

ينتج نوعين من الانماط تعرف بالانماط الطولية longitudinal modes والانماط المستعرضة transverse modes.

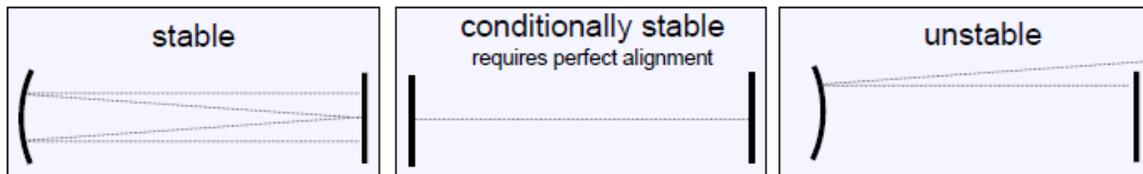
**Longitudinal modes:** only specific frequencies are possible inside the optical cavity of a laser, according to standing wave condition.

**Transverse modes:** are created in cross section of the beam, perpendicular to the optical axis of the laser.

إن السبب يعود في تكون هذه الانماط يعود إلى تكون أمواج موقوفة بين المرآتين. وكما نعلم أن الأمواج الموقوفة تتكون نتيجة لتداخل موجتين لهما نفس التردد وتنتشران في اتجاهين متعاكسين في المسافة بين المرآتين

### Resonator Stability

Need the resonator to be “stable”, i.e. the light stays in the cavity



Use ABCD matrices, resonator stable if round-trip matrix satisfies

$$0 \leq \frac{A+D+2}{4} \leq 1 \quad \text{Equality} \rightarrow \text{conditionally stable}$$

Flat mirrors, just free space,  $A = D = 1$ , gives conditionally stable

لكي يستمر الحصول على ليزر ولكن الفقد الناتج عن عدة عوامل يسبب في نقصان الربحية. ولكي نحصل على ليزر فإن الربحية لكل دورة يجب أن تكون على الأقل أكبر من الخسارة في كل دورة.

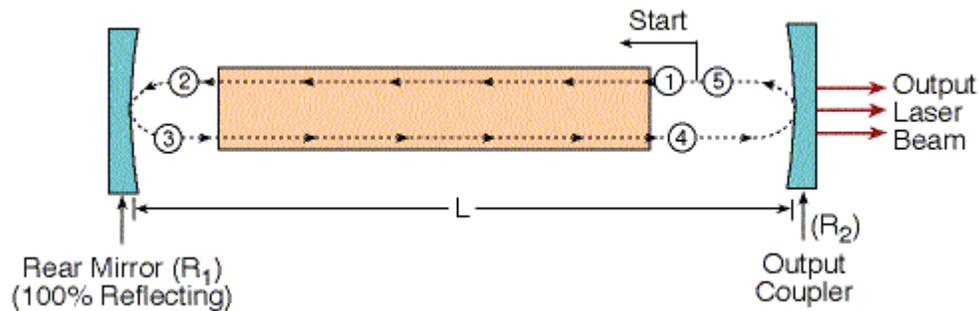
**The total losses of the laser system is due to a number of different processes these are:**

1. Transmission at the mirrors
2. Absorption and scattering by the mirrors
3. Absorption in the laser medium
4. Diffraction losses at the mirrors

All these losses will contribute to reduce the effective gain coefficient

**Round trip Gain (G)**

Figure below show the round trip path of the radiation through the laser cavity. The path is divided to sections numbered by 1-5, while point “5” is the same point as “1”.



*Round trip path of the radiation through the laser cavity. G = round trip Gain*

**Round trip gain**, determines if the output power of the laser will increase, decrease, or remain constant. It includes all the losses and amplifications that the beam has in a complete round trip through the laser.

By definition, Round trip Gain is given by:

$$G = I_5 / I_1$$

G = Round trip Gain.

$I_1$  = Intensity of radiation at the beginning of the loop.

$I_5$  = Intensity of radiation at the end of the loop.

**OPTICAL RESONATORS AND EIGENMODES**Important Concepts

- Stability
- Resonance (Eigen modes)
- Photon Lifetime

**Optical cavities**

The optical cavities (also known as optical resonators) are made to amplify the light within the cavity, so the mirrors used are highly reflective. Essentially, light enters the cavity through one mirror, reflects off the opposite mirror, and returns to the first mirror, while some of it is transmitted (exits the cavity) through each mirror. This light transmitted through the first mirror in each arm is the light that interferes at the beam splitter to form the signal.

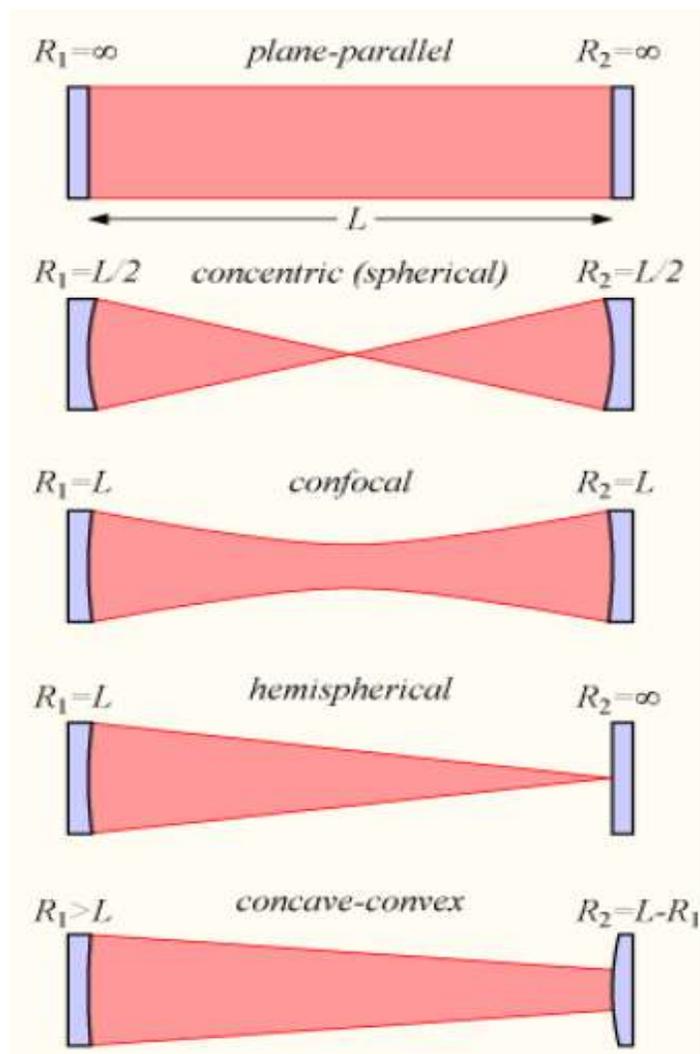
**Various types of optical resonators**

The cavity is designed so that the beam will remain entirely within the cavity's mirrors.

These are referred to as follows where  $R_1$  and  $R_2$  are the radii of curvature of the mirrors and  $L$  is the distance between the mirrors:

- |                                  |                        |
|----------------------------------|------------------------|
| <b>a) plane-parallel</b>         | $R_1=R_2=\infty$       |
| <b>b) concentric (spherical)</b> | $R_1+R_2=L$            |
| <b>c) confocal</b>               | $R_1+R_2=2L$           |
| <b>d) hemispherical</b>          | $R_1=L, R_2=\infty$    |
| <b>e) concave-convex</b>         | $R_1 \gg L, R_2=L-R_1$ |

There are five different types of stable two-mirror optical cavities, as shown in Figure 1. These types of resonators differ in their focal lengths of the mirrors (governed by the mirror's radius of curvature) and in their distance between the mirrors (cavity length). As you can see from Figure 1, some beams have different shapes within the cavity and are thus chosen for different purposes.



**Figure 1: The five possible types of stable cavities.**

There are simple mathematical formulae that indicate whether or not a cavity is stable. In its simplest form, the rule can be stated as follows:

*Given a cavity made of two spherical mirrors (of radii of curvature  $R_1$  and  $R_2$ ) separated by a distance  $L$ , the cavity is stable if*

$$0 \leq g_1 g_2 \leq 1$$

Where:

$$g_1 = 1 - \frac{L}{R_1}$$

$$g_2 = 1 - \frac{L}{R_2}$$

Graphically, Figure 3 shows a plot of the stability region of cavities. If  $g_1$  and  $g_2$  are such that their intersection lies within the shaded region of this diagram, then the cavity is stable.

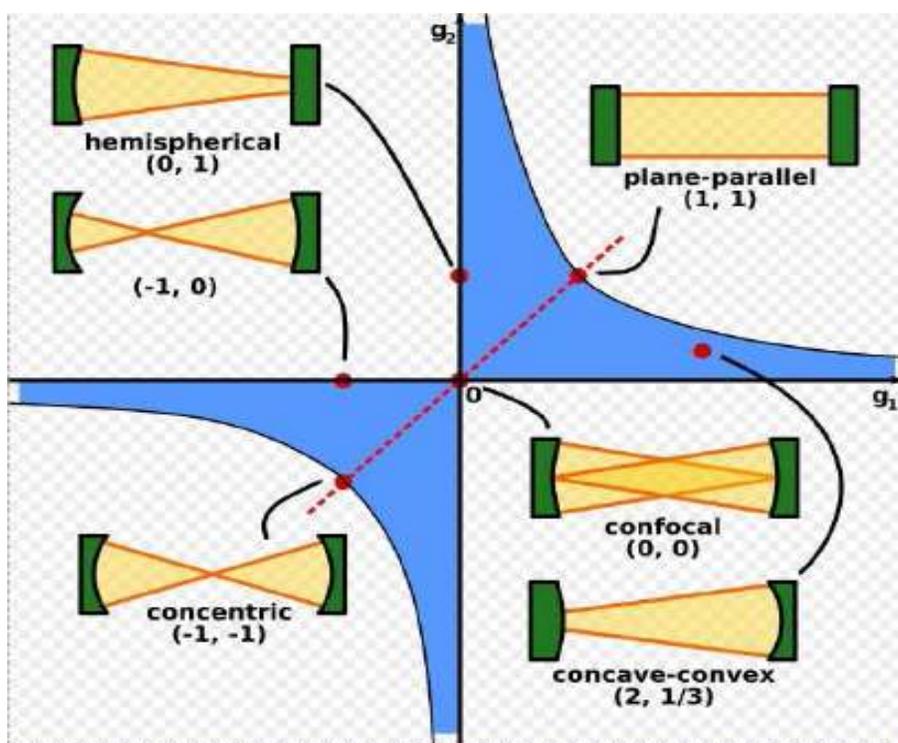


Figure 3: The stability diagram.

مركز تكور المرآة : و هو مركز الكرة التي تكون المرآة جزء منها أو التي قطعت منها المرآة  
 نصف قطر تكور المرآة : هو المسافة بين مركز تكور المرآة و أي نقطة على سطحها.  
 البعد البؤري: و هو المسافة بين البؤرة وقطب المرآة و البعد البؤري يساوى نصف نصف تكور المرآة.

**Example/** Determine the stability of the following cavity resonators?

(i)  $L=1.5\text{m}$ ,  $R_1=3\text{m}$ ,  $R_2=2\text{m}$ ?

(ii)  $L=1\text{m}$ ,  $R_1=0.5\text{m}$ ,  $R_2=2\text{m}$ ?

(iii)  $L=1\text{m}$ ,  $R_1=3\text{m}$ ,  $R_2=-2\text{m}$ ?

**Sol./** The stability condition for a laser resonator cavity is given by the expression,

$$0 \leq g_1 g_2 \leq 1$$

$$g_1 = (1 - L/R_1) \text{ and } g_2 = (1 - L/R_2)$$

$$(i) (1 - 1.5/3)(1 - 1.5/2) = (1 - 0.5)(1 - 0.75) = (0.5)(0.25) = 0.125$$

Since,  $0 < 0.125 < 1$

The cavity resonator is stable.

$$(ii) (1 - 1/0.5)(1 - 1/2) = (1 - 2)(0.5) = (-1)(0.5) = -0.5$$

Since,  $g_1 g_2 = -0.5 < 0$

The cavity resonator is unstable.

$$(iii) (1 - 1/3)(1 - 1/-2) = (1 - 2/3)(0.5) = (1/3)(3/2) = 1$$

Since,  $g_1 g_2 = 1$

The cavity resonator is marginally stable.

**Threshold Gain Coefficient:**

To sustain laser oscillations the gain coefficient must be at least large enough to overcome the losses in the laser system. The sources of loss include the following:

1. Transmission, absorption and scattering by the mirrors.
2. Diffraction around the boundary of the mirrors.
3. Absorption and scattering in the laser active medium.

The minimum or threshold gain coefficient  $k_{th}$  required from the condition that the round trip gain  $G$  in the irradiance of the beam must be at least unity.

If  $G < \text{unity}$  then the oscillations would die out, and

$G > \text{unity}$  then the oscillations would grow.

In traveling from  $M_1$  to  $M_2$  in the laser resonator cavity, fig. (1), the beam irradiance increases from  $I_0$  to  $I$ , where

$$I_1 = I_0 e^{(k-\gamma)L}$$

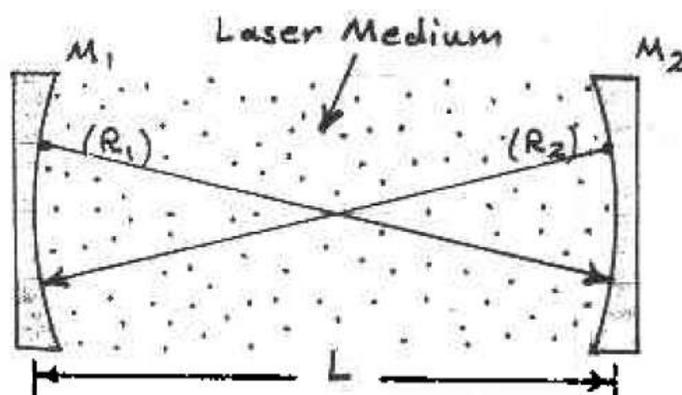


Fig. (1)

and,

$\gamma$ : is the effective volume loss coefficient which reduce the effective gain coefficient to  $(k_{th}-\gamma)$ . After reflection at  $M_2$  the beam irradiance will be  $I_0 R_2 e^{2(k-\gamma)L}$  and after a complete round trip the irradiance will be  $I_0 R_1 R_2 e^{2(k-\gamma)L}$ , so that the round trip gain is

$$G = \frac{\text{Final irradiance}}{\text{Initial irradiance}} = \frac{I_0 R_1 R_2 e^{2(k-\gamma)L}}{I_0} = R_1 R_2 e^{2(k-\gamma)L}$$

The threshold condition for laser oscillations is

$$G = R_1 R_2 e^{2(k-\gamma)L} = 1$$

Where  $k_{th}$ , the threshold gain coefficient, is given by

$$k_{th} = \gamma + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

The first term in Eq. 1 represent the volume losses while the second is the loss in the form of the useful output, thus the condition for steady-state laser operation is that the gain equals the sum of the losses.

**Example/** In a ruby laser ( $\lambda=694.3$  nm), the ruby crystal is 0.1m long and the mirror reflectances are 97% and 90%. Given that the losses are 10% per round trip. Calculate the threshold gain coefficient?

Sol.

$$k_{th} = \gamma + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

$$k_{th} = 0.1 + \frac{1}{2 \times 0.1} \ln \left( \frac{1}{(0.95)(0.90)} \right)$$

$$k_{th} = 0.88 \text{ cm}^{-1}$$