University of Technology
Building and Construction Eng. Dept.
Final Exam –2012/2013

Subject : Mathematics I **Class: 1st stage**
Branch : All Branches **Time : 3 Hours**

ملاحظة : الاجابة على اربعة اسئلة فقط

Q.1 A.) Find the area between the curves y= cosx and y= - sinx from x = 0 to π/2. (9 marks)

B.) Find the Determinant of the matrix
$$A=\begin{vmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$ (8 marks)

C.) Evaluate : $\int x^2 e^x \, dx$ (8 marks)

Q.2 A.) Find a unit vector (u) in the direction of the vector from $P_1\,(1,0,1)$ to $P_2\,(3,2,0)$. (9 marks)

B.) Find $\int \frac{dx}{4x^2+4x+2}$ (8 marks)

C.) Find dy/dx if $y = \sqrt[x]{tanhx}$ (8 marks)

Q.3 A.) Solve the following system of equations by Crammers Rule

$$2sin^2\theta + \frac{1}{y} = 1$$ (9 marks)

$$2cos^2\theta + \frac{3}{y} = 3$$

B.) Find the Domain, Range and Graph : $y = \sqrt{4-x}$ (8 marks)

C.) Find $\int \frac{dx}{\sqrt{x^2-25}}$ (8 marks)

Q.4 A.) If v_1= 2i+j-k & v_2= i+2j+2k, find the scalar (dot) and vector (cross) product (9 marks)

B.) Find $\int \frac{6x+7}{(x+2)^2}\, dx$ (8 marks)

C.) Find $\lim_{x\to-\infty} \frac{2x^2-x+3}{3x^2+5}$ (8 marks)

Q.5 A.) Solve the equation to find x, if sinhx = $-\frac{3}{4}$ (9 marks)

B.) Find the equation of the line tangent to the curve (8 marks)

$x = 2 + \sec\theta$ $\quad$ $y = 1+ 2\tan\theta$ $\quad$ at $\theta = \frac{\pi}{6}$

C.) For the complex number, find x and y if

$(3+4i)^2 - 2\,(x-yi) = x+yi$ (8 marks)

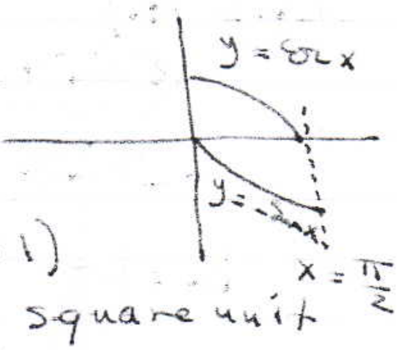
2013 رياضيات Set ①

Q.1 A.) $A = \int_0^{\pi/2} (y_2 - y_1)\,dx$

$= \int_0^{\pi/2} (\cos x + \sin x)\,dx$

$= \sin x - \cos x \Big|_0^{\pi/2} = (1-0) - (0-1)$

$= 1 + 1 = 2$ square unit



B.) $\det A = 2(-1+6) - 1(3+4) + 3(9+2)$

$= 10 - 7 + 33 = 36$

or

$$\begin{vmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 3 & -1 \\ 2 & 3 \end{matrix}$$

$= -2 - 4 + 27 + 6 + 12 - 3 = 36$

C.) $\int x^2 e^x = x^2 * e^x - 2\int e^x * x\,dx$

$= x^2 e^x - 2\left[x e^x - \int e^x\,dx\right]$

$= x^2 e^x - 2xe^x + 2e^x + c$

Q.2 A.) $\overrightarrow{P_1P_2} = 2i + 2j - k \qquad |\overrightarrow{P_1P_2}| = \sqrt{4+4+1} = 3$

$u = \frac{2i + 2j - k}{3} = \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$

B.) $\frac{1}{4}\int \frac{dx}{x^2 + x + \frac{1}{2}} = \frac{1}{4}\int \frac{dx}{x^2 + x + \frac{1}{4} - \frac{1}{4} + \frac{1}{2}} = \frac{1}{4}\int \frac{dx}{(x+\frac{1}{2})^2 + \frac{1}{4}}$

$= \frac{1}{4} \cdot \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x + \frac{1}{2}}{\frac{1}{2}}\right) = \frac{1}{2}\tan^{-1}(2x+1) + c$

C.) $y = (\tanh x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \ln \tanh x$

$\frac{1}{y} * y' = \frac{1}{x} * \frac{1}{\tanh x} \operatorname{sech}^2 x + \ln \tanh x * -\frac{1}{x^2}$

$y' = y \cdot \left[\frac{\operatorname{sech}^2 x}{x \tanh x} + \ln \tanh x * -\frac{1}{x^2}\right]$

Q.3 A.)

$2\sin^2\theta + \frac{1}{y} = 1$

$-2\sin^2\theta + \frac{3}{y} = 1$

$2(1-\sin^2\theta) + \frac{3}{y} = 3$

$2 - 2\sin^2\theta + \frac{3}{y} = 3$

$-2\sin^2\theta + \frac{3}{y} = 1$

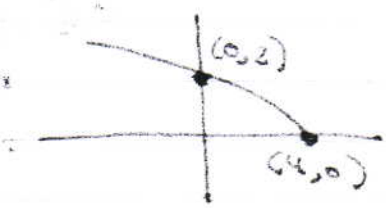
$D_0 = \begin{vmatrix} 2 & 1 \\ -2 & 3 \end{vmatrix} = 8$

$D_1 = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 2 \Rightarrow \sin^2\theta = \frac{2}{8} = \frac{1}{4} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

$D_2 = \begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} = 4 \Rightarrow \frac{1}{y} = \frac{4}{8} = \frac{1}{2} \Rightarrow y = 2$

B.) $4 - x \geq 0 \Rightarrow 4 \geq x$ or $x \leq 4$

Domain $x \leq 4$, Rang $y \geq 0$

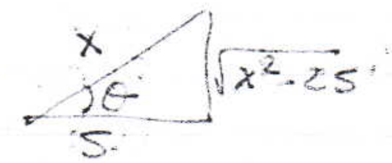


C.) $\int \frac{dx}{\sqrt{x^2-25}}$ let $x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$I = \int \frac{5\sec\theta\tan\theta\, d\theta}{\sqrt{25\sec^2\theta - 25}} = \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + c$

$\sec\theta = \frac{x}{5}$



$I = \ln\left| \frac{x}{5} + \frac{\sqrt{x^2-25}}{5} \right| + c$

Q.4 A.) $v_1 \cdot v_2 = (2*1) + (1*2) + (-1*2)$
$= 2 + 2 - 2 = 2$

$v_1 \times v_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 2 & 2 \end{vmatrix} = 4i - 5j + 3k$

B.) $\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{Ax + 2A + B}{(x+2)^2}$

$6x + 7 = Ax + 2A + B$

$A = 6$

$2A + B = 7 \Rightarrow B = -5$

$I = \frac{6}{x+2} - \frac{5}{(x+2)^2} = 6\ln|x+2| - 5\frac{(x+2)^{-1}}{-1}$

$= 6\ln|x+2| + \frac{5}{x+2} + c$

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Q.4 C.) $\lim_{x \to -\infty} \dfrac{\frac{2x^2}{x^2} - \frac{x}{x^2} + \frac{3}{x^2}}{\frac{3x^2}{x^2} + \frac{5}{x^2}} = \lim_{x \to -\infty} \dfrac{2 - \frac{1}{x} + \frac{3}{x^2}}{3 + \frac{5}{x^2}} = \dfrac{2}{3}$

Q.5 A.) $\frac{1}{2}(e^x - e^{-x}) = -\frac{3}{4}$

$2e^x - 2e^{-x} = -3 \Rightarrow \left(2e^x - \frac{2}{e^x} = -3\right)e^x$

$2(e^x)^2 + 3e^x - 2 = 0$

$e^x = \dfrac{-3 \mp \sqrt{9 - 4 * 2 * -2}}{4} = \dfrac{-3 \mp \sqrt{25}}{4} = \dfrac{-3 \mp 5}{4}$

either $e^x = 0.5$ or $e^x = -2$ ~~(dr.)~~

$x = \ln 0.5 = -0.693$

B.) $\dfrac{dx}{d\theta} = \sec\theta \tan\theta$, $\dfrac{dy}{d\theta} = 2\sec^2\theta$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{2\sec^{\cancel{2}}\theta}{\cancel{\sec\theta}\tan\theta} = 2 \dfrac{\cancel{\ }\cos\theta}{\cos\theta\,\cancel{\ }\sin\theta}$

$= 2\csc\theta$

at $\theta = \frac{\pi}{6}$ $\quad \dfrac{dy}{d\cancel{x}} = 2 * \dfrac{1}{\sin 30} = 4$

$x = 2 + \dfrac{1}{\cos 30} = 2 + \dfrac{1}{\frac{\sqrt{3}}{2}} = 2 + \dfrac{2}{\sqrt{3}} =$

$y = 1 + 2\tan 30 = \dfrac{1}{\sqrt{3}}$

$y - y_0 = m(x - x_0)$

$y - \dfrac{1}{\sqrt{3}} = 4\left(x - \left(2 + \dfrac{2}{\sqrt{3}}\right)\right)$

Q.5 c.)

$9 + 24i - 16 - 2x + 2yi = x + yi$

$-7 - 2x + (24 + 2y)i = x + yi$

$-7 - 2x = x \implies -7 = 3x \implies x = \frac{-7}{3}$

$24 + 2y = y \implies y = -24$