Chapter Three Motion in One Dimension

2-1 Motion

Everything in the world moves. Even seemingly stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies. The classification and comparison of motions (called kinematics) is often challenging.

2-2 Position and Displacement

To locate an object means to find its position relative to some reference point, often the origin (or zero point) of an axis such as the x axis in Fig. 2.1. The positive direction of the axis is in the direction of increasing numbers (coordinates), which is toward the right in Fig. 2.1. The opposite direction is the negative direction.

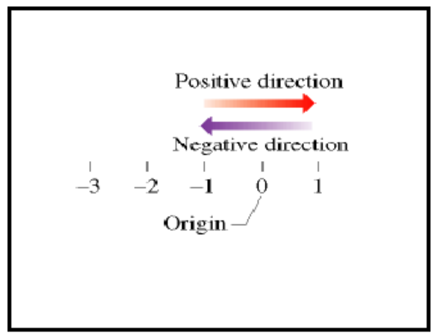


Fig. 2.1

For example, a particle might be located at x = 5 m, which means that it is 5 m in the positive direction from the origin. If it were at x = -5 m, it would be just as far from the origin but in the opposite direction. A plus sign for a coordinate need not be shown, but a minus sign must always be shown.

A change from one position x_1 to another position x_2 is called a displacement □x, where

$$\Delta x = x_2 - x_1 \qquad ....2.1$$

When numbers are inserted for the position values x_1 and x_2, a displacement in the positive direction (toward the right in Fig. 2-1) always comes out positive, and one in the opposite direction (left in the figure), negative. For example, if the particle moves from x_1 = 5 m to x_2

= 12 m, then Δx = (12 m) - (5 m) = +7 m. A plus sign for a displacement need not be shown, but a minus sign must always be shown.

2-3 **Average Velocity and Average Speed**

A compact way to describe position is with a graph of position x plotted as a function of time t—a graph of x(t). As a simple example, Fig. 2.2 shows the position function x(t) for a stationary armadillo (which we treat as a particle) at x= -2 m.

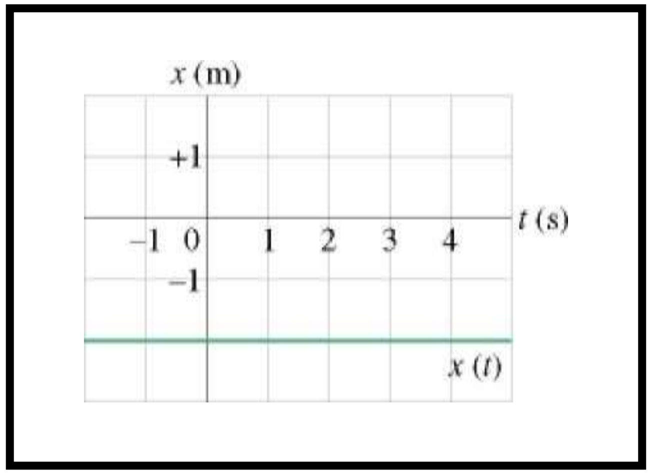


Figure 2.3a, also for an armadillo, is more interesting, because it involves motion. The armadillo is apparently first noticed at t = 0 when it is at the position x = -5 m. It moves toward x = 0, passes through that point at t = 3 s, and then moves on to increasingly larger positive values of x. Figure 2-3b depicts the actual straight-line motion of the armadillo and is something like what you would see. The graph in Fig. 2-3a is more abstract and quite unlike what you would see, but it is richer in information. It also reveals how fast the armadillo moves.

The x-component of average velocity, or average x-velocity, is the x-component of displacement, Δx, divided by the time interval Δt during which the displacement occurs. We use the symbol v_{av-x} for average x-velocity

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad \text{(average } x\text{-velocity, straight-line motion)}$$ …….2.2

As an example, for the dragster x_1 = 19 m. x_2 = 277 m. t_1 = 1.0 s, and t_2 = 4.0 s, so

$$v_{av\text{-}x} = \frac{277\text{ m} - 19\text{ m}}{4.0\text{ s} - 1.0\text{ s}} = \frac{258\text{ m}}{3.0\text{ s}} = 86\text{ m/s}$$

As an example, for the dragster x_1 = 277 m. x_2 = 19 m. t_1 = 16.0 s, and t_2 = 25.0 s, so

$$v_{av\text{-}x} = \Delta x/\Delta t = (-258\text{ m})/(9.0\text{ s}) = -29\text{ m/s}.$$

A common unit for $v_{av\text{-}x}$ is the meter per second (m/s).

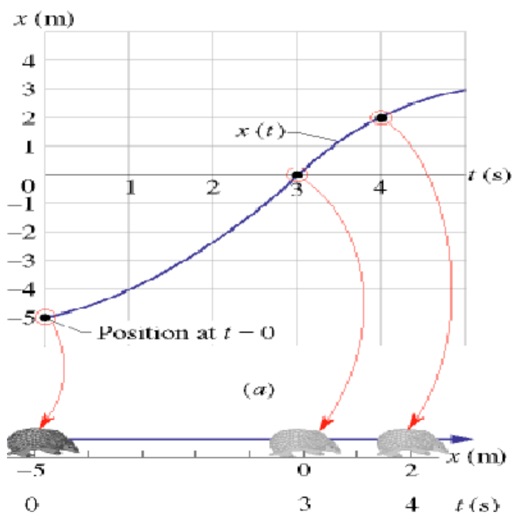


Fig. 2.3

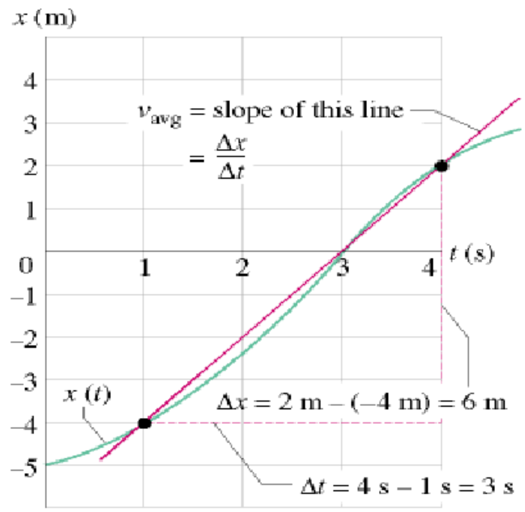


Fig. 2.4

Like displacement, $v_{av\text{-}x}$ has both magnitude and direction. Its magnitude is the magnitude of the line's slope. The average velocity v_{avg} always has the same sign as the displacement Δx because Δt is always positive.

Average speed s_{avg} is a different way of describing "how fast" a particle moves. Whereas the average velocity involves the particle's displacement Δx, the average speed involves the total distance covered (for example, the number of meters moved), independent of direction; that is,

$$s_{ava} = \frac{\text{total distance}}{At} \quad \textbf{...2.3}$$

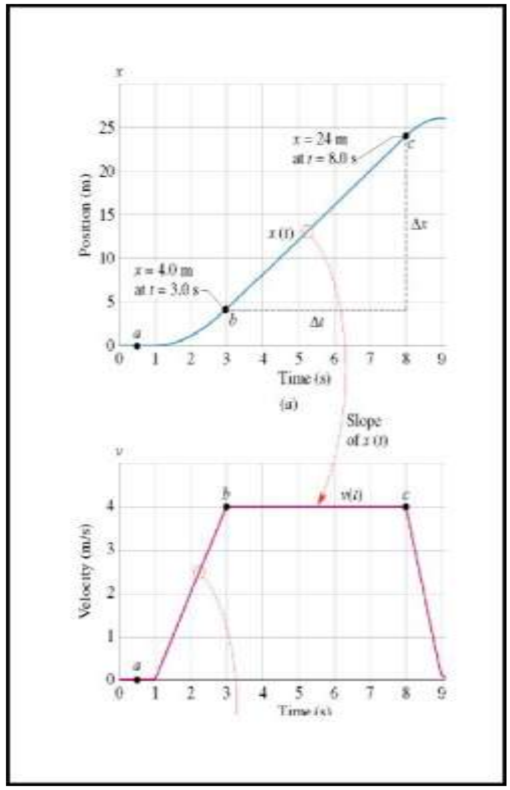
Because average speed does not include direction, it lacks any algebraic sign. Sometimes s_{avg} is the same (except for the absence of a sign) as v_{avg}.

2-4 <u>Instantaneous Velocity and Speed</u>

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval Δt. However, the phrase "how fast" more commonly refers to how fast a particle is moving at a given instant—and that is its instantaneous velocity (or simply velocity) v.

The velocity at any instant is obtained from the average velocity by shrinking the time interval Δt closer and closer to 0. As Δt dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \text{.......}\mathbf{2.4}$$



This equation displays two features of the instantaneous velocity u.

Example

Figure 2-5a is an x(t) plot for an elevator that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Plot u as a function of time.

Solution:

Fig. 2.5

During the interval bc the slope is constant and nonzero; so then the cab moves with constant velocity. We calculate the slope of x(t) then as

$$\frac{\Delta \sim}{\Delta t} = v = \frac{24\ \text{m} - 4\ \text{m}}{8\ \text{s} - 3\ \text{s}} = +4\ \text{m/s}$$

These intervals (where u = 0 and u = 4 m/s) are plotted in Fig. 2-5b. In addition, as the cab initially begins to move and then later slows to a stop, u varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig.2-5b is the required plot.

To find such a change in x during any interval, we must calculate the area "under the curve" on the u(t) graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in x is

$$\Delta x = \frac{4\ \text{m/s}}{8\ \text{s} - 3\ \text{s}} = +20\ \text{m}$$

Example

The position of a particle moving on an x axis is given by

$$x = 7.8 + 9.2\, t - 2.1\, t^3$$

with x in meters and t in seconds. What is its velocity at t = 3.5 s? Is the velocity constant, or is it continuously changing?

Solution:

$$v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2\, t - 2.1\, t^3)$$

which becomes $= 0 + 9.2 - (3)(2.1)\, t^2$

At t = 3.5 s, $= 9.2 - (6.3)(3.5)^2 = -68$ **m/s**.

2-5 **Acceleration**

When a particle's velocity changes, the particle is said to undergo acceleration. For motion along an axis, the average acceleration a_{avg} over a time interval At is

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad \text{...... 2.5}$$

where the particle has velocity u_1 at time t_1 and then velocity u_2 at time t_2. The instantaneous acceleration is the derivative of the velocity with respect to time:

$$a = \frac{dv}{dt} \quad \text{....2.6}$$

Combine Eq. 2-6 with Eq. 2-4, yield:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad \text{..................2.7}$$

Example

Suppose the x-velocity u_x of the car in Fig. 2.6 at any time t is given by the equation

$$v_x = 60\ \text{m/s} + (0.50\ \text{m/s}^3)t^2$$

(a) Find the change in x-velocity of the car in the time interval between t_1 = 1.0 s and t_2 = 3.0 s. (b) Find the average x-acceleration in this time interval. (c) Find the instantaneous x-acceleration at time t_1 = 1.0 s by taking At to be first 0.1 s, then 0.01 s, then 0.001 s. (d) Derive an expression for the instantaneous x-acceleration at any time, and use it to find the x-acceleration at t = 1.0 s and t= 3.0s.

Solution

a) at t_1 = 1.0 s

$$v_{1x} = 60\ \text{m/s} + (0.50\ \text{m/s}^3)(1.0\ \text{s})^2 = 60.5\ \text{m/s}$$

At t_2 = 3.0 s

$$v_{2x} = 60\ \text{m/s} + (0.50\ \text{m/s}^3)(3.0\ \text{s})^2 = 64.5\ \text{m/s}$$

The change in x-velocity Au_x is

$$\Delta v_x = v_{2x} - v_{1x} = 64.5\ \text{m/s} - 60.5\ \text{m/s} = 4.0\ \text{m/s}$$

The time interval is $\Delta t = 3.0\ \text{s} - 1.0\ \text{s} = 2.0\ \text{s}.$

(b) The average x-acceleration during this time interval is

$$a_{av\text{-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{4.0\ \text{m/s}}{2.0\ \text{s}} = 2.0\ \text{m/s}^2$$

(c) When At = 0.1 s, t_2 = 1.1 s and we find

$$v_{2x} = 60\ \text{m/s} + (0.50\ \text{m/s}^3)(1.1\ \text{s})^2 = 60.605\ \text{m/s}$$

$$\Delta v_x = 0.105\ \text{m/s}$$

$$a_{av\text{-}x} = \frac{\Delta v_x}{\Delta t} = \frac{0.105\ \text{m/s}}{0.1\ \text{s}} = 1.05\ \text{m/s}^2$$

(d) The instantaneous x-acceleration is a_x = $du_j dt$. The derivative of a constant is zero and the derivative of t^2 is 2t. so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[60\ \text{m/s} + (0.50\ \text{m/s}^3)t^2]$$
$$= (0.50\ \text{m/s}^3)(2t) = (1.0\ \text{m/s}^3)t$$

When t = 1.0 s,

$$a_x = (1.0\ \text{m/s}^3)(1.0\ \text{s}) = 1.0\ \text{m/s}^2$$

When t = 3 s,

$$a_x = (1.0\ \text{m/s}^3)(3.0\ \text{s}) = 3.0\ \text{m/s}^2$$

H.W.

A particle's position on the x axis of Fig. 2-1 is given by **$x = 4 - 27t + t^3$** with x in meters and t in seconds. (a) Find the particle's velocity function u(t) and acceleration function *a*(t). (b) Is there ever a time when u = 0?

2-6 **Constant Acceleration: A Special Case**

In many types of motion, the acceleration is either constant or approximately so. For example, you might accelerate a car at an approximately constant rate when a traffic light turns from red to green. Then graphs of your position, velocity, and acceleration would resemble those in Fig. 2-6.

When the acceleration is constant, the average acceleration and instantaneous acceleration are equal and we can write Eq. 2.5, with some changes in notation, as

$$a = a_{avg} = \frac{v - v_o}{t - 0} \quad \text{............. 2.8}$$

Here u_o is the velocity at time t = 0, and v is the velocity at any later time t. We can recast this equation as

$$v = v_o - a\,t \quad \text{..................... 2.9}$$

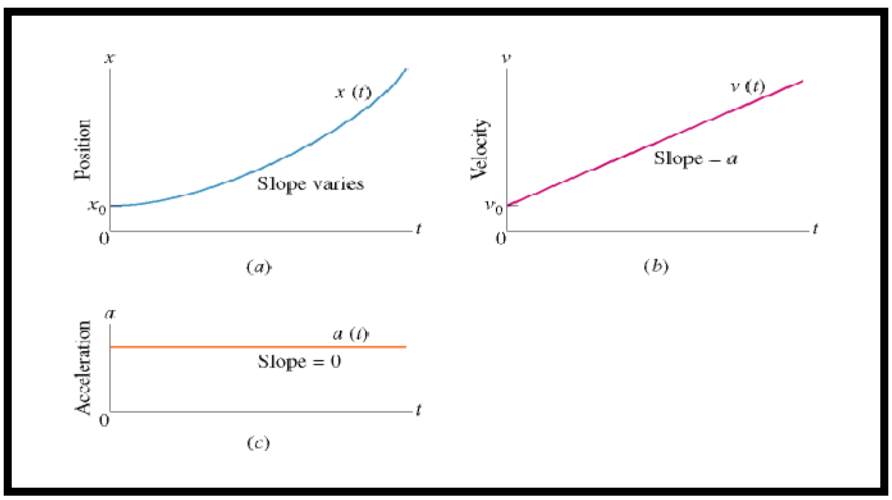


Fig. 2.6

As a check, note that this equation reduces to u = u_o for t = 0, as it must. As a further check, take the derivative of Eq. 2.9 .Doing so yields du/dt = *a*, which is the definition of a. Figure 2- 6b shows a plot of Eq. 2.9

, the u(t) function; the function is linear and thus the plot is a straight line. In a similar manner we can rewrite Eq. 2.2 (with a few changes in notation) as

∼~~~ = ~~ ~~
~~~ and then as ~ = ~~ + ~~~~ ~................. 2.10

For the interval from t = 0 to the later time t then, the average velocity is

$V_{avg} = \frac{1}{2}(v_o + v)$ 2.11

Substituting the right side of Eq. 2.9 for u yields, after a little rearrangement,

∼~~~ = ~~ + ~~ ~ ~ 2.12

Finally, substituting Eq. 2-12 into Eq. 2-10 yields

$X - X_0 = V_0 t + \frac{1}{2} a t^2$ 2.13

$V^2 = V_0^2 + 2a(x - x_0)$ ....2.14 (with constant acceleration only)

Eliminate the acceleration a between Eqs. 2-9 and 2-13 to produce an equation in which *a* does not appear:

$X - x_0 = \frac{1}{2}(v - v_o)t$ ....2.15 (with constant acceleration only)

Finally, we can eliminate $1)_o$, obtaining

$x - x_o = vt - \frac{1}{2} a t^2$ 2.16

Example

A motorcyclist heading east through a small Iowa city accelerates after he passes the signpost marking the city limits (Fig. 2.7). His acceleration is a constant 4.0 m/s^2. At time t = 0 he is 5.0 m east of the signpost, moving east at 15 m/s. (a) Find his position and velocity at time t = 2.0 s. (b) Where is the motorcyclist when his velocity is 25 m/s?

Solution a)

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$= 5.0\,m + (15\,m/s)(2.0\,s)$$
$$= 43\,m$$

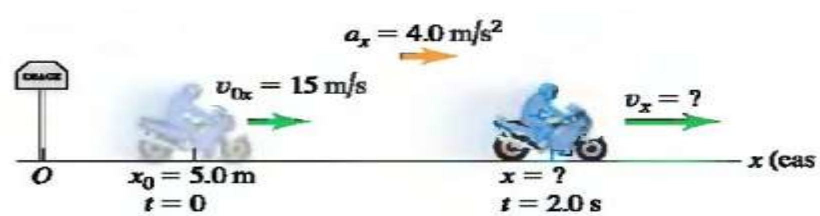


Fig. 2.7
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$$v_x = v_{0x} + a_x t$$
$$= 15\ \text{m/s} + (4.0\ \text{m/s}^2)(2.0\ \text{s}) = 23\ \text{m/s}$$

b)

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x = x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x}$$
$$= 5.0\ \text{m} + \frac{(25\ \text{m/s})^2 - (15\ \text{m/s})^2}{2(4.0\ \text{m/s}^2)}$$
$$= 55\ \text{m}$$

$$v_x = v_{0x} + a_x t \quad \text{so}$$
$$t = \frac{v_x - v_{0x}}{a_x} = \frac{25\ \text{m/s} - 15\ \text{m/s}}{4.0\ \text{m/s}^2} = 2.5\ \text{s}$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$= 5.0\ \text{m} + (15\ \text{m/s})(2.5\ \text{s}) + \frac{1}{2}(4.0\ \text{m/s}^2)(2.5\ \text{s})^2$$
$$= 55\ \text{m}$$

2-7 **Another Look at Constant Acceleration**

The equations (2-9) and (2-13) are the basic equations from which the others are derived. Those two can be obtained by integration of the acceleration with the condition that *a* is constant. To find Eq. 2-9, we rewrite the definition of acceleration (Eq.2.6) as dv = a dt We next write the indefinite integral (or anti-derivative) of both sides:

$$1\ dv = 1\ a\ dt$$

Since acceleration *a* is a constant, it can be taken outside the integration. Then we obtain

$$1 dv = a1\ dt$$

v= a t +C 2.17

To evaluate the constant of integration C, we let t = 0, at which time u = □$_o$. Substituting these values into Eq. 2-17 (which must hold for all values of t, including t = 0) yields

$$v_0 = (a)(0) + C = C$$

Substituting this into Eq. 2-17 gives us Eq. 2-9.

To derive Eq. 2-13, we rewrite the definition of velocity (Eq. 2-4) as

$$dx = v\, dt$$

and then take the indefinite integral of both sides to obtain

$$f\ dx = f v\ dt$$

Generally □ is not constant, so we cannot move it outside the integration. However, we can substitute for v with Eq. 2-9:

f dx= ~(~~ + ~ ~) ~~

Since □$_o$ is a constant, as is the acceleration a, this can be rewritten as

$$f\ dx = v_0 f\ dt + a\ f t dt$$

Integration now yields

$x = v_0\, t + $ ~~ $a\, t2 + C^*$ 2.18

where C* is another constant of integration. At time t = 0, we have x = x_o. Substituting these values in Eq. 2-18 yields x_o = C*. Replacing C´ with x_o in Eq. 2-18 gives us Eq. 2-13.

Example

Sally is driving along a straight highway in her classic 1965 Mustang. At time t = 0, when Sally is moving at 10 m/s in the positive x-direction, she passes a signpost at x = 50 m. Her x-acceleration is a function of time: $a_x = 2.0 m/s^2 - (0.10 m/s^3)t$, (a) Find her x-velocity and position as functions of time. (b) When is her x-velocity greatest? (c) What is the maximum x-velocity? (d) Where is the car when it reaches the maximum x-velocity?

Solution: Fig. 2.8 show the details of example above

a)

$$v_x = 10\,\text{m/s} + \int_0^t [2.0\,\text{m/s}^2 - (0.10\,\text{m/s}^3)t]\,dt$$
$$= 10\,\text{m/s} + (2.0\,\text{m/s}^2)t - \frac{1}{2}(0.10\,\text{m/s}^3)t^2$$

$$x = 50\,\text{m} + \int_0^t \left[10\,\text{m/s} + (2.0\,\text{m/s}^2)t - \frac{1}{2}(0.10\,\text{m/s}^3)t^2\right] dt$$
$$= 50\,\text{m} + (10\,\text{m/s})t + \frac{1}{2}(2.0\,\text{m/s}^2)t^2 - \frac{1}{6}(0.10\,\text{m/s}^3)t^3$$

b)

$$0 = 2.0\,\text{m/s}^2 - (0.10\,\text{m/s}^3)t$$
$$t = \frac{2.0\,\text{m/s}^2}{0.10\,\text{m/s}^3} = 20\,\text{s}$$

c)

$$v_{max\text{-}x} = 10\,\text{m/s} + (2.0\,\text{m/s}^2)(20\,\text{s}) - \frac{1}{2}(0.10\,\text{m/s}^3)(20\,\text{s})^2$$
$$= 30\,\text{m/s}$$

d)

$$x = 50\,\text{m} + (10\,\text{m/s})(20\,\text{s}) + \frac{1}{2}(2.0\,\text{m/s}^2)(20\,\text{s})^2$$
$$- \frac{1}{6}(0.10\,\text{m/s}^3)(20\,\text{s})^3$$
$$= 517\,\text{m}$$

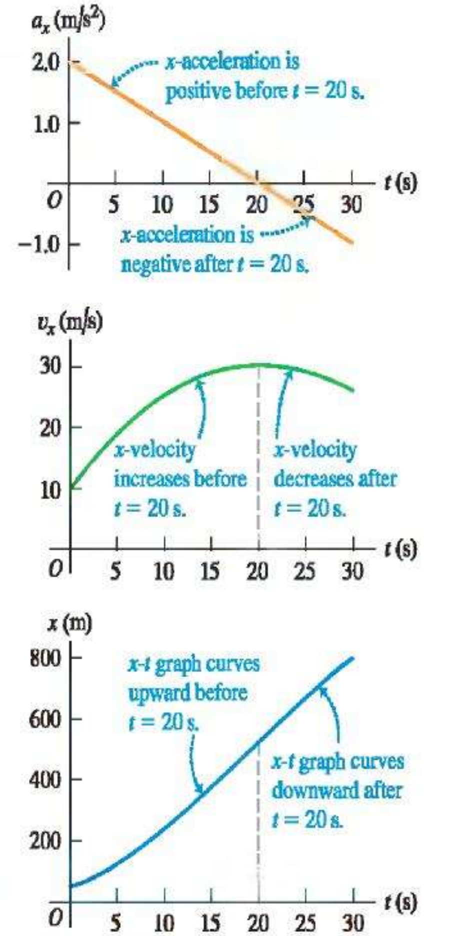

Fig. 2.8

2-8 **Free-Fall Acceleration**

If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate. That rate is called the **free-fall acceleration**, and its magnitude is represented by g. At sea level in Earth's midlatitudes the value is **9.8 m/s^2** which is what you should use for the problems. However, note that for free fall:

(1) The directions of motion are now along a vertical y axis instead of the x axis, with the positive direction of y upward.

(2) The free-fall acceleration is now negative—that is, downward on the y axis, toward Earth's center—and so it has the value -g in the equations.

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ m/s^2$, and the magnitude of the acceleration is $g = 9.8\ m/s^2$.

Example

In Fig. 2-9, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

$t = \frac{V - V_o}{a} = \frac{0 - 12}{-9.8} = 1.2\ s$

Solution:

(b) What is the ball's maximum height above its release point?

Solution:

$Y = \frac{V^2 - V_o^2}{2a} = \frac{0 - (12)^2}{2(-9.8)} = 7.3\ m$

(c) How long does the ball take to reach a point 5.0 m above its release point? **Solution:**

$Y = V_o t - \frac{1}{2} g t^2$ or $5 = (12)t - \frac{1}{2}(9.8)t^2$

If we temporarily omit the units, we can rewrite this as $4.9 t^2 - 12 t + 5 = 0$

Solving this quadratic equation for t yields **t = 0.53 s and t = 1.9 s**

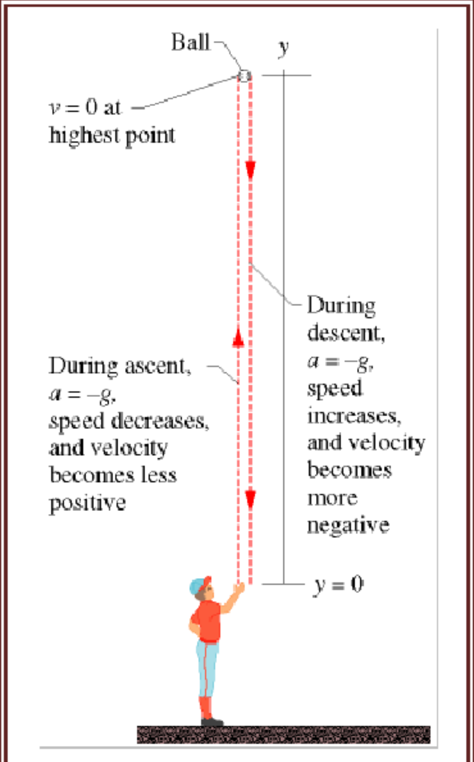

Fig. 2.9

There are two such times! This is not really surprising because the ball passes twice through y = 5.0 m, once on the way up and once on the way down.

Example

A one-euro coin is dropped from the Leaning Tower of Pisa(Fig. 2.10). It Starts from rest and falls freely. Compute its position and velocity after 1.0s,2.0s,and 3.0s.

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}(-g)t^2 = (-4.9\ \text{m/s}^2)t^2$$

$$v_y = v_{0y} + a_y t = 0 + (-g)t = (-9.8\ \text{m/s}^2)t$$

When t = 1.0s, y = (-4.9 m/s^2)(1.0 s)^2 = -4.9m and v_y = (-9.8m/s^2)(1.0s) = -9.8m/s; after 1s, the coin is 4.9 m below the origin (y is negative) and has a downward velocity (v_y is negative) with magnitude 9.8 m/s.

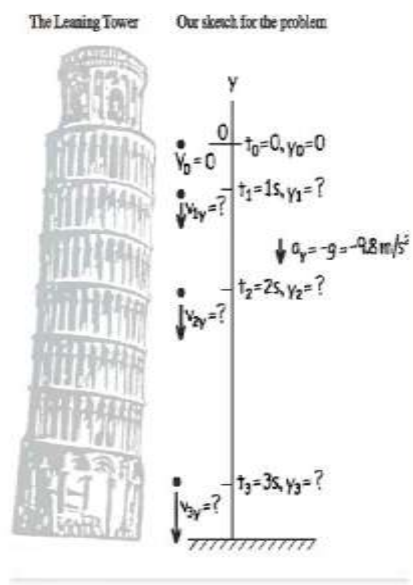

Fig. 2.10

H. W. You throw a ball vertically upward from the roof of a tall building (Fig. 2.11). The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. At the location of the building, g = 9.80 m/s^2. Find (a) the position and velocity of the ball 1.00 s and 4.00 s after leaving your hand; (b) the velocity when the ball is 5.00 m above the railing; (c) the maximum height reached and the time at which it is reached; and (d) the acceleration of the ball when it is at its maximum height.

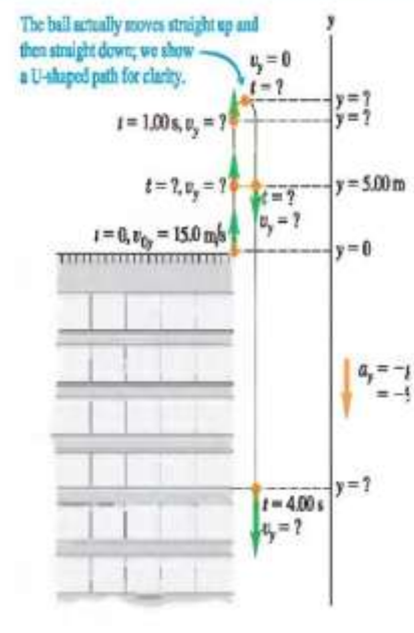

Fig. 2.11