Subject: A/C Engines and Jet Propulsion — **الموضوع : محركات طائرة و دفع**

Weekly Hours : Theoretical :2 **Units : 5** — **الوحدات : 5** **الساعات الأسبوعية : نظري : 2**

Tutorial : 1 — **مناقشة : 1**

Experimental : 1 — **عملي : 1**

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AEROENGINES

Assisting books:
- Gas Turbine Theory — by Cohen
- Mechanics and Thermodynamics of Propulsion systems
- Gas Turbine Engineering — by Harman
- Thermodynamics of Turbomachinery — by Dixon
- solved problems in Turbomachinery — by Dixon

Course contents:

(1) Introduction

(2) Thermodynamics of A/c engines

(3) Compressors:
   C.F.C. , A.F.C.

(4) Turbines:
   R.F.T , A.F.T

(5) Combustion systems and fuel injection

(6) Air intakes
   subsonic , Supersonic

(7) Nozzles and thrust reversals
   Convergent and C-D nozzles.

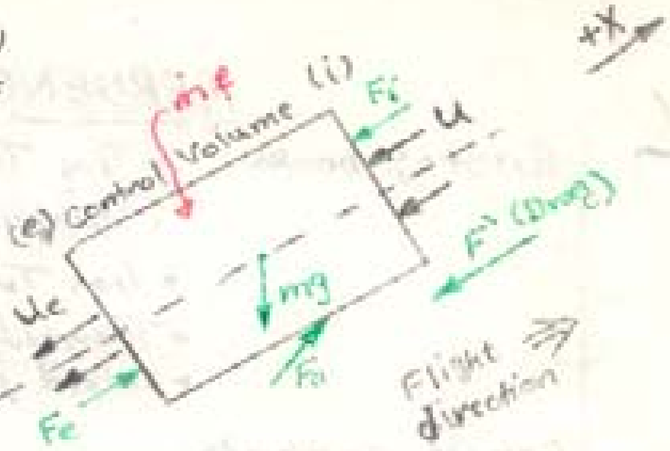
(8) Engine Fuel systems.

(I) Introduction; classification:

A/c Engines (Air breathing engines)

- Piston (intermittent combustion)
- Jet engines (continuous combustion)
  - With gas generator
    - Turbojet
      - single spool
      - twin spool
    - Bypass
      - Front fan
      - Aft fan
    - Turbshaft
      - turboprop
      - Helicopter
  - without gas generator
    - Athodydes
      - ramjet
      - pulse jet
    - Rockets
      - solid fuel
      - liquid fuel

The Thrust EQUATION



$F_i = P_i A_i$

$F_e = P_e A_e$

$F_a = P_a (A_i - A_e)$

$\dot{m}_e = (\dot{m}_a + \dot{m}_f)$

$\quad = \dot{m}_a \left[1 + \frac{\dot{m}_f}{\dot{m}_a}\right]$

$\dot{m}_e = \dot{m}_a (1+f)$ ------ (1) where f = fuel/air ratio.

Change in momentum (momentum $= \dot{m}V$)

$\Sigma F_x = \frac{\partial}{\partial t}(m V_x)_{cv} + \Sigma(\dot{m}V_x)_e - \Sigma(\dot{m}V_x)_i$

$F_e + F_a - F_i - F' - mg\sin\theta = \frac{d}{dt}(mU)_{cv} + \dot{m}_e(-u_e) - \dot{m}_i(-u_i)$

$\frac{d}{dt}(mu) = m\frac{du}{dt} + \cancel{u\frac{dm}{dt}}^{\,= \text{zero}}$

$\underbrace{F' + mg\sin\theta + m\frac{du}{dt}}_{\text{Thrust}(T)} = F_e + F_a - F_i + \dot{m}_e u_e - \dot{m}_i u_i$

$T = \dot{m}_e u_e - \dot{m}_i u_i + P_e A_e - P_i A_i + P_a(A_i - A_e)$

$T = \dot{m}_e u_e - \dot{m}_i u_i + A_e(P_e - P_a) - A_i(P_i - P_a)$

If no compression in inlet $P_i = P_a$; let $u_i = u$

$T = \dot{m}_e u_e - \dot{m}_i u_i + A_e(P_e - P_a) \qquad \dot{m}_i = \dot{m}_a$

$T = \dot{m}_a[(1+f)u_e - u_i] + A_e(P_e - P_a)$ ---- (2)

For ideal expansion $P_e = P_a$, for $f \ll 1$

$T = \dot{m}_a(u_e - u)$ ------------ (3)

Engine Performance

- propulsive efficiency: is the ratio of thrust power (TU) to the rate of production of propellant kinetic energy :-

For single propellant stream:-

$\eta_p = \frac{TU}{\frac{1}{2}\dot{m}_a u_e^2 - \frac{1}{2}\dot{m}_a u^2} = \frac{2TU}{\dot{m}_a(u_e^2 - u^2)} = \frac{2TU}{\dot{m}_a(u_e - u)(u_e + u)}$

but $T = \dot{m}_a(u_e - u)$ for ideal expansion

$$\eta_p = \frac{2u}{u_e+u} = \frac{2\frac{u}{u_e}}{1+\frac{u}{u_e}} \quad \text{---------(4)}$$

Eq (4) shows that $\eta_p = 1 = max$ when $\frac{u}{u_e} = 1$ but from eq (3) $T = zero$ so u_e must be $> u$ & η_p always < 1.

- Thermal efficiency: is the ratio of rate of addition of kinetic energy to the propellant to the total energy consumption rate ($\dot{m}_f Q_R$). where Q_R = heat of reaction of fuel.

$$\eta_{th} = \frac{\frac{1}{2}\dot{m}_a u_e^2 - \frac{1}{2}\dot{m}_a u^2}{\dot{m}_f Q_R} = \frac{u_e^2 - u^2}{2 f Q_R} \quad \text{----(5a) [for Ram, TF \& TJ]}$$

$$\eta_{th} = \frac{P_s}{\dot{m}_f Q_R} \quad \text{-----(5b) for TS where } P_s \text{ -- shaft power.}$$

$$\eta_{th} = \frac{P_{es}}{\dot{m}_f Q_R} \quad \text{-----(5c) for T.P. where } P_{es} = \text{equivalent shaft power}$$

- Propeller efficiency: - is the ratio of thrust power to shaft power.

$$\eta_{pr} = \frac{T_{pr}\, u}{P_s} \quad \text{---(6a)} \qquad T_{pr} = \text{propeller thrust.}$$

$$\eta_{pr} = \frac{T\, u}{P_{es}} \quad \text{----(6b)}$$

- overall efficiency: - $\eta_o = \eta_p \eta_{th}$ or $= \eta_{pr} \eta_{th}$

$$\eta_o = \eta_p \eta_{th} = \frac{T u}{KE} \cdot \frac{KE}{\dot{m}_f Q_R} = \frac{T u}{\dot{m}_f Q_R} \quad \text{---(7a)}$$

$$\eta_o = \frac{\dot{m}_a (u_e - u)\, u}{\dot{m}_f Q_R} = \frac{(u_e - u)\, u}{f Q_R} \quad \text{-------(7b)}$$

For double propellant streams: $T =$

$$T = \dot{m}_{aH}[(1+f) u_{eH} - u] + \dot{m}_{ac}[u_{ec} - u] \quad \text{---(7c)}$$

let $\bar{u}_e$ be the (thrust averaged) exhaust velocity then

$$\bar{u}_e = \frac{\dot{m}_{aH}(1+f) u_{eH} + \dot{m}_{ac} u_{ec}}{\dot{m}_{aH}(1+f) + \dot{m}_{ac}}$$

$$T = \dot{m}_e [\bar{u}_e - u]$$

$$\eta_o = \eta_p \eta_{th}$$

$$\eta_o = \frac{\dot m_e(\bar u_e - u)u}{\dot m_f Q_R} = \frac{\dot m_e}{\dot m_f}\left[\frac{(\bar u_e - u)u}{Q_R}\right]$$

but $\dot m_e = \dot m_{ac} + \dot m_{aH}$

$$\eta_o = \frac{\dot m_{aH} + \dot m_{ac}}{\dot m_f}\left[\frac{(\bar u_e - u)u}{Q_R}\right] \times \frac{\dot m_{aH}}{\dot m_{aH}}$$

$$= \left(1 + \frac{\dot m_{ac}}{\dot m_{aH}}\right)\left(\frac{(\bar u_e - u)u}{f Q_R}\right)$$

To find $\eta_{o\,max}$:

$$\eta_o = \frac{\dot m_e}{\dot m_f Q_R}[(\bar u_e - u)u] = K[(\bar u_e - u)u]$$

$$\frac{d\eta_o}{du} = K\frac{d}{du}[\bar u_e u - u^2] = K[\bar u_e - 2u] = 0$$

as $K \neq 0$ $\quad \bar u_e - 2u = 0 \Rightarrow u = \frac{\bar u_e}{2}$

Takeoff thrust: For stationary A/C $u = 0$

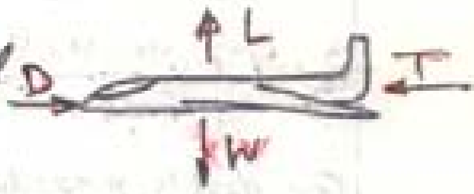
$T_s = \dot m_a u_e$ ......(7d)

$\eta_{th} = \frac{\frac{1}{2}\dot m_a u_e^2}{\dot m_f Q_R} \Rightarrow \dot m_a = \frac{2\dot m_f Q_R \eta_{th}}{u_e^2}$

or $T_s = \frac{2\dot m_f Q_R \eta_{th}}{u_e}$ ......(7e)

Eq (7e) shows that for a given $\dot m_f$ & η_{th} $T_s \propto \frac{1}{u_e}$ for given rate of energy consumption.
Thus to $\uparrow T_s \Rightarrow \downarrow u_e$ but $T_s = \dot m_a u_e \Rightarrow \dot m_a$ must $\uparrow$
So we accelerate a large $\dot m_a$ to low u_e we get high $T_s \Rightarrow$ T.F. engines.

A/C Range: In level flight at constant speed



$T = D$ & $L = W$

$$T = D = L \times \frac{D}{L} = \frac{mg}{L/D}$$

m = instantaneous mass of A/c, g = acc. of gravity L = lift D = Drag

Thrust power $= TU = \frac{mgu}{L/D}$

$\eta_o = \frac{Tu}{\dot m_f Q_R} \Rightarrow \dot m_f = Tu/\eta_o Q_R$

$\dot m_f = \frac{mgu}{\eta_o Q_R (L/D)}$ ......(8a) but $\dot{m}_f = -\frac{dm}{dt} = -u\frac{dm}{ds}$ where s – distance

$$\frac{dm}{ds} = -\frac{mg}{\eta_o Q_R (L/D)} \quad \text{------ (8b)}$$

Assume $\eta_o Q_R (L/D) = $ Constant

$$\int_0^s ds = -\frac{\eta_o Q_R (L/D)}{g}\int_{m_1}^{m_2} \frac{dm}{m}$$

$$S = \frac{\eta_o Q_R (L/D)}{g} \ln \frac{m_1}{m_2} \quad \text{------ (8c)}$$ **Brequet's** range formula

Where m_1 & m_2 = initial & final mass of A/c

eq(8c) shows that $S \propto \eta_o$

specific fuel consumption

$$TSFC = \dot{m}_f / T \quad \text{------ (9a)} \quad \text{for TJ \& TF}$$

$$= \dot{m}_f / \dot{m}_a[(1+f)u_e - u] \quad \text{------ (9b)}$$

For shaft power engines (TS)

$$BSFC = \dot{m}_f / P_s \quad \text{------ (9c)}$$ brake spec. fuel consumption

$$EBSFC = \dot{m}_f / P_{es}$$ equivalent brake spec. fuel consumption

$$= \dot{m}_f / (P_s + T_c U) \quad \text{------ (9d)}$$

Where: T_c = exhaust thrust; P_s = Shaft power of propellant

<u>CLASIFICATION OF AEROENGINES</u>

Aircraft power plants (engines) are internal combustion engines and are of two types:

<u>Type one</u>: *piston engines* or intermittent combustion engines

<u>Type two</u>: *air breathing engines* or continuous combustion engines

Type two can be subdivided into engines:

- **<u>Without gas generator</u>** which include:

  * **Athodyd** (Aero thermodynamic ducts) which includes:

    The *Ram* jets, and

    The *Pulse* jets.

  * **Rockets** which could be:

    Solid fuel, or

    Liquid fuel.

- **<u>With gas generator</u>** which includes:

  * **Turbojet engines** that can be of:

    Single spool, or

    Double spool.

  * **Bypass engines** that can have:

    Front fan, or

    After fan.

  * **Turbo shaft engines** that can be:

    Helicopter engine, or

    Turbo propeller engine.

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Definitions

2.3.2: Engine Performance: This includes the evaluation of the following parameters.

Propulsive efficiency: is the ratio of thrust power (TU) to the rate of production of propellant kinetic energy.

Thermal efficiency: is the ratio of rate of addition of kinetic energy to the propellant to the total energy consumption.

Propeller efficiency: is the ratio of thrust power to the shaft power.

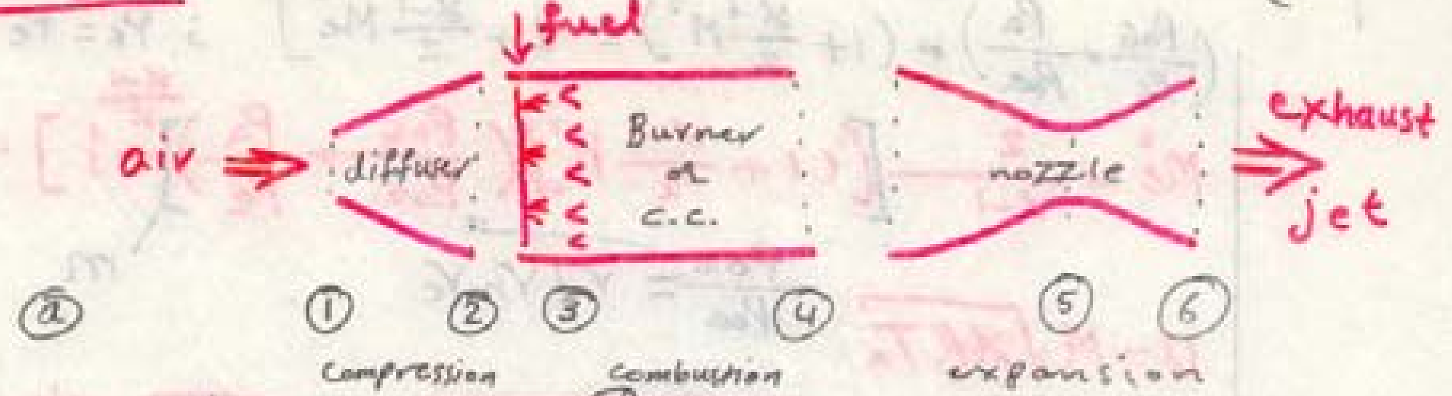
Overall efficiency: is the product of propulsive efficiency by the thermal efficiency.

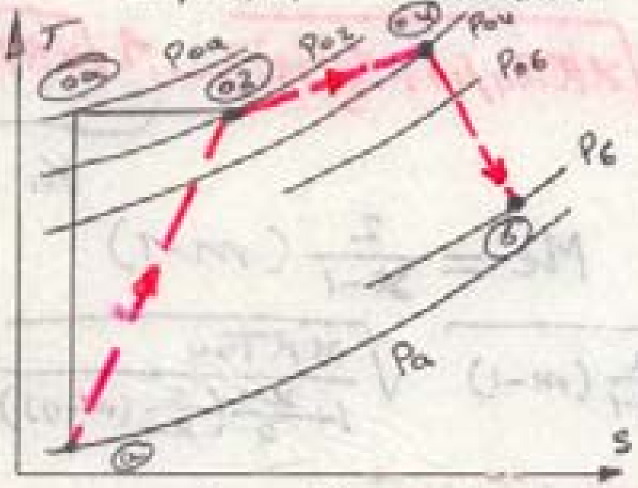
A/C range

Specific fuel consumption (SFC): is the ratio of mass of fuel consumed to the thrust produced (TSFC). Or the ratio of mass of fuel consumed to the shaft power (BSFC). Or the ratio of mass of fuel consumed to the equivalent shaft power (EBSFC).

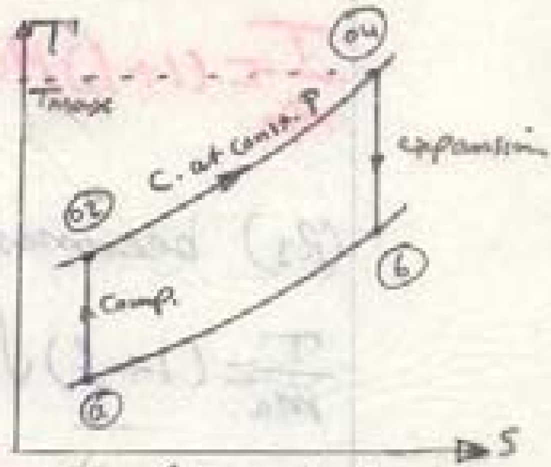
2.3: Examples

Ram jet





Actual ram jet



ideal ram jet

$r_d = \dfrac{P_{02}}{P_{0a}} < 1$ .. diffuser total press.

$r_c = \dfrac{P_{04}}{P_{02}} < 1$ .. burner " "

$r_n = \dfrac{P_{06}}{P_{04}} < 1$ .. nozzle " "

$$T = \dot{m}_a\left[(1+f)u_e - u\right] + A_e(P_e - P_a) \quad \cdots (R_1)$$

$$u_e = M_e\sqrt{\varkappa R T_e} \qquad T_e = T_{04}\Big/\left(1+\frac{\varkappa-1}{2}M_e^2\right)$$

$$= M_e\sqrt{\varkappa R T_{04}\Big/\left(1+\frac{\varkappa-1}{2}M^2\right)} \quad \cdots\cdots (R_2)$$

overall press. ratio: $\dfrac{P_{06}}{P_{0a}} = \dfrac{P_{02}}{P_{0a}} \cdot \dfrac{P_{04}}{P_{02}} \cdot \dfrac{P_{06}}{P_{04}} = r_d\, r_c\, r_n$

$$\frac{P_{0a}}{P_a} = \left(1+\frac{\varkappa-1}{2}M^2\right)^{\varkappa/\varkappa-1} ; \quad \frac{P_{06}}{P_6} = \frac{P_{06}}{P_e} = \left(1+\frac{\varkappa-1}{2}M_e^2\right)^{\frac{\varkappa}{\varkappa-1}} ; \ \varkappa = \text{const.}$$

$$\frac{P_{06}}{P_6} \cdot \frac{P_a}{P_{0a}} \cdot \frac{P_{0a}}{P_a} = \left[1+\frac{\varkappa-1}{2}M_e^2\right]^{\frac{\varkappa}{\varkappa-1}}$$

1

$$\left(\frac{P_{o6}}{P_6}\cdot\frac{P_a}{P_{oa}}\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(1+\frac{\gamma-1}{2}M^2\right) = 1+\frac{\gamma-1}{2}M_e^2 \Big] \quad ; \; P_6 = P_e$$

$$M_e^2 = \frac{2}{\gamma-1}\left[\underbrace{\left[\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{P_{o6}}{P_{oa}}\cdot\frac{P_a}{P_e}\right)^{\frac{\gamma-1}{\gamma}}\right]}_{m} - 1\right] \cdots (R_3)$$

$$\frac{P_{o6}}{P_{oa}} = r_d\, r_n\, r_c$$

$$U = M\sqrt{\gamma R T_a}$$

$$\frac{T}{\dot m_a} = (1+f)M_e\sqrt{\gamma R T_{o4}\Big/\left(1+\frac{\gamma-1}{2}M_e^2\right)} - \underbrace{M\sqrt{\gamma R T_a}}_{K_1} + \underbrace{\frac{P_e A_e}{\dot m_a}\left(1-\frac{P_a}{P_e}\right)}_{K_2}$$

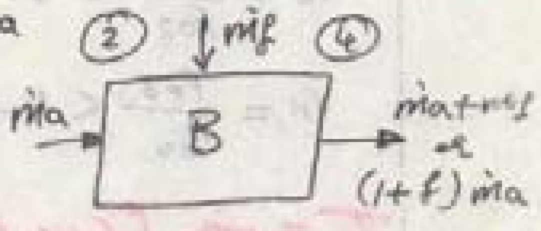
(R_3) becomes $\quad M_e^2 = \frac{2}{\gamma-1}(m-1)$

$$\frac{T}{\dot m_a} = (1+f)\sqrt{\frac{2}{\gamma-1}(m-1)}\,\sqrt{\frac{\gamma R T_{o4}}{1+\frac{\gamma-1}{2}\left(\frac{2}{\gamma-1}(m-1)\right)}} - K_1 + K_2$$

$$\frac{T}{\dot m_a} = (1+f)\sqrt{\frac{2\gamma R T_{o4}(m-1)}{(\gamma-1)m}} - M\sqrt{\gamma R T_a} + \frac{P_e A_e}{\dot m_a}\left(1-\frac{P_a}{P_e}\right)$$

--- (R_4)

$$\dot m_a h_{o2} + \dot m_f Q_R = (1+f)\dot m_a h_{o4} \quad \div \dot m_a$$



$$(1+f)h_{o4} = h_{o2} + f Q_R$$

$$(1+f)c_p T_{o4} = c_p T_{o2} + f Q_R \quad \div c_p T_{oa}$$

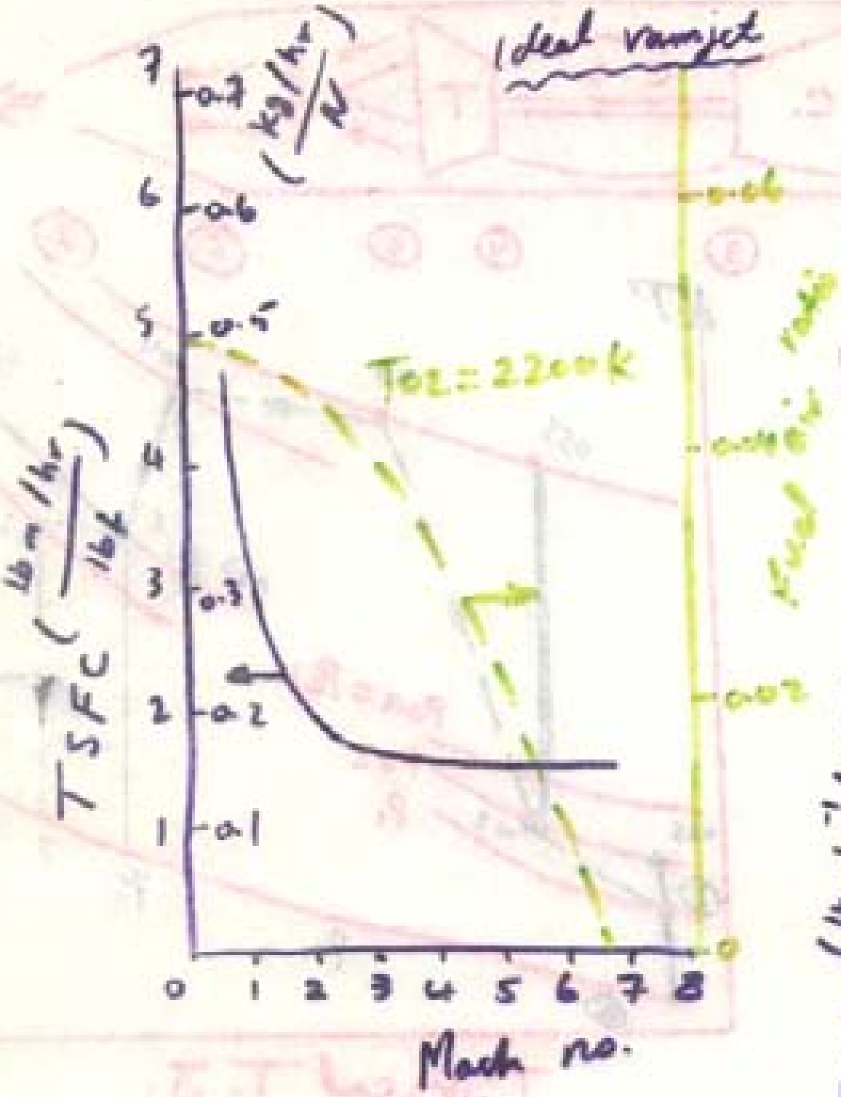
$$(1+f)\frac{T_{o4}}{T_{oa}} = \frac{T_{o2}}{T_{oa}} + \frac{f Q_R}{c_p T_{oa}}$$

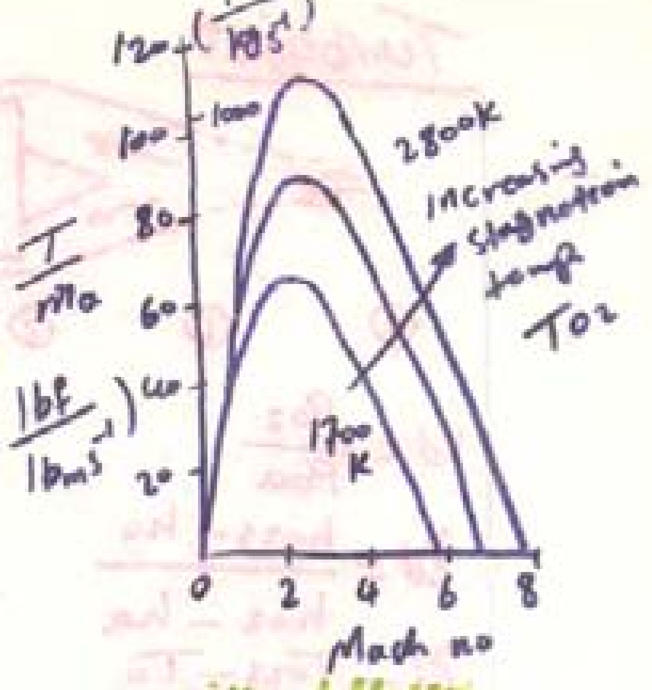
$$\frac{T_{o4}}{T_{oa}} + f\left(\frac{T_{o4}}{T_{oa}}\right) = \frac{T_{o2}}{T_{oa}} + \frac{f Q_R}{c_p T_{oa}} \qquad \text{let } T_{o2} = T_{oa}$$

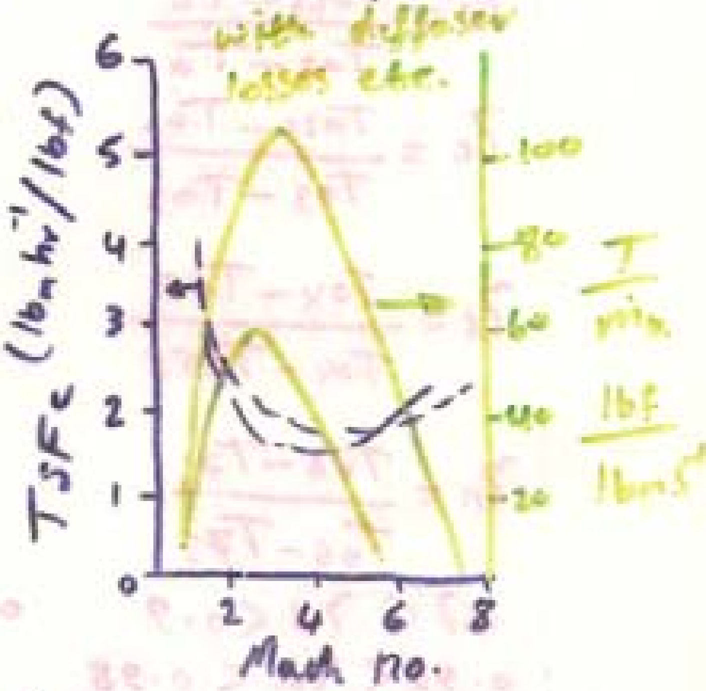
$$f\left[\left(\frac{Q_R}{c_p T_{oa}}\right) - \frac{T_{o4}}{T_{oa}}\right] = \frac{T_{o4}}{T_{oa}} - 1$$

$$f = \frac{\frac{T_{o4}}{T_{oa}} - 1}{\left(\frac{Q_R}{c_p T_{oa}}\right) - \left(\frac{T_{o4}}{T_{oa}}\right)} \quad ------ (R5)$$

Typical performance







The specific impulse is defined by:-

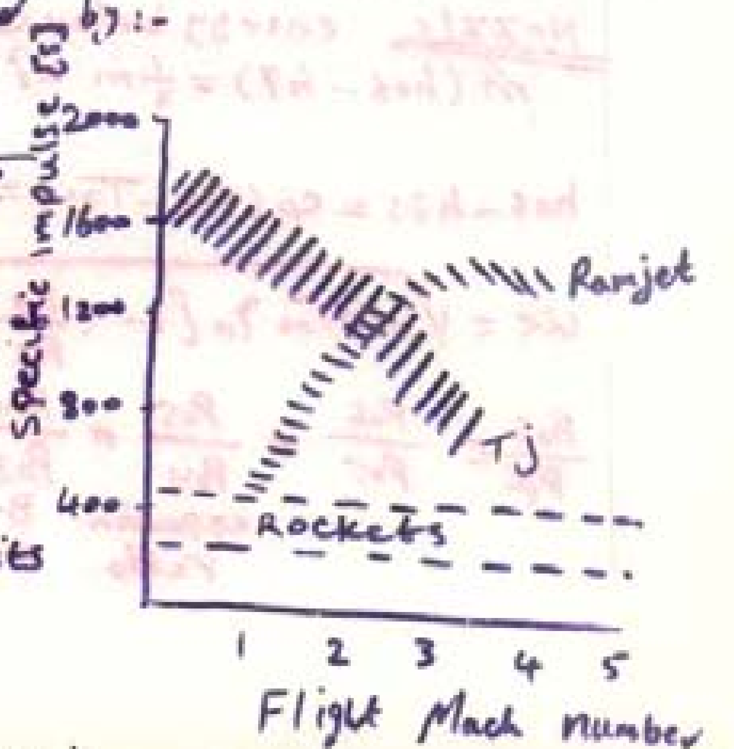
$$I = \frac{T}{\dot{m}_f\, g_0} = \frac{T}{\dot{m}_a\, g_0} \cdot \frac{1}{f}$$

For a hydrocarbon fuel $f^{-1} \sim 16$

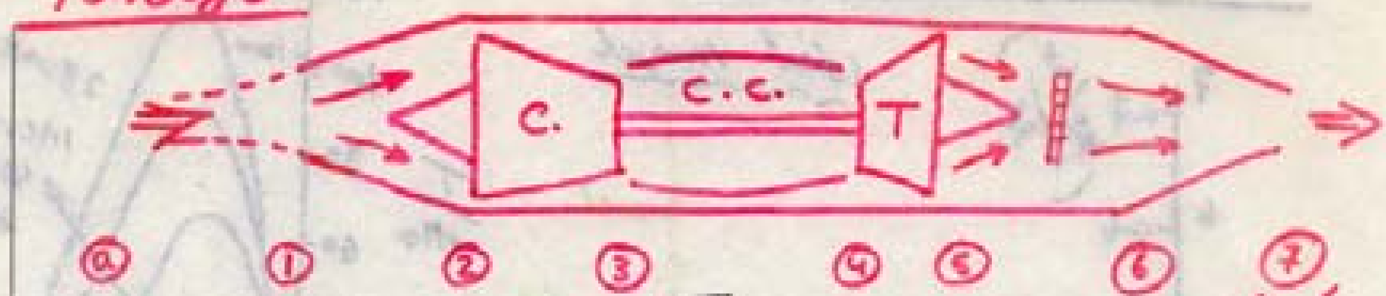
$M = 2.5$, $\frac{T}{\dot{m}_a g_0} \sim 80$ seconds

$\Rightarrow I = 1280s$

The ramjet offers substantial benefits over Rockets & T.J.s at high Mach numbers.



Turbojet



$$r_d = \frac{P_{02}}{P_{0a}}$$

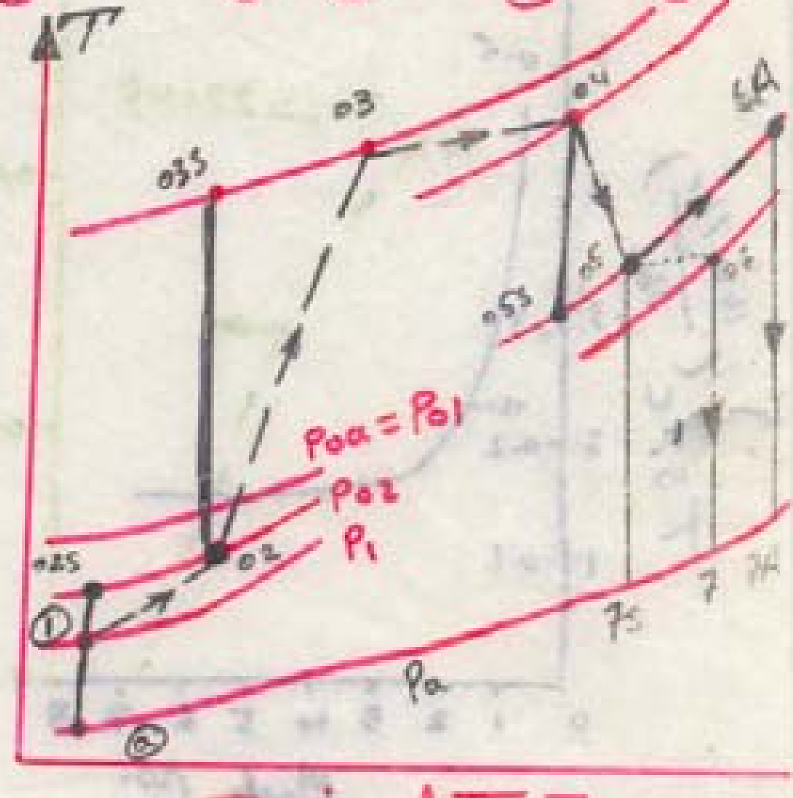
$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a}$$

$$\eta_d = \frac{T_{02s} - T_a}{T_{02} - T_a}$$

$$\eta_c = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}}$$

$$\eta_t = \frac{T_{04} - T_{05}}{T_{04} - T_{05s}}$$

$$\eta_n = \frac{T_{06} - T_7}{T_{06} - T_{7s}}$$



Typical T. J.

$0.7 < \eta_d < 0.9$ $\quad 0.85 < \eta_c < 0.9$ $\quad 0.9 < \eta_t < 0.95$

$0.95 < \eta_n < 0.98$

Nozzle energy balance

$\dot m (h_{06} - h_7) = \frac{1}{2} \dot m\, u_e^2$ $\qquad \frac{u_e^2}{2} = h_{06} - h_7 = \eta_n (h_{06} - h_{7s})$ --(1)

$$h_{06} - h_{7s} = c_p (T_{06} - T_{7s}) = c_p T_{06} \left(1 - \frac{T_{7s}}{T_{06}}\right) = c_p T_{06}\left[1 - \left(\frac{P_7}{P_{06}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$u_e = \sqrt{2 c_p T_{06}\, \eta_n \left[1 - \left(\frac{P_7}{P_{06}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad \text{.. . . - - - . (2)}$$

$$\frac{P_{06}}{P_7} = \frac{P_{06}}{P_{05}} * \frac{P_{05}}{P_{04}} * \frac{P_{04}}{P_{03}} * \frac{P_{03}}{P_{02}} * \frac{P_{02}}{P_a} * \frac{P_a}{P_7}$$

expansion ratio $\quad$ B. ratio $\quad$ comp. ratio $\quad$ diffuser $\quad$ nozzle expansion procedure

$\frac{p_a}{p_7}$, $\frac{p_{o4}}{p_{o3}}$, $\frac{p_{o6}}{p_{o5}}$ ... assumed or given (ideally = 1)

$\frac{p_{o3}}{p_{o2}}$ = comp. ratio ... assumed or given

$$T_{o2} = T_{o1} = T_{oa} \qquad \frac{T_o}{T_a} = 1 + \frac{\varkappa-1}{2} M^2$$

$$\eta_d = \frac{T_{o2s} - T_a}{T_{o2} - T_a} = \frac{\frac{T_{o2s}}{T_a} - 1}{\frac{T_{o2}}{T_a} - 1} = \frac{(p_{o2}/p_a)^{\frac{\varkappa-1}{\varkappa}} - 1}{1 + \frac{\varkappa-1}{2} M^2 - 1}$$

$$\frac{p_{o2}}{p_a} = \left(1 + \eta_d \frac{\varkappa-1}{2} M^2\right)^{\varkappa/\varkappa-1}$$

$$\frac{W_c}{\dot m} = h_{o3} - h_{o2} = c_p (T_{o3} - T_{o2}) = \frac{c_p}{\eta_c} (T_{o3s} - T_{o2})$$

$$= \frac{c_p T_{o2}}{\eta_c} \left[\left(\frac{T_{o3s}}{T_{o2}}\right) - 1\right] = \frac{c_p T_{o2}}{\eta_c} \left[\left(\frac{p_{o3}}{p_{o2}}\right)^{\frac{\varkappa-1}{\varkappa}} - 1\right]$$

$$\frac{W_t}{\dot m} = h_{o4} - h_{o5} = c_p (T_{o4} - T_{o5}) = \eta_t c_p (T_{o4} - T_{o5s})$$

$$= \eta_t c_p T_{o4} \left\{1 - \frac{T_{o5s}}{T_{o4}}\right\} = \eta_t c_p T_{o4} \left[1 - \left(\frac{p_{o5}}{p_{o4}}\right)^{\frac{\varkappa-1}{\varkappa}}\right]$$

$$\frac{W_c}{\dot m} = \frac{W_t}{\dot m}$$

$$\frac{c_p T_{o2}}{\eta_c} \left[\left(\frac{p_{o3}}{p_{o2}}\right)^{\frac{\varkappa-1}{\varkappa}} - 1\right] = \eta_t c_p T_{o4} \left[1 - \left(\frac{p_{o5}}{p_{o4}}\right)^{\frac{\varkappa-1}{\varkappa}}\right]$$

rearranging & noting that $T_{o2} = T_a \left(1 + \frac{\varkappa-1}{2} M^2\right)$

$$\left(\frac{p_{o5}}{p_{o4}}\right)^{\frac{\varkappa-1}{\varkappa}} = 1 - \frac{T_a}{\eta_c \eta_t T_{o4}} \left(1 + \frac{\varkappa-1}{2} M^2\right) \left[\left(\frac{p_{o3}}{p_{o2}}\right)^{\frac{\varkappa-1}{\varkappa}} - 1\right]$$

$$r_d = \frac{p_{o2}}{p_{oa}} = \frac{p_{o2}/p_a}{p_{oa}/p_a} = \frac{p_{o2}/p_a}{\left(1 + \frac{\varkappa-1}{2} M^2\right)^{\frac{\varkappa}{\varkappa-1}}}$$

$$\left(\frac{p_{o2}}{p_a}\right)^{\frac{\varkappa-1}{\varkappa}} = \eta_d \frac{\varkappa-1}{2} M^2 + 1$$

or $$\frac{p_{o2}}{p_a} = \left[\eta_d \frac{\varkappa-1}{2} M^2 + 1\right]^{\frac{\varkappa}{\varkappa-1}}$$

$$r_d = \left[\frac{1 + \eta_d \frac{\varkappa-1}{2} M^2}{1 + \frac{\varkappa-1}{2} M^2}\right]^{\varkappa/\varkappa-1}$$

Turbofan

$W_t = W_c + W_f$

$$\dot m_e (h_{04} - h_{05}) = \dot m_a (h_{03} - h_{02}) + \beta \dot m_a (h_{08} - h_{0a})$$

$$(1+f)\dot m_a C_{ph}(T_{04} - T_{05}) = \dot m_a C_{pc}(T_{03} - T_{02}) + \beta \dot m_a C_{pc}(T_{08} - T_{0a})$$

after rearranging & $T_{02} = T_{0a}$

$$\frac{T_{05}}{T_{04}} = 1 - \left[\frac{\left(\frac{T_{03}}{T_{02}} - 1\right) + \beta\left(\frac{T_{08}}{T_{0a}} - 1\right)}{(1+f)\left(\frac{C_{ph}}{C_{pc}}\right)\left(\frac{T_{04}}{T_{0a}}\right)}\right]$$

$$\eta_t = \frac{h_{04} - h_{05}}{h_{04} - h_{05s}} \Rightarrow h_{04} - h_{05} = \eta_t (h_{04} - h_{05s})$$

$$T_{04} - T_{05} = \eta_t (T_{04} - T_{05s}) \quad \div T_{04}$$

$$1 - \frac{T_{05}}{T_{04}} = \eta_t \left(1 - \frac{T_{05s}}{T_{04}}\right) = \eta_t \left(1 - \left(\frac{P_{05}}{P_{04}}\right)^{\gamma-1/\gamma}\right)$$

$$\frac{T_{05}}{T_{04}} = 1 - \eta_t \left\{1 - \left(\frac{P_{05}}{P_{04}}\right)^{\gamma-1/\gamma}\right\} = 1 - \eta_t - \eta_t \left(\frac{P_{05}}{P_{04}}\right)^{\gamma-1/\gamma}$$

$$\frac{P_{05}}{P_{04}} = \frac{1 - \eta_t - \frac{T_{05}}{T_{04}}}{\eta_t} \qquad \Longrightarrow \left(\frac{P_{05}}{P_{04}}\right) =$$

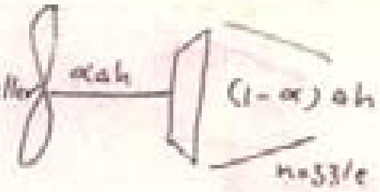
$$\frac{T_{05}}{T_{04}} = 1 - \left[\frac{\left(\frac{T_{03}}{T_{02}}\right) - 1 + \beta\left(\left(\frac{T_{08}}{T_{0a}}\right) - 1\right)}{(1+f)\left(\frac{C_{ph}}{C_{pc}}\right)\left(\frac{T_{04}}{T_{0a}}\right)}\right]$$

$$\frac{T_{03}}{T_{02}} = 1 - \frac{1}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\frac{T_{08}}{T_{0a}} = 1 - \frac{1}{\eta_f}\left[\left(\frac{P_{08}}{P_{0a}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Turbo Prob 2 Turboshaft

Δh = work , α = percentage work to drive the propeller



Propeller thrust power $T_{pr} u$

$$T_{pr} u = \eta_{pr} \eta_g \eta_{pt} \alpha \Delta h \dot{m}$$

$$T_{pr} = \frac{\eta_{pr} \eta_g \eta_{pt} \alpha \Delta h \dot{m}}{u}$$

$$T_n = \dot{m}(u_e - u)$$

$$\eta_n = \frac{\frac{1}{2} \dot{m} u_e^2}{\dot{m}(1-\alpha)\Delta h} \Rightarrow u_e = \sqrt{2 \eta_n (1-\alpha)\Delta h}$$

$$T = \frac{\eta_{pr} \eta_g \eta_{pt} \alpha \Delta h \dot{m}}{u} + \dot{m}\left[\sqrt{2\eta_n(1-\alpha)\Delta h} - u\right]$$

To find α for max T ; $\frac{dT}{d\alpha} = 0$

$$\frac{dT}{d\alpha} = \frac{\eta_{pr} \eta_g \eta_{pt} \Delta h}{u} + \frac{d}{d\alpha}\left[2\eta_n \Delta h - 2\eta_n \Delta h \alpha\right]^{\frac{1}{2}}$$

$$= \frac{\eta_{pr} \eta_g \eta_{pt} \Delta h}{u} - 2 \times \frac{1}{2} \eta_n \Delta h = 0$$

$$\Rightarrow \alpha = 1 - \frac{u^2}{2\Delta h}\left[\frac{\eta_n}{\eta_{pr}^2 \eta_g^2 \eta_{pt}^2}\right] \quad \alpha \alpha \alpha$$