrow G(x) = A \cdot e^{\frac{\lambda x}{2}} \quad [A = e^{c_1/}$$

$$\frac{-3\,dF/dy}{F} = \lambda \Rightarrow \frac{-3\,dF}{F} = \lambda \cdot dy$$

$$-3\ln F = \lambda \cdot y + c_2$$

$$\ln F = \frac{\lambda}{-3}y + \frac{c_2}{-3}$$

$$F(y) = e^{\frac{\lambda}{-3}y} \cdot B \qquad B = e^{-\frac{c_2}{3}}$$

$$\therefore u = A e^{\frac{\lambda x}{2}} \cdot B e^{\frac{\lambda}{-3}y} = D \cdot e^{\frac{\lambda x}{2} - \frac{\lambda y}{3}}$$

$\Rightarrow$ cont.

Q4 / cont.

$$U = D \cdot e^{\frac{\lambda x}{2} - \frac{\lambda y}{3}}$$

① $U(0,0) = 1$

$\therefore 1 = D e^{0} \Rightarrow D = 1$

$U(x,y) = e^{\frac{\lambda x}{2} - \frac{\lambda y}{3}}$

② $U(2,-3) = e^{2}$

$e^{2} = e^{\lambda + \lambda} \Rightarrow 2\lambda = 2 \Rightarrow \lambda = 1$

$\therefore U(x,y) = e^{\frac{x}{2} - \frac{y}{3}}$

Q5/ $2x_1 + x_2 + x_3 = 5$

$x_1 + 3x_2 + 2x_3 = 1$

$3x_1 - 2x_2 - 4x_3 = -4$

Rearange the equations

$x_1 + 3x_2 + 2x_3 = 1$
$2x_1 + x_2 + x_3 = 5$
$3x_1 - 2x_2 - 4x_3 = -4$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 2 & 1 & 1 & 5 \\ 3 & -2 & -4 & -4 \end{array}\right] \begin{array}{l} \\ r_2 - 2r_1 \\ r_3 - 3r_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -5 & -3 & 3 \\ 0 & -11 & -10 & -7 \end{array}\right]$$

$r_3 / -11$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -5 & -3 & 3 \\ 0 & 1 & .909 & .636 \end{array}\right] 5r_3 + r_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -5 & -3 & 3 \\ 0 & 0 & 1.545 & 6.18 \end{array}\right]$$

$\therefore x_3 = \frac{6.18}{1.545} = 4$

$-5x_2 = 12 + 3 \Rightarrow x_2 = -3$

$x_1 = 9 - 8 + 1 \Rightarrow x_1 = 2$

Q6 /

		Δf	$\Delta^2 f$	$\Delta^3 f$
10	227.04			
		27.148		
15	362.78		0.2766	
		29.914		0.0454
20	512.35		0.8445	
		36.248		
22.5	602.97			

$$V(t) = 227.04 + 27.148(t-10) + 0.2766(t-10)(t-15)$$
$$+ 0.0454(t-10)(t-15)(t-20)$$

$$\therefore V(16) = 227.04 + 27.148(16-10) + 0.2766(16-10)(16-15)$$
$$+ 0.0454(16-10)(16-15)(16-20)$$

$$= 227.04 + 162.888 + 1.6596 - 1.0896$$

$$= 390.498 \quad m/sec$$

Q7 :

$$\frac{1}{2} y' \cdot y = \cos x \qquad y(0.5) = 2.1, \quad h = 0.4$$

find $y(0.9) = ?$

$$y' = \frac{2 \cos x}{y}$$

1^{st} stage: $y(0.5) = 2.1$, $x_0 = 0.5$, $y_0 = 2.1$

$$k_1 = y'(0.5, 2.1) * 0.4 = 0.3343$$

$$k_2 = y'\left(0.5 + \frac{0.4}{2}, 2.1 + \frac{0.3343}{2}\right) * 0.4 = 0.2699$$

$$k_3 = y'\left(0.5 + \frac{0.4}{2}, 2.1 + \frac{0.2699}{2}\right) * 0.4 = 0.2738$$

$$k_4 = y'(0.5 + 0.4, 2.1 + 0.2738) * 0.4 = 0.2095$$

$$\therefore \Delta y = \frac{1}{6}\left[0.3343 + 2*0.2699 + 2*0.2738 + 0.2095\right]$$

$$= \frac{1.6312}{6} = 0.2719$$

$$\therefore y(0.9) = y(0.5) + \Delta y$$

$$= 2.1 + 0.2719$$

$$y(0.9) = 2.3719$$

Q8/ at $i=0$ $y_0=0$, $M_0=0$

$$\frac{M_0}{EI}=\frac{d^2y}{dx^2}=\frac{y_1-2y_0+y_{-1}}{h^2}$$

$$h^2*0=y_1+y_{-1} \Rightarrow y_{-1}=-y_1$$

at $i=3$ $y_3=0$ $\theta_3=0$

$$(\theta)_3=\frac{dy}{dx}=\frac{y_4-y_2}{2h} \Rightarrow y_4=y_2$$

then: at $i=1 \Rightarrow \omega=-g$ (given $y_2=1$)

$$\frac{-g}{EI}=\frac{1}{h^4}\left(y_3-4y_2+6y_1-4y_0+y_{-1}\right)$$

$$\frac{-gh^4}{EI}=(0-4y_1+6y_1-0-y_1)$$

$$\therefore y_1=\frac{-gh^4}{EI}$$

$$\text{at } i=2: \frac{(-g*\frac{h}{2})+(-Q*\frac{h}{2})}{h}=\frac{-g-Q}{2}$$

$$\frac{-g-Q}{2EI}=\frac{1}{h^4}\left(y_4-4y_3+6y_2-4y_1+y_0\right)$$

$$\frac{(-g-Q)*h^4}{2EI}=\left[y_1-0+6y_1-4y_1+0\right]$$

$$\therefore y_1=\frac{(-g-Q)*h^4}{6EI}$$

Q8/ cont.

$$\therefore \frac{-q h^4}{EI} = \frac{-(q+Q) * h^4}{6EI} \quad \left(* = \frac{EI}{h^4}\right)$$

$$q = \frac{q+Q}{6} \Rightarrow 6q = q + Q$$

$$\therefore Q = 5q$$

$$\frac{Q}{q} = 5$$