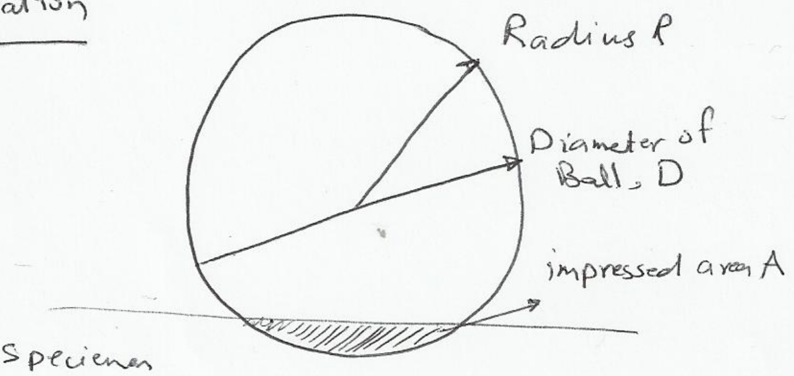
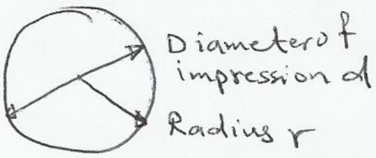
Brinell hardness Number Derivation



$$BHN = \frac{P}{\pi \frac{D}{2}\left(D-\sqrt{D^2-d^2}\right)}$$

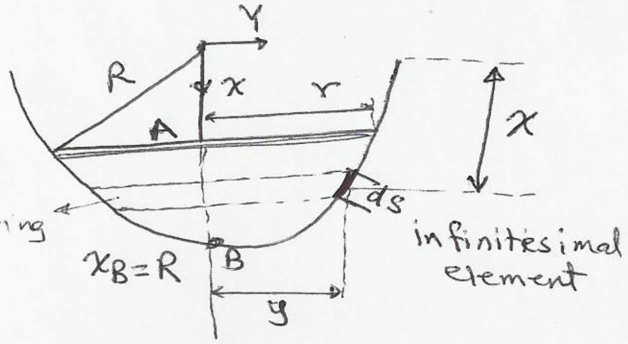


Brinell test consiste of pressing a 10 mm diameter ball into a material under a specified load (~ 3000 kg) in a specific amount of time (~10 sec)

The Brinell hardness Number (BHN) is the ratio of load to the impressed area, so: $BHN = L/A$

The impressed Area is part of the curved surface area of the sphere.

Lets find A using integration



$x_A = \sqrt{R^2-r^2}$

Area of infinitesimal element (ds).

$a = (2\pi y)\, ds$

↳ circumference of ring (avg.)

So $a = 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

dx, dy, ds

$ds = \sqrt{(dx)^2+(dy)^2}$

$ds = \left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right) dx$

Now $y = \sqrt{R^2-x^2} \Rightarrow$

$$\frac{dy}{dx} = -\frac{x}{\sqrt{R^2-x^2}}$$

$$a = 2\pi\sqrt{R^2-x^2}\cdot\sqrt{\frac{R^2}{R^2-x^2}}\, dx = 2\pi R\, dx$$

$$A = \int_A^B a = \int_A^B 2\pi R\, dx \Rightarrow A = \int_{x_A}^{x_B} 2\pi R dx = \int_{\sqrt{R^2-r^2}}^{R} 2\pi R\, dx$$

$$A = 2\pi R\left(R-\sqrt{R^2-r^2}\right) \overset{OR}{\Longrightarrow} A = 2\pi \frac{D}{2}\left(\frac{D}{2}-\sqrt{\frac{D^2}{4}-\frac{d^2}{4}}\right)$$

$A = \pi\frac{D}{2}\left(D-\sqrt{D^2-d^2}\right)$ (impressed Area) $\therefore BHN = \frac{P}{\pi\frac{D}{2}\left(D-\sqrt{D^2-d^2}\right)}$

Q.2:(b)

$$N_v = N \exp\left(-\frac{Q_v}{kT}\right)$$

$$N_v = 8\times10^{28}\ \frac{atoms}{m^3} \exp\left(-\frac{0.9\ ev}{8.62\times10^{-5}\ \frac{ev}{k}\times 1073k}\right)$$

$$N_v = 8\times10^{28} \exp(5.947\times10^{-5}) = 4.757\times10^{24}\ \text{Vacancies.}$$

(c) I - False

II - (b)

Q.3:(a) $E = \frac{\sigma}{\epsilon} \Rightarrow \epsilon = \frac{\sigma}{E} = \frac{345\times10^{6}}{103\times10^{9}} = 0.003349$

$\because \epsilon = \frac{\Delta l}{l_o} \Rightarrow \Delta l = l_o \epsilon = 0.00349 \times 76mm = 0.254mm$

$\because \Delta l = l - l_o \Rightarrow l = l_o + \Delta l = 76mm + 0.254 = 76.254mm$

(b):

1- Atomic size factor

2- crystal structure

3- Electronegativity

4- Valances.

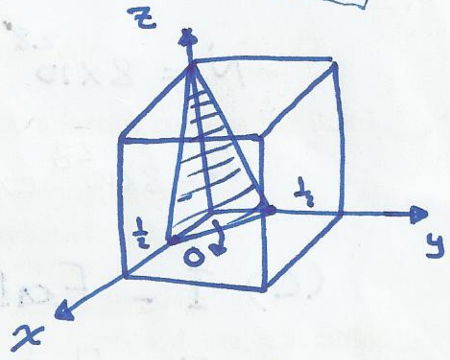
(c): (I): False

(II): False

1st Mid. Term Exam for Materials properties (234)
2nd year.

Q.1: (a) (I)

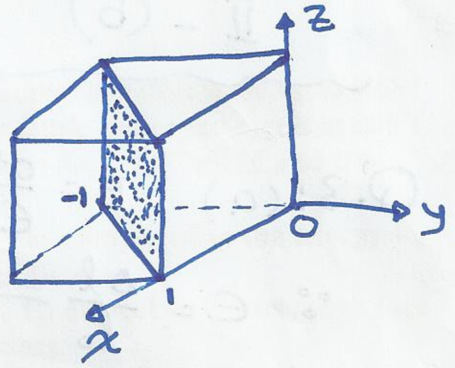
Miller indicies	2	2	1
reciprocals	$\frac{1}{2}$	$\frac{1}{2}$	1
intercepts	$x=\frac{1}{2}$	$y=\frac{1}{2}$	$z=1$



(II)

Miller indicies	1	$\bar{1}$	0
reciprocals	1	-1	∞
intercepts	$x=1$	$y=-1$	

Plane parallel to Z axis



(b) $d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{0.361\,nm}{\sqrt{2^2+2^2+1^2}} = \frac{0.361\,nm}{\sqrt{9}} = 0.12\,nm$

(c) I-(b) . II - True

Q.2 (a): $APF = \frac{\text{Vol. of atoms in unit cell}}{\text{Vol. of unit cell}}$

Since there are two atoms per BCC unit cell.

∴ Vol. of atoms in unit cell $= 2 \times \frac{4}{3}\pi R^3 = 8.373\,R^3$

Vol. of unit cell $= a^3$

∵ Lattice constant a in BCC unit structure $= 4R/\sqrt{3}$

∴ $V_{u.cell} = a^3 = \left(\frac{4R}{\sqrt{3}}\right)^3 = 12.32\,R^3$

∴ $APF = \frac{8.373\,R^3}{12.32\,R^3} = 0.68$