University of Technology
Building and Construction Engineering Department
Final Exam-First Attempt / 2013-2014

Subject: Engineering Statistics **Year: 2nd**
Division: All divisions **Time:3 hours**
Examiner: Committee **Date: 2/6 /2014**

[Answer 5 questions only]- [أجب عن خمسة اسئلة فقط]

Q1: The frequency table below shows the compressive strength of concrete cubes results.

a) Construct a histogram.
b) Calculate mean and median.
c) Calculate the variance and the Standard Deviation.
d) Calculate the percentage of the compressive strength results <34.3 N/mm^2.

Class interval (N / mm^2)	31 – 31.9	32 – 32.9	33 – 33.9	34 – 34.9	35 – 35.9
Frequency	30	45	55	40	25

(20 marks)

Q2: Batches that consist of 55 concrete blocks from a production process are checked for conformance to building requirements. The mean number of nonconforming concrete blocks in a batch is 4. Assume that the number of nonconforming concrete blocks in a batch, denoted as X, is a binomial random variable.
(a) What are n and p? (b) What is $P(X \leq 2)$? (c) What is $P(X < 53)$?

(20 marks)

Q3: The diameter of a shaft in an optical storage drive is normally distributed with `mean 0.2508 inch and standard deviation 0.0005 inch. The specifications on the shaft are 0.2500 ± 0.001 inch. What proportion of shafts conforms to specifications?

(20 marks)

Q4: The analysis of cement sample for a concrete mixture is summarized by conformance to specifications.

		Chemical conforms	
		Yes	No
Physical Conforms	Yes	**30**	**1**
	No	**3**	**2**

(a) If a cement sample is selected at random, what is the probability that the sample conforms to physical requirements?
(b) What is the probability that the selected cement sample conforms to physical requirements or to chemical requirements?
(c) What is the probability that the selected shaft either conforms to physical requirements or does not conform to chemical requirements?
(d) What is the probability that the selected shaft conforms to both physical and chemical requirements?

(20 marks)

Q5: Aircrew escape systems are powered by a solid propellant. The burning rate of this propellant is an important product characteristic. Specifications require that the mean burning rate must be 50 cm/s. We know that the standard deviation of burning rate is 2 cm/s. significance level of 0.05 and. The selected random sample size n= 25 and obtained sample average burning rate of $\bar{X}$= 51.3 cm/s. What conclusions should be drawn?

(a) Test the hypothesis Ho: $\mu = 50$ versus Ha: $\mu \neq 50$ using $\alpha = 0.01$

(b) Test the hypothesis Ho: $\mu = 50$ versus Ha: $\mu \neq 50$ using $\alpha = 0.05$

(20 marks)

Q6: The diameter of a concrete pipes manufactured for a waste line is known to have a normal distribution with $\sigma = 0.1$ cm. A random sample of size 8 yields an average diameter of 30 cm.

a) Find a 95% two-sided confidence interval on the mean pipe diameter.
b) If the bound of error in estimation E is one-half of the length of 95% CL, find the value of sample size?

(20 marks)

بعض القوانين المفيدة

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad or \; Z = \frac{\bar{x} - \mu}{\sigma} \quad or \quad Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2 f_i}{n} \; ; \; \mu = \bar{x} \pm Z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$P(X = x_i) = \binom{n}{x} p^x (1-p)^{n-x} \; ; P(X = x_i) = \frac{e^{-\lambda} \lambda^x}{x_i}$$

$$n = \left[\frac{z_\alpha/2\sigma}{E}\right]^2$$

$$P = 2[1 - \emptyset(|z|)]$$
$$P = 1 - \emptyset(z)$$
$$P = \emptyset(z)$$

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

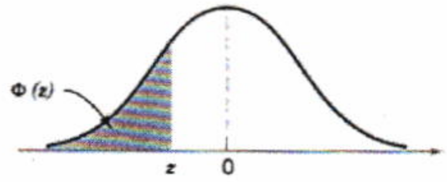


Table II Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

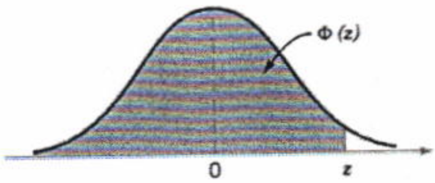
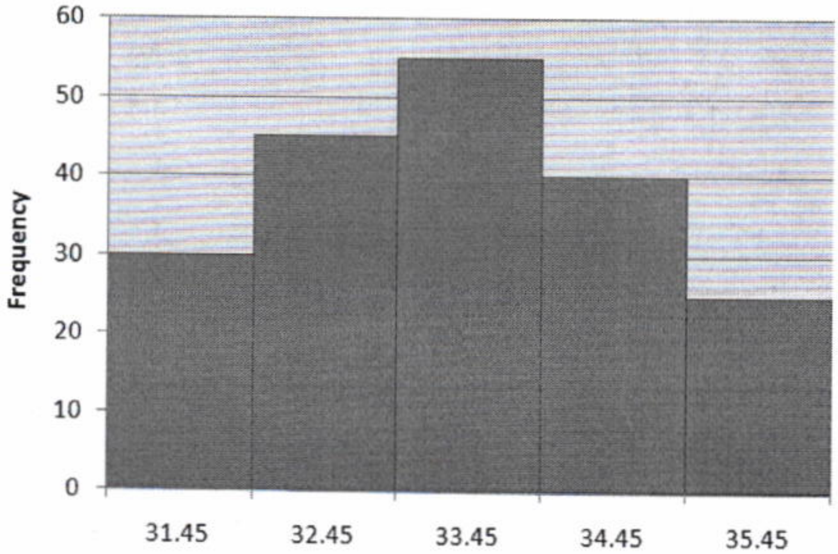


Table II Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

Q1)
The Answer
a-



b)

Class	fi	xi	$fixi$	$(xi-\overline{x})^2 fi$	Fi
31 – 31.9	30	31.45	943.5	110.592	0.153846
32 – 32.9	45	32.45	1460.24	38.088	0.230769
33 – 33.9	55	33.45	1839.75	0.352	0.282051
34 – 34.9	40	34.45	1378	46.656	0.205128
35 – 35.9	25	35.45	886.25	108.16	0.128205
	$\sum = 195$		$\sum = 6507.75$	$\sum = 303.848$	$\sum = 1$

$$mean = \bar{X} = \frac{\sum fixi}{\sum fi} = \frac{6507.75}{195} = 33.37$$

$$median = a + \frac{\frac{n}{2} - n_1}{fm} \Delta$$

$$median = 32.95 + \frac{\frac{195}{2} - 75}{55} 1 = 33.36$$

c)

$$S^2 = \frac{(xi - \bar{x})^2 fi}{n} = \frac{303.848}{195} = 1.55819$$

Standard deviation = $\sqrt{S^2} = \sqrt{1.558} = 1.248$

d)

طريقة القانون

$$P(X < 34.3)$$

$$pi = a + \frac{np - n_1}{f} \Delta$$

$$34.3 = 33.95 + \frac{195p - 130}{40} (1)$$

$$p = \frac{144}{195} = 0.738 \approx 75\%$$

او طريقة المساحات

$$P(X < 39.5) = \frac{Area\ of\ Histogram\ less\ than\ 34.3}{total\ area}$$

$$= \frac{\Delta_5 f_5 + \Delta_4 f_4 + \Delta_3 f_3 + \Delta_2 f_2 + \Delta_1 f_1}{\Delta \sum fi}$$

$$P(X < 39.5) = \frac{1 * 30 + 1 * 45 + 1 * 55 + 0.35 * 40}{1 * 195} = \frac{144}{195} = 0.738 \approx 74\%$$

Q2)

The Answer

(a) What are n and p?

n=55 , p= 0.072

(b) What is P(X ≤ 2)?

$$P(X \leq 2) = \sum_{x}^{55} \binom{55}{x} (0.072)^x (1 - 0.072)^{55-x}$$

$$= 0.0164 + 0.07 + 0.14670 = 0.2331$$

(c) What is P(X < 53)?

$$P(X < 53) = 1 - P(X \leq 2) = 1 - 0.2331 = 0.7669$$

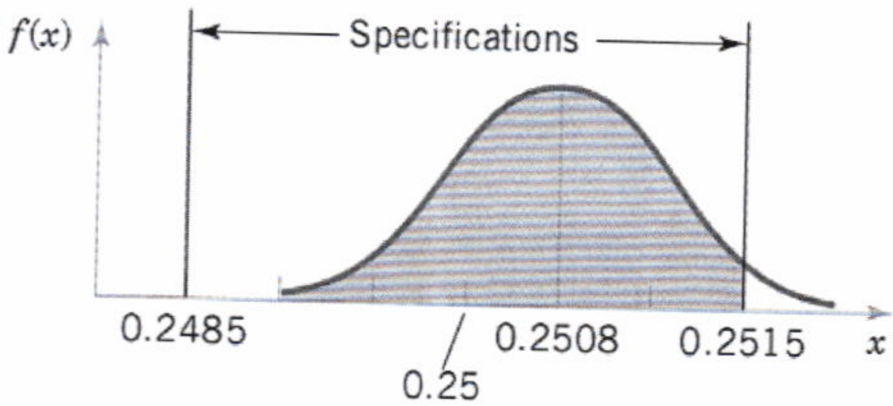
Q3)

The Answer

$$P(0.2485 < X < 0.2515) = P\left(\frac{0.2485 - 0.2508}{0.0005} < Z < \frac{0.2515 - 0.2508}{0.0005}\right)$$
$$= P(-4.6 < Z < 1.4) = P(Z < 1.4) - P(Z < -4.6)$$
$$= 0.91924 - 0.0000 = 0.91924$$

$$P(0.2485 < X < 0.2515) = P\left(\frac{0.2485 - 0.2500}{0.0005} < Z < \frac{0.2515 - 0.2500}{0.0005}\right)$$

$$= P(-3 < Z < 3)$$
$$= P(Z < 3) - P(Z < -3)$$
$$= 0.99865 - 0.00135$$
$$= 0.9973$$



Q4)

(a) If a cement sample is selected at random, what is the probability that the sample conforms to physical requirements?

$$P(A) = \frac{31}{36} = 0.86$$

(b) What is the probability that the selected cement sample conforms to physical requirements or to chemical requirements?

$$P(B) = \frac{33}{36} = 0.91$$

$$P(A \cap B) = \frac{30}{36} = 0.83$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.94$$

(c) What is the probability that the selected shaft either conforms to physical requirements or does not conform to chemical requirements?

$P(A \cup B') = ?$

$$P(A \cap B') = \frac{2}{36} = 0.055$$

$$P(B') = \frac{5}{36} = 0.138$$

$P(A \cup B') = 0.86 + 0.138 - 0.055 = 0.94$, $P(A \oplus B) = 0.94 - 0.055 =$ $= 0.885$

(d) What is the probability that the selected shaft conforms to both physical and chemical requirements?

$$P(A \cap B) = \frac{30}{36} = 0.83$$

Q5)

a) Test the hypothesis Ho: μ = 50 versus Ha: μ ≠ 50 using α = 0.01

n=25

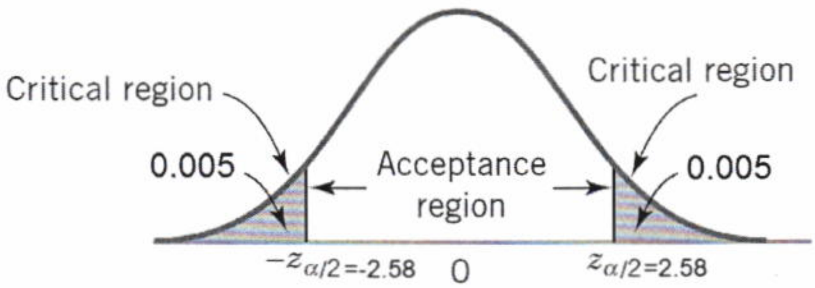
$\bar{x} = 51.3$

$\sigma = 2$

a) Test $H_0: \mu = 50$

$H_1: \mu \neq 50$

$\alpha = 0.01$



$$Z = \frac{\bar{x} - \mu_o}{\sigma/\sqrt{n}} = \frac{51.3 - 50}{2/\sqrt{25}} = \frac{1.3}{0.4} = 3.25 \therefore \text{reject } H_0$$

b) Test the hypothesis Ho: μ = 50 versus Ha: μ ≠ 50 using α = 0.05

1. The parameter of interest is μ, the mean burning rate.
2. H_0: $\mu = 50$ centimeters per second
3. H_1: $\mu \neq 50$ centimeters per second
4. $\alpha = 0.05$
5. The test statistic is

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

6. Reject H_0 if $z_0 > 1.96$ or if $z_0 < -1.96$. Note that this results from step 4, where we specified $\alpha = 0.05$, and so the boundaries of the critical region are at $z_{0.025} = 1.96$ and $-z_{0.025} = -1.96$.
7. Computations: Since $\bar{x} = 51.3$ and $\sigma = 2$,

$$z_0 = \frac{51.3 - 50}{2/\sqrt{25}} = 3.25$$

8. Conclusion: Since $z_0 = 3.25 > 1.96$, we reject H_0: $\mu = 50$ at the 0.05 level of significance. Stated more completely, we conclude that the mean burning rate differs from 50 centimeters per second, based on a sample of 25 measurements. In fact, there is strong evidence that the mean burning rate exceeds 50 centimeters per second.

Q6)

n=8

$\bar{x} = 30$

$\sigma = 0.1$

a)

$Z_{\alpha/2}$=1.96

$$\mu = \bar{x} \pm Z_c \frac{\sigma}{\sqrt{n}}$$

$$\mu = 30 \pm 1.96 \frac{0.1}{\sqrt{8}} = \begin{pmatrix} 29.93 \\ 30.069 \end{pmatrix} \begin{matrix} lower \\ upper \end{matrix}$$

b)

E=L/2=0.0695

$$n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{1.96 * 0.1}{0.0695}\right)^2 \approx 8$$ المطلوب b لغرض تدقيق نتائج a