



University Of Technology
Building and Construction Eng. Dept.
Final Exam – 2013/2014



Subject : Mathematics II

Class: 2nd

Branch : All branches

Time : 3 Hours

Date : 4/6/ 2014

Note: Answer only **eight** questions.

Q1: The discharge of water (Q) over a horizontal weir is given by $Q = C_d L H^{3/2}$, where H is the head over the weir, L is the length of the weir and assume C_d is constant. What is the percentage error in Q caused by an error in measuring H of $\pm 0.2\%$ and L of $\pm 1\%$ if the average value of $H_0 = 2\text{m}$ and $L_0 = 4\text{m}$.

Q2: In which direction is the directional derivative of the function $w(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ at the point (1,1): a- equal to zero, b- Increase must rapidly.

Q3: Solve the following differential equation using undetermined coefficient method.
 $y'' + 3y' + 2y = e^{-x} + e^{2x}$

Q4: Show that the vectors $A = 3i - 2j + k$, $B = i - 3j + 5k$ and $C = 2i + j - 4k$ form a right triangle.

Q5: The acceleration of a particle in the plane as a function of time (t) is $a(t) = -3ti$, when the particle's velocity and position are $v(0) = 2j$ and $R(0) = 4i$. Find the position (x,y) of this particle when $t = 2\text{sec}$.

Q6: Find the position \bar{x} of the center of gravity for a thin plate bounded by the curve $y = x^2$ and $x = 2$ and $y = 0$ has a variable density $\delta = 1 + x + xy$. *At the first quadrant*

Q7: Use the spherical coordinates to solve the following integral for the solid bounded above by the spherical surface $z = (4 - x^2 - y^2)^{1/2}$ and below by the xy-plane.

$$\iiint z^2 \cdot (x^2 + y^2 + z^2)^{\frac{1}{2}} \cdot dv$$

Q8: Find the inverse of the matrix [A] given by $\begin{vmatrix} 6 & -1 & 5 \\ 2 & 1 & 3 \\ 4 & 0 & -2 \end{vmatrix}$ and show that:
 $[A]^{-1} \cdot [A] = I$.

Q9: Find the nature of the following series:

a- $\sum_{n=1}^{\infty} \frac{n! x^{-n}}{(n+1)!}$

b- $\sum_{n=1}^{\infty} \ln \frac{n}{(n+1)}$

Q10: A: Find $W = (i)^{\ln i}$

B: Show that $\cos Z = \cos x \cdot \cosh y - i \sin x \cdot \sinh y$

Good Luck

$Q_1 //$

$$Q = C_d L H^{1.5}$$

$$Q = f(L, H)$$

$$dQ = \frac{\partial Q}{\partial L} dL + \frac{\partial Q}{\partial H} dH$$

$$\frac{\partial Q}{\partial L} = C_d H^{1.5}$$

$$\frac{\partial Q}{\partial H} = 1.5 C_d L H^{0.5}$$

$$dL \approx \Delta L = \pm \frac{1}{100}$$

$$dH \approx \Delta H = \pm \frac{0.2}{100}$$

$$dQ = \pm C_d H^{1.5} \left(\frac{1}{100}\right) \pm 1.5 C_d L H^{0.5} \left(\frac{0.2}{100}\right)$$

for $H = 2 \text{ m}$ and $L = 4 \text{ m}$

$$dQ = \pm \frac{C_d (2)^{1.5}}{100} \pm 1.5 C_d (4) (2)^{0.5} \left(\frac{0.2}{100}\right)$$

$$dQ = \pm 0.02828 C_d \pm 0.00169 C_d$$

$$dQ = \pm 0.03 C_d = \Delta Q$$

$$Q_{H=2, L=4} = C_d (4) (2)^{1.5} = 11.3137 C_d$$

$$\text{Percentage error} = \frac{\Delta Q}{Q} = \frac{\pm 0.003 C_d}{11.3137 C_d} = 0.027\%$$

Q2/

$$w(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$

$$\frac{\partial w}{\partial x} = \frac{2x(x^2 + y^2) - 2x(x^2 - y^2)}{(x^2 + y^2)^2} \Rightarrow \left. \frac{\partial w}{\partial x} \right|_{\substack{x=1 \\ y=1}} = \frac{2(1)(1^2+1^2) - 2(1)(1^2-1)}{(1^2+1^2)^2} = 1$$

$$\frac{\partial w}{\partial y} = \frac{-2y(x^2 + y^2) - 2y(x^2 - y^2)}{(x^2 + y^2)^2} \Rightarrow \left. \frac{\partial w}{\partial y} \right|_{\substack{x=1 \\ y=1}} = \frac{-2(1)(1^2+1^2) - 2(1)(1^2-1)}{(1^2+1^2)^2} = -1$$

$$\frac{dw}{ds} = \frac{\partial w}{\partial x} \cos \theta + \frac{\partial w}{\partial y} \sin \theta$$

$$\therefore \frac{dw}{ds} = \cos \theta - \sin \theta$$

a- for $\frac{dw}{ds} = 0 \Rightarrow \cos \theta - \sin \theta = 0 \Rightarrow \tan \theta = 1$
 $\theta = \tan^{-1}(1) \Rightarrow \theta = \frac{\pi}{4}$

b- for $\frac{dw}{ds} = \max \Rightarrow \frac{d(\frac{dw}{ds})}{d\theta} = 0$

$$\frac{d(\frac{dw}{ds})}{d\theta} = -\sin \theta - \cos \theta = 0 \Rightarrow \tan \theta = -1$$

$$\theta = \tan^{-1}(-1) \Rightarrow \theta = -\frac{\pi}{4}$$

$$Q3// \ddot{y} + 3\dot{y} + 2y = e^{-x} + e^{2x} \quad \text{--- (1)}$$

$$\text{Sol/ } r^2 + 3r + 2 = 0$$

$$(r+2)(r+1) = 0$$

$$r_1 = -1 \quad \& \quad r_2 = -2$$

$$y_h = C_1 e^{r_1 x} + C_2 e^{r_2 x}, \quad r_1 \neq r_2$$

$$y_h = C_1 e^{-x} + C_2 e^{-2x}$$

$$y_p = Ax e^{-x} + B e^{2x}$$

$$\dot{y}_p = A e^{-x} - Ax e^{-x} + 2B e^{2x}$$

$$\ddot{y}_p = -A e^{-x} - A e^{-x} + Ax e^{-x} + 4B e^{2x}$$

$$\ddot{y}_p = -2A e^{-x} + Ax e^{-x} + 4B e^{2x}$$

sub in (1)

$$\begin{aligned} & \underline{-2A e^{-x}} + \underline{Ax e^{-x}} + \underline{4B e^{2x}} + \underline{3A e^{-x}} - \underline{3Ax e^{-x}} + \underline{6B e^{2x}} \\ & \quad + \underline{2Ax e^{-x}} + \underline{2B e^{2x}} = e^{-x} + e^{2x} \end{aligned}$$

$$\underline{A e^{-x}} + \underline{12B e^{2x}} = \underline{e^{-x}} + \underline{e^{2x}}$$

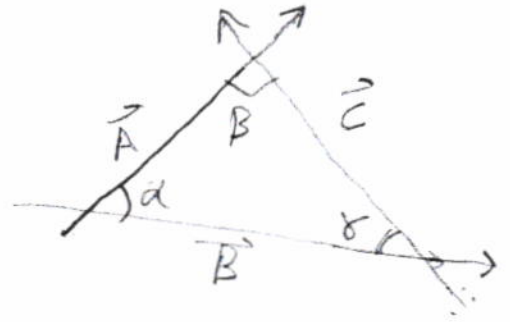
$$\therefore A = 1 \quad \& \quad B = \frac{1}{12}$$

$$y_p = x e^{-x} + \frac{1}{2} e^{2x}$$

$$\therefore y = C_1 e^{-x} + C_2 e^{-2x} + x e^{-x} + \frac{1}{2} e^{2x}$$

Q4

$$\vec{A} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$
$$\vec{B} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$$
$$\vec{C} = 2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$$



$$\alpha = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\alpha = \cos^{-1} \frac{3(1) - 2(-3) + 1(5)}{\sqrt{3^2 + 2^2 + 1^2} \sqrt{1^2 + 3^2 + 5^2}} \Rightarrow \alpha = \cos^{-1} \left(\frac{14}{\sqrt{14} \sqrt{35}} \right)$$

$$\alpha = 50.76^\circ$$

$$\beta = \cos^{-1} \frac{\vec{A} \cdot \vec{C}}{|\vec{A}| |\vec{C}|} = \cos^{-1} \frac{3(2) - 2(1) + 1(-4)}{\sqrt{14} \sqrt{2^2 + 1^2 + 4^2}} = \cos^{-1} \left(\frac{0}{\sqrt{14} \sqrt{21}} \right)$$

$$\beta = 90^\circ$$

$$\gamma = \cos^{-1} \frac{\vec{B} \cdot \vec{C}}{|\vec{B}| |\vec{C}|} = \cos^{-1} \frac{1(2) - 3(1) + 5(-4)}{\sqrt{35} \sqrt{21}} = \cos^{-1} \left(\frac{-21}{\sqrt{35} \sqrt{21}} \right)$$

$$\gamma = 39.32^\circ$$

$$\alpha + \beta + \gamma = 50.76^\circ + 90^\circ + 39.32^\circ = 180^\circ$$

Q5//

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Q5//

$$a(t) = -3t\mathbf{i} = \frac{d\mathbf{v}}{dt}$$

$$\mathbf{v}(t) = \int a(t) dt \Rightarrow \mathbf{v}(t) = \int (-3t\mathbf{i}) dt + C_1$$

$$\mathbf{v}(t) = -\frac{3}{2}t^2\mathbf{i} + C_1$$

$$\text{For } \mathbf{v}(0) = 2\mathbf{j} \Rightarrow 2\mathbf{j} = -\frac{3}{2}(0)^2\mathbf{i} + C_1 \Rightarrow C_1 = 2\mathbf{j}$$

$$\therefore \mathbf{v}(t) = -\frac{3}{2}t^2\mathbf{i} + 2\mathbf{j} = \frac{d\mathbf{R}}{dt}$$

$$\mathbf{R}(t) = \int \mathbf{v}(t) dt$$

$$\mathbf{R}(t) = \int \left(-\frac{3}{2}t^2\mathbf{i} + 2\mathbf{j}\right) dt + C_2$$

$$\mathbf{R}(t) = -\frac{3}{2} \frac{t^3}{3}\mathbf{i} + 2\mathbf{j}t + C_2$$

$$\mathbf{R}(t) = -\frac{t^3}{2}\mathbf{i} + 2\mathbf{j}t + C_2$$

$$\text{For } \mathbf{R}(0) = 4\mathbf{i} \Rightarrow 4\mathbf{i} = -\frac{(0)^3}{2}\mathbf{i} + 2\mathbf{j}(0) + C_2 \Rightarrow C_2 = 4\mathbf{i}$$

$$\mathbf{R}(t) = -\frac{t^3}{2}\mathbf{i} + 2t\mathbf{j} + 4\mathbf{i} \Rightarrow \mathbf{R}(t) = \left(4 - \frac{t^3}{2}\right)\mathbf{i} + 2t\mathbf{j}$$

$$\therefore \mathbf{R}(2) = \left(4 - \frac{2^3}{2}\right)\mathbf{i} + 2(2)\mathbf{j} \Rightarrow \mathbf{R}(2) = (0)\mathbf{i} + 4\mathbf{j}$$

$$\mathbf{R}(t) = X\mathbf{i} + Y\mathbf{j} \Rightarrow \boxed{X=0, Y=4}$$

$$Q6 // \bar{x} = \frac{My}{m}$$

$$* m = \iint \delta(x,y) dx dy$$

$$m = \int_0^4 \int_{\sqrt{y}}^2 (1+x+xy) dx dy$$

$$m = \int_0^4 \left[x + \frac{x^2}{2} + \frac{x^2}{2} y \right]_{\sqrt{y}}^2 dy$$

$$m = \int_0^4 \left[\left(2 + \frac{2^2}{2} + \frac{2^2}{2} y \right) - \left(y^{\frac{1}{2}} + \frac{y}{2} + \frac{y^2}{2} \right) \right] dy$$

$$m = \int_0^4 \left(4 + \frac{3}{2} y - y^{\frac{1}{2}} - \frac{y^2}{2} \right) dy$$

$$m = \left[4y + \frac{3}{4} y^2 - \frac{2}{3} y^{\frac{3}{2}} - \frac{y^3}{6} \right]_0^4 \Rightarrow m = \left[4(4) + \frac{3(4)^2}{4} - \frac{2}{3} 4^{\frac{3}{2}} - \frac{4^3}{6} \right]$$

$$m = 12$$

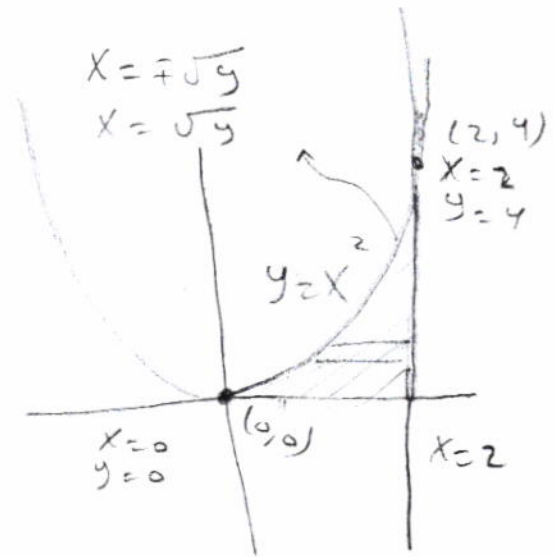
$$* My = \iint x \delta(x,y) dx dy$$

$$My = \int_0^4 \int_{\sqrt{y}}^2 x(1+x+xy) dx dy \Rightarrow My = \int_0^4 \int_{\sqrt{y}}^2 (x+x^2+x^2y) dx dy$$

$$My = \int_0^4 \left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^3}{3} y \right]_{\sqrt{y}}^2 dy$$

$$My = \int_0^4 \left[\left(\frac{2^2}{2} + \frac{2^3}{3} + \frac{2^3}{3} y \right) - \left(\frac{y}{2} + \frac{y^{\frac{3}{2}}}{3} + \frac{y^{\frac{3}{2}}}{3} y \right) \right] dy$$

$$My = \int_0^4 \left(2 + \frac{8}{3} + \frac{8}{3} y \right) - \left(\frac{y}{2} + \frac{y^{\frac{3}{2}}}{3} + \frac{y^{\frac{5}{2}}}{3} \right) dy$$



$$M_y = \int_0^4 \left(4.667 + 2.6667y - \frac{y^{\frac{3}{2}}}{3} - \frac{y^{\frac{5}{2}}}{3} \right) dy$$

$$M_y = \left[4.667y + \frac{2.6667y^2}{2} - \frac{2}{15}y^{\frac{5}{2}} - \frac{2}{21}y^{\frac{7}{2}} \right]_0^4$$

$$M_y = \left[4.667(4) + \frac{2.6667(4^2)}{2} - \frac{2}{15}(4)^{\frac{5}{2}} - \frac{2}{21}(4)^{\frac{7}{2}} \right] - 0$$

$$M_y = 23.54$$

$$\bar{X} = \frac{23.54}{12} \Rightarrow \boxed{\bar{X} = 1.96 \text{ units}}$$

Q 7 //

$$Z = (4 - x^2 - y^2)^{\frac{1}{2}}$$

$$Z^2 = 4 - x^2 - y^2 \Rightarrow x^2 + y^2 + Z^2 = 4 \Rightarrow \rho^2 = 4 \Rightarrow \rho = 2$$

$$Z = \rho \cos \phi \quad \& \quad \rho = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\begin{aligned} \iiint Z^2 (x^2 + y^2 + z^2)^{\frac{1}{2}} dV &= \iiint \rho^2 \cos^2 \phi \cdot \rho dV \\ &= \iiint \rho^3 \cos^2 \phi \sin \phi d\rho d\phi d\theta = \int_0^\pi \int_0^\pi \int_0^2 \rho^3 \cos^2 \phi \sin \phi d\rho d\phi d\theta \\ &= \int_0^\pi \int_0^\pi \left[\frac{\rho^4}{4} \right]_0^2 \cos^2 \phi \sin \phi d\phi d\theta = \frac{64}{6} \int_0^\pi \int_0^\pi \cos^2 \phi \sin \phi d\phi d\theta \\ &= \frac{64}{6} \int_0^\pi \left[-\frac{\cos^3 \phi}{3} \right]_0^\pi d\theta = \frac{64}{18} \int_0^\pi \left[-\frac{1}{3} - (-\frac{1}{3}) \right] d\theta \\ &= \frac{64}{18} \int_0^\pi 2 d\theta = \frac{64}{9} \int_0^\pi \theta \end{aligned}$$

$$\boxed{= \frac{64}{9} \pi}$$