Chapter Four: Mechanical Properties of Materials

STRESS AND STRAIN

INTRODUCTION

Many materials, when in service, are subjected to forces or loads; examples include the aluminum alloy from which an airplane wing is constructed and the steel in an automobile axle. In such situations it is necessary to know the characteristics of the material and to design the member from which it is made such that any resulting deformation will not be excessive and fracture will not occur. The mechanical behavior of a material reflects the relationship between its response or deformation to an applied load or force. Important mechanical properties are strength, hardness, ductility, and stiffness.

NORMAL STRESS

To introduce the concepts of stress and strain, we begin with the relatively simple case of a straight bar undergoing **axial loading**, as shown in Fig1. In this section we consider the stress in the bar.

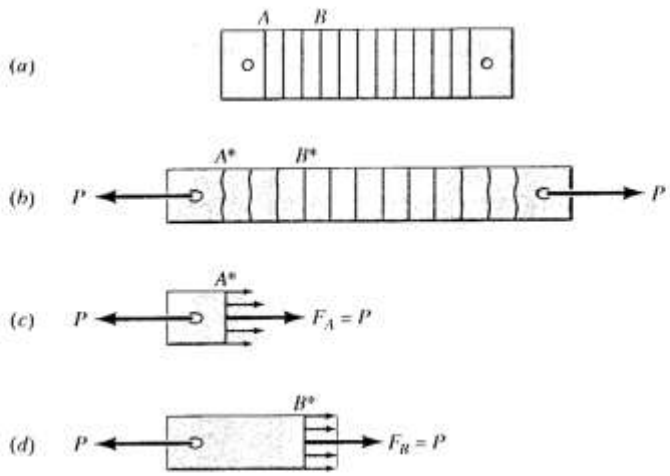


Fig.1 A straight bar undergoing axial loading (a) the undeformed bar, with vertical lines indicating cross sections. (b) The deformed bar. (c) The distribution of internal force at section A. (d) The distribution of internal force at section B.

Equal and opposite forces of magnitude P acting on a straight bar cause it to *elongate*, and also to get narrower, as can be seen by comparing Figs. la and lb. The bar is said to be in *tension*. If the external

forces had been applied in the opposite sense, that is, pointing toward each other, the bar would have shortened and would then be said to be in *compression*.

Definition of Normal Stress: The thin arrows in Figs. 1c and ld represent the distribution of force on cross sections at A and B, respectively. (A cross section is a plane that is perpendicular to the axis of the bar.) Near the ends of the bar, for example at section A, the resultant normal force, FA, is not uniformly distributed over the cross section; but at section B, farther from the point of application of force P, the force distribution is uniform. In mechanics, the term stress is used to describe the distribution of a force over the area on which it acts and is expressed as force intensity, that is, as force per unit area.

$$Stress = \frac{Force}{Aera}$$

The units of stress are units of force divided by units of area. In the International System of units (SI), stress is specified using the basic units of force (Newton) and length (meter) as Newtons per meter squared (N/m^2). This unit, called the Pascal (1 Pa = 1 N/m^2), is quite small, so in engineering work stress is normally expressed in kilopascals (1 kPa = 10^3 N/m^2), megapascals (1 MPa = 10^6 N/m^2), or gigapascals (1 GPa = 10^9 N/m^2). For example,1 psi = 6895 Pa = 6.895 kPa. . In the U.S. Customary System of units (USCS), stress is normally expressed in pounds per square inch (psi) or in kips per square inch, that is, kilopounds per square inch (Ksi).

There are two types of stress, called normal stress and shear stress. In this section we will consider only normal stress. In words, normal stress is defined by

$$Normal \cdot\cdot stress = \frac{Force \cdot normal(i.e., perpendicular) to.an.area}{Aera.on.which.the.force.acts}$$

The symbol used for normal stress is the lowercase Greek letter sigma (σ). The normal stress at a point is defined by the equation

$$\sigma(x, y, z) = \lim_{\Delta A \to 0} (\frac{\Delta F}{\Delta A}) \quad \textbf{Normal stress} \quad \ldots\ldots\ldots(1)$$

Where, as shown in Fig..2a, ΔF is the normal force (assumed positive in tension) acting on an elemental area ΔA containing the point (x, y, z) where the stress is to be determined.

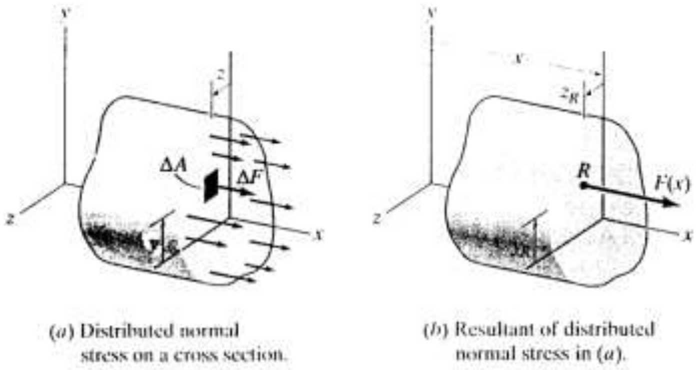


Fig. 2 Normal force on a cross section

The sign convention for normal stress is as follows:

- A positive value for σ indicates **tensile stress**, that is, the stress due to a force ΔF that pulls on the area on which it acts.
- A negative value for σ indicates **compressive stress**.

Average Normal Stress: Even when the normal stress varies over a cross section, as it does in Fig. 1c, we can compute the average normal stress on the cross section by letting

$$\sigma_{avg} = \frac{F}{A} \quad \text{Average Normal Stress} \qquad \ldots\ldots(2)$$

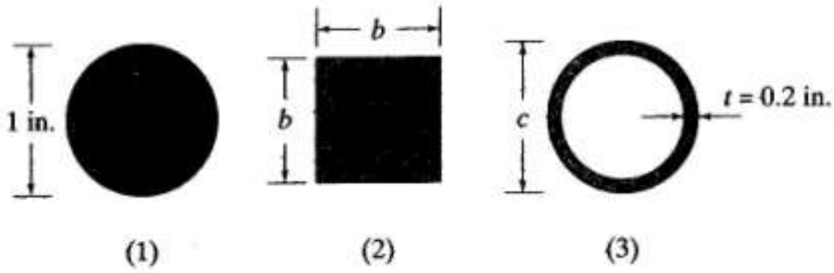
Thus, for Figs. 1c and ld we get

$$(\sigma_{avg})_A = \frac{F_A}{A} = \frac{P}{A} \quad , \quad (\sigma_{avg})_B = \frac{F_B}{A} = \frac{P}{A}$$

Much of the rest of on going discussion is devoted to determining how stress is distributed on cross sections of structural members under various loading conditions. However, in many situations the normal stress on a cross section is either constant or very nearly constant, as we will see in the next examples.

Prob. 1. **A l-in.-diameter solid bar (1), a square solid bar (2), and a circular tubular member with 0.2-in. wall thickness (3), each supports an axial tensile load of 5 kips, (a) Determine the**

axial stress in bar (1). (b) If the axial stress in each of the other bars is 6 ksi, what is the dimension, b, of the square bar, and what is the outer diameter, c, of the tubular member?

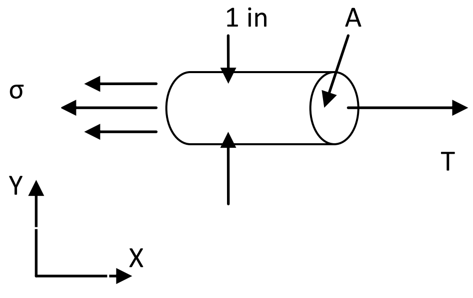


P-1

Solution

(a) Axial stress

FBD: cylindrical solid bar



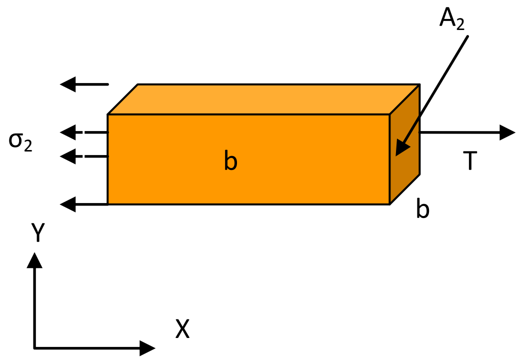
Definition:

$$\sigma_1 = \frac{T}{A_1} = \frac{5kips}{0.7854in^2} = 6.366ksi$$

$$A_1 = \frac{\pi d_1}{4} = \frac{\pi(1in)^2}{4} = 0.7854in^2$$

(b) Dimensions

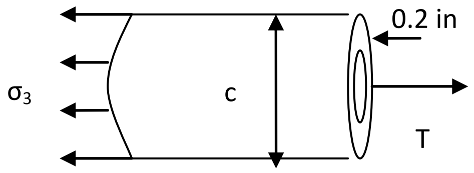
FBD: square solid bar



Definition: normal stress

$$\sigma_2 = \frac{T}{A_2} = 6ksi. \quad A_2 = b^2{}_2$$

$$b = \sqrt{\frac{T}{\sigma_2}} = 0.9129in$$



B=0.91 in

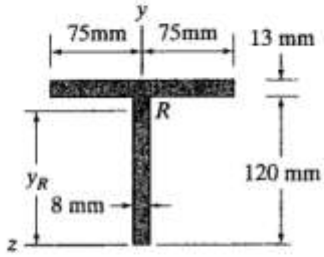
C=1.526 in

Definition: normal stress

$$\sigma_3 = \frac{T}{\sigma_3}$$
$$A_3 = \frac{\pi}{4}[c^2 - (c - 0.4)^2]$$
$$= \frac{\pi}{4}(0.8c - 0.16)$$
$$c = [\frac{T}{\sigma_3}(\frac{4}{\pi}) + 0.16]/0.8$$
$$= 1.526in$$

Prob. 2. The structural tee is shown in Fig. P-2 supports a compressive load P = 200 kN. (a) Determine the coordinate y_R of the point R in the cross section where the load must act in order to produce uniform compressive axial stress in the member, and (b) determine the magnitude of that compressive stress.

(a)



(b)

P-2

$$\sigma_3 = \frac{T}{\sigma_3}$$

$$A_3 = \frac{\pi}{4}[c^2 - (c - 0.4)^2]$$

$$= \frac{\pi}{4}(0.8c - 0.16)$$

$$c = [\frac{T}{\sigma_3}(\frac{4}{\pi}) + 0.16]/0.8$$

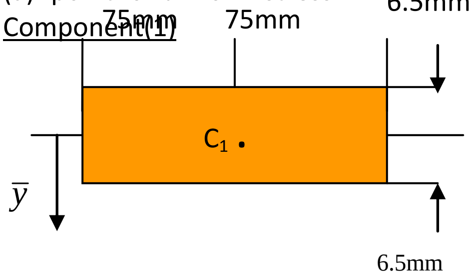
$$= 1.526in$$

$$where A_3 = \frac{T}{\sigma_3} = \frac{5kips}{6ksi} = \frac{\pi}{4}(0.8c - 0.16)$$

Solution:

(a) point for uniform stress

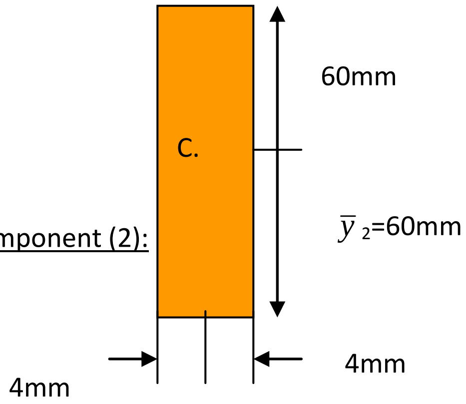
Component (1)



If P is applied at the cross-sectional centroid, then the resulting axial stress will be uniform. Therefore, R is the centroid.

component	$\bar{y}_i$ mm	A_i mm^2	$\bar{y}_i A_i$ mm^3
1	126.5	1950	246675
2	60	960	57600
Sum	Σ	2910	304275

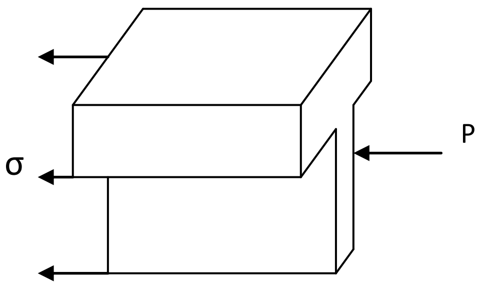
Component (2):



Centroid of cross sectional area

$$y_R = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = 104.56mm$$

y_R = 105 mm



Stress Resultant: Internal resultants were introduced in previous section, and Examples show how equilibrium is used to relate these resultants on a cross section to the external loads. Equation 2 relates the average normal stress on a cross section to the normal force on the cross section. Let us now examine in greater detail the relationship between the distributed normal stress on a cross section and its resultant. Based on the definition of normal stress in Eq. 1, we can replace the ΔF in Fig. 2a by an elemental force dF =σ dA. Referring again to Fig. 2a and following the right-hand rule for moments, we can see that this elemental force dF contributes a moment zdF about the +y axis and ydF about the -z axis. In Fig. 2b, the resultant normal force on the cross section at x is labeled F(x),

and it acts at point (y_R, z_R) in the cross section. Given the distribution of normal stress on a cross section, σ = σ(x, y, z), we can integrate over the cross section to determine the magnitude and point of application of the **resultant normal force**

$$\sum F_x : \qquad F(x) = \int_A \sigma dA$$

$$\sum M_y : \qquad z_R F(x) = \int_A z\sigma dA \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

$$\sum M_z : \qquad -y_R F(x) = -\int_A y\sigma dA$$

The two moment equations are used to locate the line of action of the force F(x). Note that the sign convention for or implies that the force F in Eq. 3 is to be taken positive in tension.

Resultant of Constant Normal Stress on a Cross Section: Let us determine the resultant of normal stress on the cross section at x (Fig. 2a) if the normal stress is constant over the cross section. We will prove that **normal stress that is constant on a cross section corresponds to an axial force F(x) = A σ (x) acting through the centroid of the cross section at x.** (you will learn the conditions under which σ (x) is constant over the cross section in later section.)

Let the resultant be assumed to be a force F(x) acting parallel to the x axis and passing through point (y_R, z_R), as in Fig. 2b. We must show that

$$F(x) = A(x)\, \sigma(x), \qquad y_R = \bar{y}, \qquad z_R = \bar{z}$$

For this we can use Eqs. 3. Substituting the condition σ (x, y, z) = σ (x) into Eqs. 3, we get

$$\sum F_x : \qquad F(x) = \sigma(x)\int_A dA = \sigma(x) A$$

$$\sum M_y : \qquad z_R F(x) = \sigma(x)\int_A z dA = \sigma(x)\, \bar{z}\, A$$

$$\sum M_z : \qquad -y_R F(x) = -\sigma(x)\int_A y dA = -\sigma(x)\, \bar{y}\, A$$

Therefore, if the normal stress is uniform over a cross section, the normal stress over the cross section, also called the **axial stress**, and is given by

$$\sigma(x) = \frac{F(x)}{A(x)} \quad \textbf{Axial Stress Equation} \qquad .....(4)$$

Which corresponds to a force F(x) (tension positive) acting at the centroid of the cross section, that is, at $z_R = \bar{z}$, $y_R = \bar{y}$.

Uniform Normal Stress in an Axially Loaded Bar: tender certain assumptions, an axially loaded bar will have the same uniform normal stress on every cross section; that is, σ_r (x, y, z) = σ = constant. These assumptions are:

- The bar is prismatic; that is, the bar is straight and it has the same cross section throughout its length.
- The bar is homogeneous; that is, the bar is made of the same material throughout.
- The load is applied as equal and opposite uniform stress distributions over the two end cross sections of the bar.

The uniform, prismatic bar in Fig. 3a is labeled as member "i" and is subjected to equal and opposite axial forces Ft acting through the centroids at its ends. Its cross-sectional area is A_i.

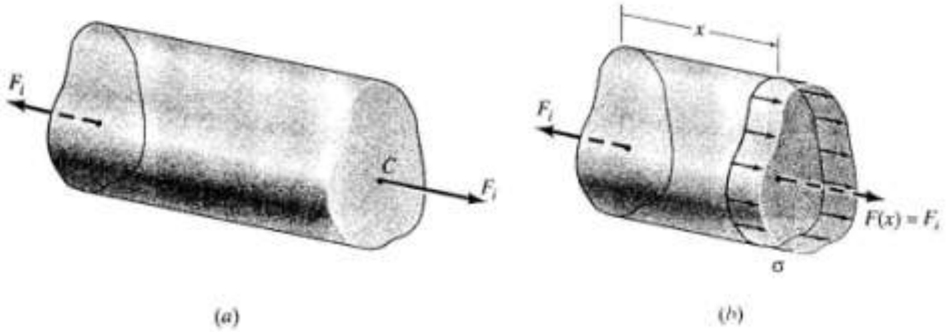
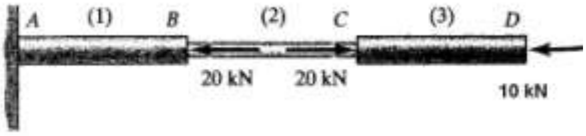


Figure 3 Uniform stress in an axially loaded prismatic bar.

The normal stress on cross sections of an axially loaded member, like the one in Fig. 3, is called the axial stress. Since, from the free-body diagram in Fig. 3b, the resultant force, F(x), on every cross section of the bar is equal to the applied load F_i, and since the cross-sectional area is constant, from Eq. 4 we get the fllowing formula for the **uniform axial stress:**

$$\sigma_i = \frac{F_i}{A_i} = const. \quad \textbf{Axial stress Equation} \quad \textbf{(5)}$$

Prob. 3. The diameter of the central one-third of a 50mm-diameter steel rod is reduced to 20 mm, forming a three segment rod, as shown in Fig. P-3. For the loading shown, Determine the axial stresses (σ_1, σ_2 and σ_3 in each of the three respective segments.

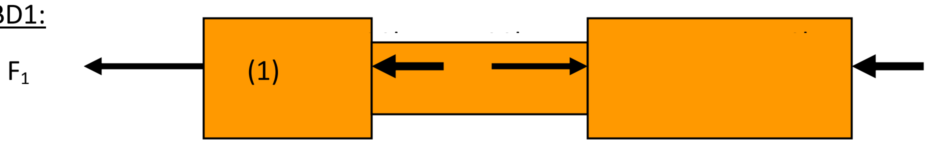


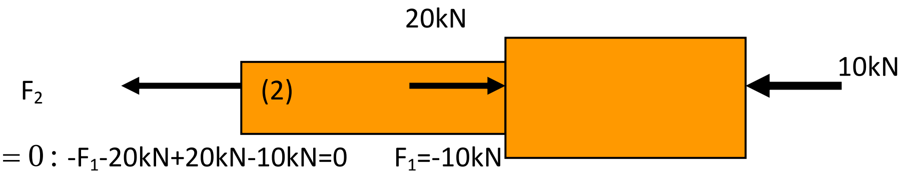
P-3

Solution

Determine axial stress σ_1, σ_2, and σ_3.

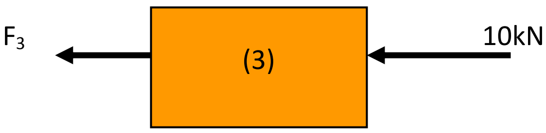
<u>Equilibrium for FBD1:</u>





$\underset{\rightarrow}{+}\sum F = 0$: $-F_1-20kN+20kN-10kN=0$ $F_1=-10kN$

<u>Equilibrium for FBD2</u>

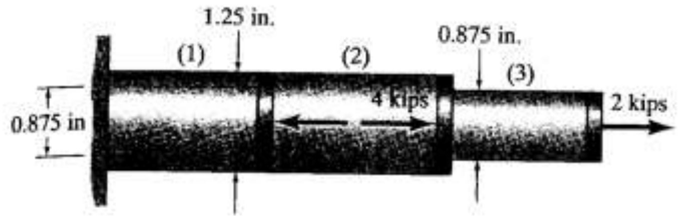


$\underset{\rightarrow}{+}\sum F = 0$: $-F_2+20kN-10kN=0$ $F_2=10kN$

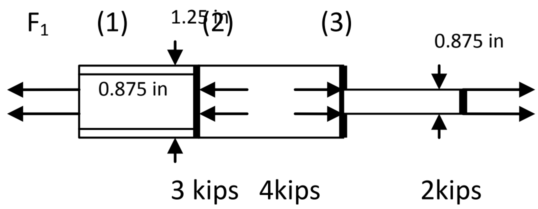
<u>Equilibrium for FBD2</u>

p-4

Prob. 4. The three-part axially loaded member in Fig.P- 4 consists of a tubular segment (1) with outer diameter $(d_o)_1$=1.25 in. and inner diameter $(d_i)_1$= 0.875 in., a solid circular rod segment (2) with diameter d_2 = 1.25 in., and another solid circular rod segment (3) with diameter d_3 = 0.875 in. The line of action of each of the three applied loads is along the centroidal axis of the member. Determine the axial stresses σ_1, σ_2, and σ_3 in each of the three respective segments.



FBD: section (1), (2) and (3)



Equilibrium

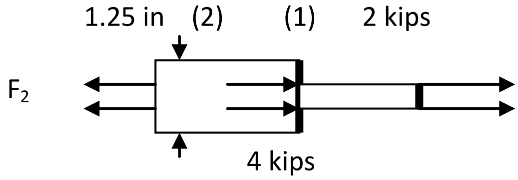
$$\xrightarrow{+} \Sigma F_x = 0:$$

$-F_1$ -3kips +4kips +2kips= 0

F_1= 3kips

$$A_1 = \frac{\pi}{4}[(1.25in)^2 - (0.875in)^2]$$

$$\sigma_1 = \frac{F_1}{A_1} = 4.793ksi$$



Equilibrium

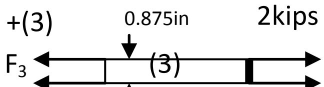
$$\xrightarrow{+} \Sigma F_x = 0:$$

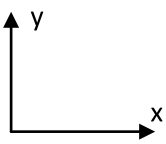
$-F_2$+4kips +2kips= 0, F_2 =6kips

$$A_2 = \frac{\pi}{4}(1.25in)^2$$

$$\sigma_2 = \frac{F_2}{A_2} = 4.889ksi$$

FBD: section (2) +(3)





σ_1= 4.79ksi

σ_2=4.89ksi

Equilibrium

$$\xrightarrow{+} \Sigma F_x = 0:$$

$-F_3$+2kips= 0, F_3 =2kips

$$A_3 = \frac{\pi}{4}(0.875in)^2$$

$$\sigma_3 = \frac{F_3}{A_3} = 3.326ksi$$

Engineering strain:

Engineering strain is defined according to

$$\in = \frac{l_i - l_o}{l_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which l_o is the original length before any load is applied, and l_i is the instantaneous length. Sometimes the quantity $l_i - l_o$ is denoted as Δl and is the deformation elongation or change in length at some instant, as referenced to the original length. Engineering strain (subsequently called just strain) is unitless. Sometimes strain is also expressed as a percentage, in which the strain value is multiplied by 100.

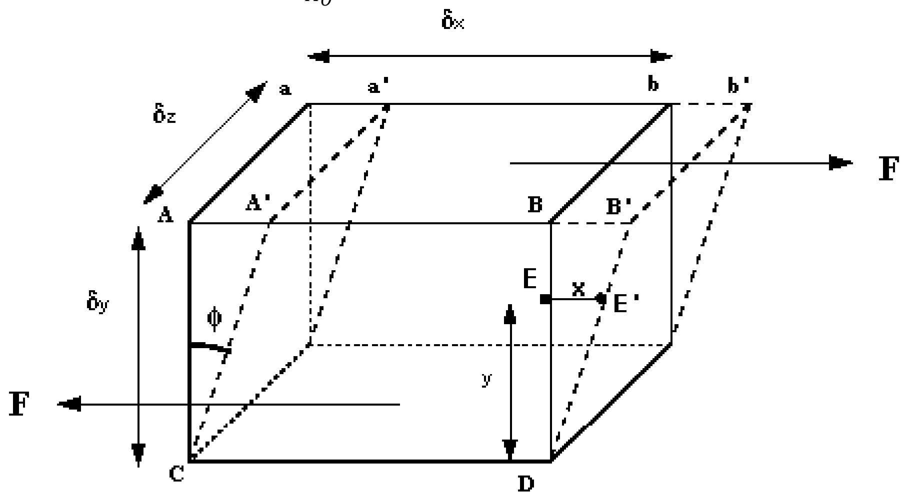
Compression Tests:

A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress. Equations 5 and 6 are utilized to compute compressive stress and strain, respectively. By convention, a compressive force is taken to be negative, which yields a negative stress. The strain is positive if the materials are stretched or negative if they are compressed.
Tensile tests are more common because they are easier to perform; also, for most materials used in structural applications, very little additional information is obtained from compressive tests. Compressive tests are used when the material is brittle in tension.

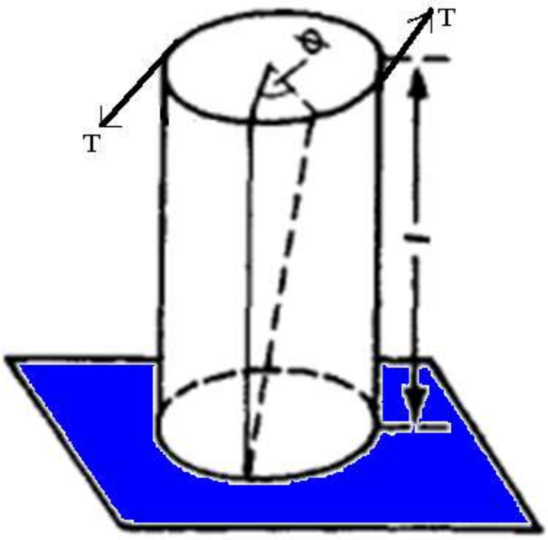
Shear and Torsional Tests:

For tests performed using a pure shear force as shown in Figure below, the shear stress is computed according to

$$\tau = \frac{F}{A_O} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$



Where *F* is the load or force imposed parallel to the upper and lower faces, each of which has an area of A_o. The shear strain γ is defined as the tangent of the strain angle , as indicated in the figure. The unit for shear stress is Pascal.
Torsion is a variation of pure shear, wherein a structural member is twisted in the manner of Figure below; torsional forces produce a rotational motion about the longitudinal axis of one end of the member relative to the other end. Examples of torsion are found for machine axles and drive shafts, and also for twist drills. Torsional tests are normally performed on cylindrical solid shafts or tubes. A shear stress is a function of the applied torque *T*, whereas shear strain is related to the angle of twist, in Figure below.



Geometric Considerations of the Stress State:

Consider the cylindrical tensile specimen of Figure below that is subjected to a tensile stress σ applied parallel to its axis. Furthermore, consider also the plane $p - p'$ that is oriented at some arbitrary angle relative to the plane of the specimen end-face. Upon this plane $p - p'$, the applied stress is no longer a pure tensile one. Rather, a more complex stress state is present that consists of a tensile (or normal) stress σ' that acts normal to the $p - p'$ plane and, in addition, a shear stress τ' that acts parallel to this plane; both of these stresses are represented in the figure below. Using mechanics of materials principles, it is possible to develop equations for σ' and τ' in terms of σ and θ, as follows:

$$\sigma' = \sigma cos^2\theta = \sigma\left(\frac{1+\cos 2\theta}{2}\right) \text{........... (8 a)}$$

$$\tau' = \sigma \sin\theta \cos\theta = \sigma\left(\frac{sin 2\theta}{2}\right) \text{......... (8 b)}$$

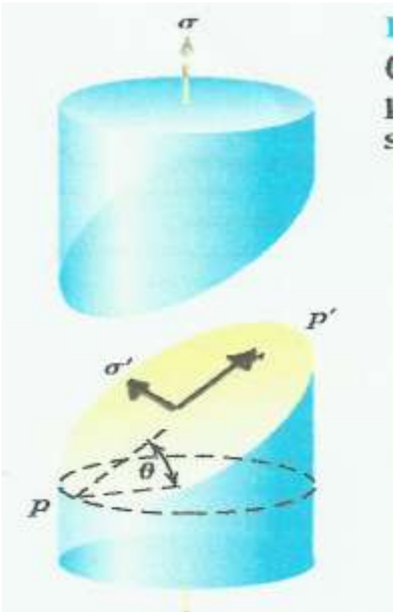


Figure Schematic representation showing normal (σ') and shear (τ') stresses that act on a plane oriented at an angle θ relative to the plane taken perpendicular to the direction along which a pure tensile stress (σ) is applied.

Elastic Deformation
Stress-Strain Behavior

The degree to which a structure deforms or strains depends on the magnitude of an imposed stress. For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship:

$\boldsymbol{\sigma = E \in}$ **…………………..** **(9) [**Hooke's law— relationship between engineering stress and engineering strain for elastic deformation (tension and compression)]

Note: **Hooke's Law** describes only the initial linear portion of the stress-strain curve for a bar subjected to uniaxial extension.

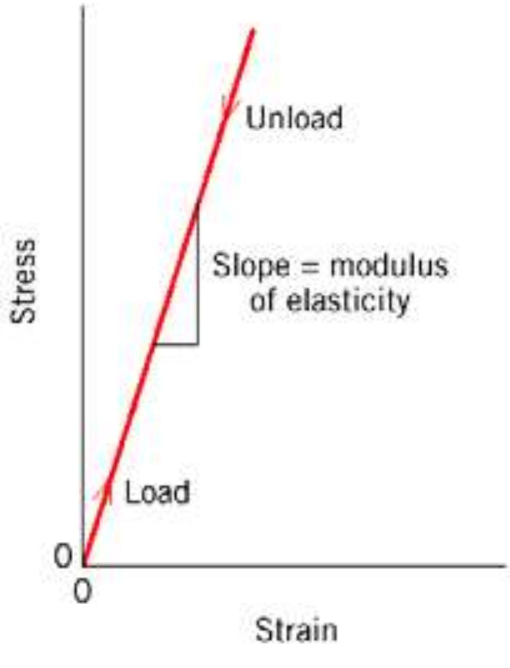
The constant of proportionality *E* (GPa or psi), The slope of the straight-line portion of the stress-strain diagram is called the **Modulus of Elasticity** or **Young's Modulus**.

The SI unit for the modulus of elasticity is gigapascal, GPa, where 1 GPa $N/m^2 = 10^3$ MPa. The moduli of elasticity are slightly higher for ceramic materials, Polymers have modulus values that are smaller than both metals and ceramics.

Room temperature modulus of elasticity values for a number of metals, ceramics, and polymers are presented in Table below.

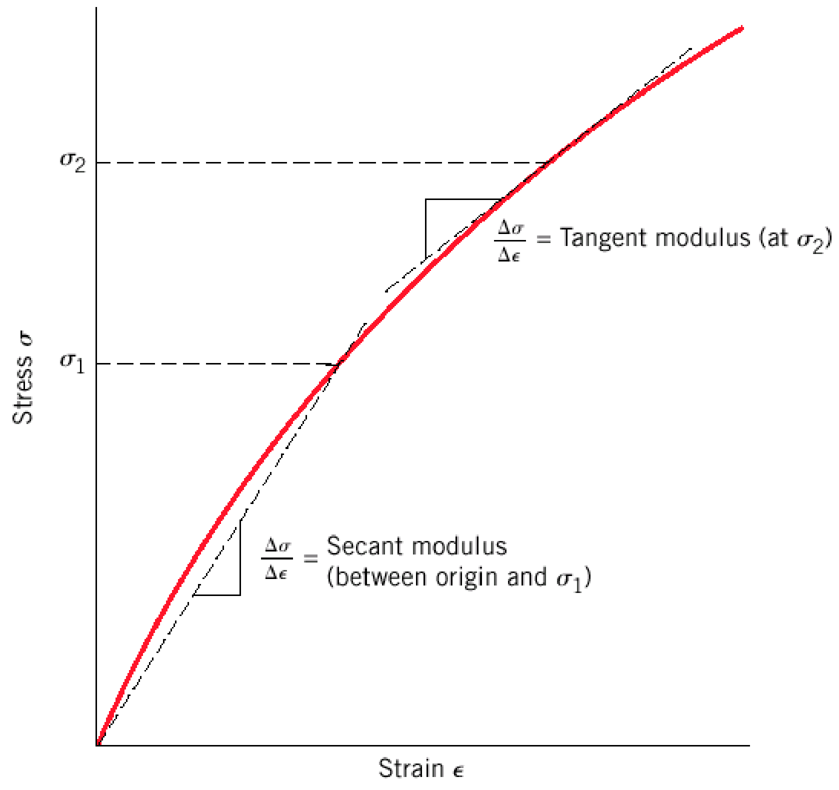
A plot of stress (ordinate) versus strain (abscissa) results in a linear relationship, as shown in Figure below. Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape. As shown in the stress–strain plot.

Schematic stress–strain diagram showing linear elastic deformation for loading and unloading cycles.



There are some materials (e.g., gray cast iron, concrete, and many polymers) for which this elastic portion of the stress–strain curve is not linear (Figure below); hence, it is not possible to determine a modulus of elasticity as described above. For this nonlinear behavior, either ***tangent*** **or** ***secant modulus*** is normally used. **Tangent modulus** is taken as the slope of the stress–strain curve at some specified level of stress, while **secant modulus** represents the slope of a secant drawn from the origin to some given point of the stress– strain curve. The determination of these moduli is illustrated in Figure below

.
Schematic stress–strain diagram showing non-linear elastic behavior, and how secant and tangent moduli are determined.



The stress–strain characteristics at low stress levels are virtually the same for both tensile and compressive situations, to include the magnitude of the modulus of elasticity. Shear stress and strain are proportional to each other through the expression
(

$G = \tau / \gamma$ (shear stress – strain) [Relationship between shear stress and shear strain for elastic deformation]
Where G is the ***shear modulus or Modulus of Rigidity***, the slope of the linear elastic r egion of the shear stress–strain curve.

Anelasticity:

So far we have assumed that elastic deformation is time independent (i.e. applied stress produces instantaneous elastic strain)

However, in reality elastic deformation takes time (finite rate of atomic/molecular deformation processes) - continues after initial loading, and after load release. This time dependent elastic behavior is known as **anelasticity**.

The effect is normally small for metals but can be significant for polymers (“visco-elastic behavior”).

Example: **Elongation (Elastic) Computation**

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). The magnitude of *E* for copper is 110 GPa ($16X10^6$ psi). If the deformation is entirely elastic, what will be the resultant elongation?

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right)E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

$$\Delta l = \frac{(276\text{ MPa})(305\text{ mm})}{110 \times 10^3\text{ MPa}} = 0.77\text{ mm } (0.03\text{ in.})$$

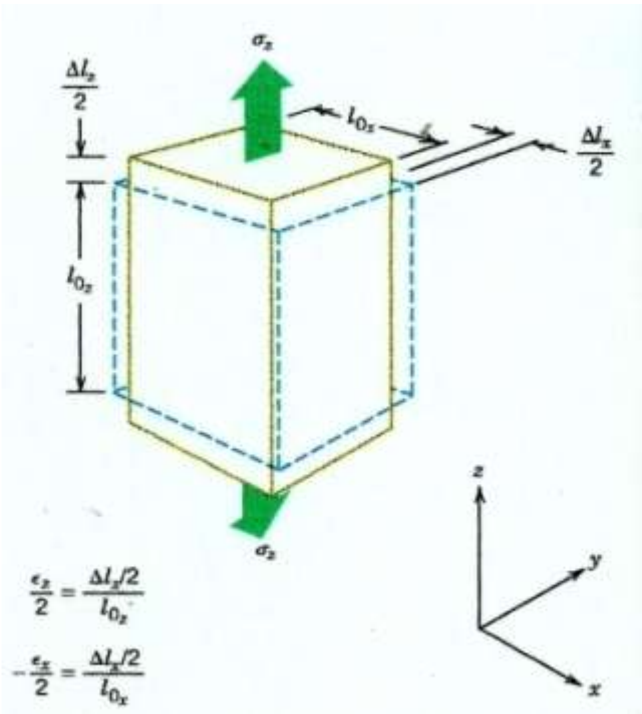
Elastic Deformation: Poisson's ratio

When a tensile stress is imposed on a metal specimen, an elastic elongation and accompanying strainϵ_z result in the direction of the applied stress (arbitrarily taken to be the *z* direction), as indicated in Figure below. As a result of this elongation, there will be constrictions in the lateral (*x* and *y*) directions perpendicular to the applied stress. If the applied stress is uniaxial (only in the *z* direction), and the material is isotropic, then$\epsilon_x = \epsilon_y$. A parameter termed **Poisson's ratio (ν)** is defined as the ratio of the lateral and axial strains, or

$\nu = \varepsilon_{lateral}/\varepsilon_{axial}$

$$\upsilon = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \quad \ldots\ldots\ldots\ldots\ldots \ (10)$$

υ is dimensionless .The negative sign is included in the expression so that will always be positive, since ϵ_x and $\epsilon_z$will always be of opposite sign or (sign shows that lateral strain is in opposite sense to longitudinal strain). Theoretically, Poisson's ratio for isotropic materials should be 0.25 and maximum value: 0.50, Typical value: 0.24 - 0.30.



For isotropic materials, shear and elastic moduli are related to each other and to Poisson's ratio according to

E = 2G (1+υ**)** → In most metals G ~ 0.4E

G is Shear Modulus (Units: N/m^2)

(Note: most materials are elastically anisotropic: the elastic behavior varies with crystallographic direction)
For these materials the elastic properties are completely characterized only by the specification of several elastic constants, their number depending on characteristics of the crystal structure. Even for isotropic materials, for complete characterization of the elastic properties, at least two constants must be given. Since the grain orientation is random in most polycrystalline materials, these may be considered to be isotropic; inorganic ceramic glasses are also isotropic. The remaining discussion of mechanical behavior assumes isotropy and polycrystallinity because such is the character of most engineering materials.

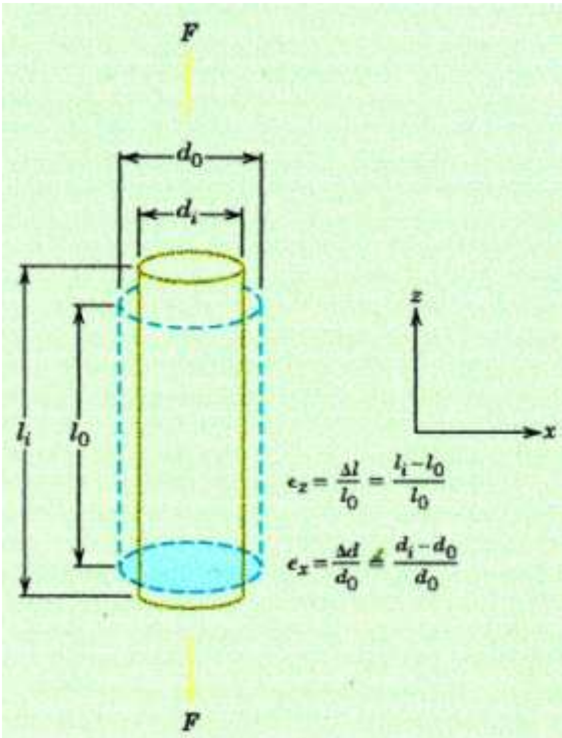
Example: Computation of Load to Produce Specified Diameter Change

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5x10^{-3} mm (10^{-4} in.) change in diameter if the deformation is entirely elastic. The value for Poisson's ratio for brass is 0.34

When the force *F* is applied, the specimen will elongate in the *z* direction and at the same time experience a reduction in diameter,Δd of 2.5x10^{-3} mm in the *x* direction. For the strain in the *x* direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3}\ \text{mm}}{10\ \text{mm}} = -2.5 \times 10^{-4}$$

Which is negative, since the diameter is reduced



It next becomes necessary to calculate the strain in the z direction
The value for Poisson's ratio for brass is 0.34

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3 \text{ MPa}) = 71.3 \text{ MPa}$$

the applied force may be determined as

$$F = \sigma A_0 = \sigma\left(\frac{d_0}{2}\right)^2 \pi$$

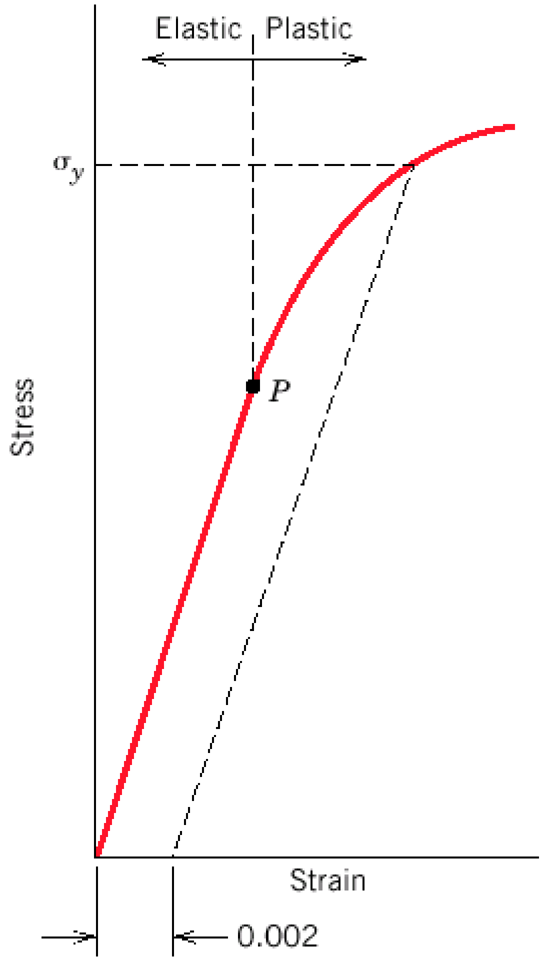
$$= (71.3 \times 10^6 \text{ N/m}^2)\left(\frac{10 \times 10^{-3} \text{ m}}{2}\right)^2 \pi = 5600 \text{ N} (1293 \text{ lb}_f)$$

Plastic deformation:
- Stress and strain are not proportional
- The deformation is not reversible
- Deformation occurs by breaking and re-arrangement of atomic bonds (in crystalline materials primarily by motion of dislocations)

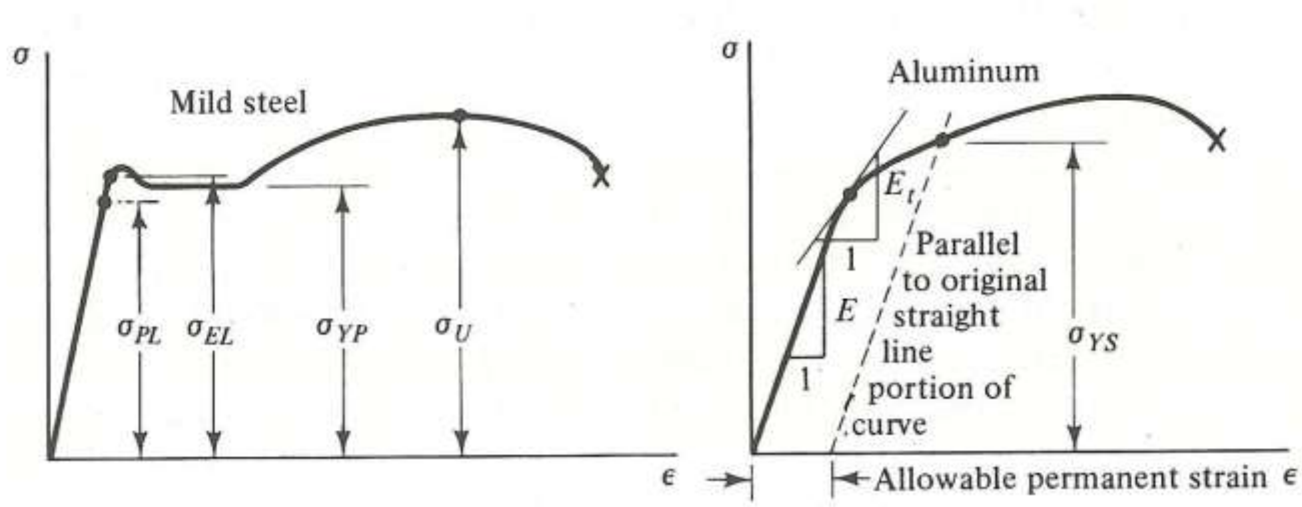
Yield strength σ_y - is chosen as that causing a permanent strain of 0.002

Yield point P - the strain deviates from being proportional to the stress (the proportional limit)

The yield stress *is a measure of resistance to plastic deformation*



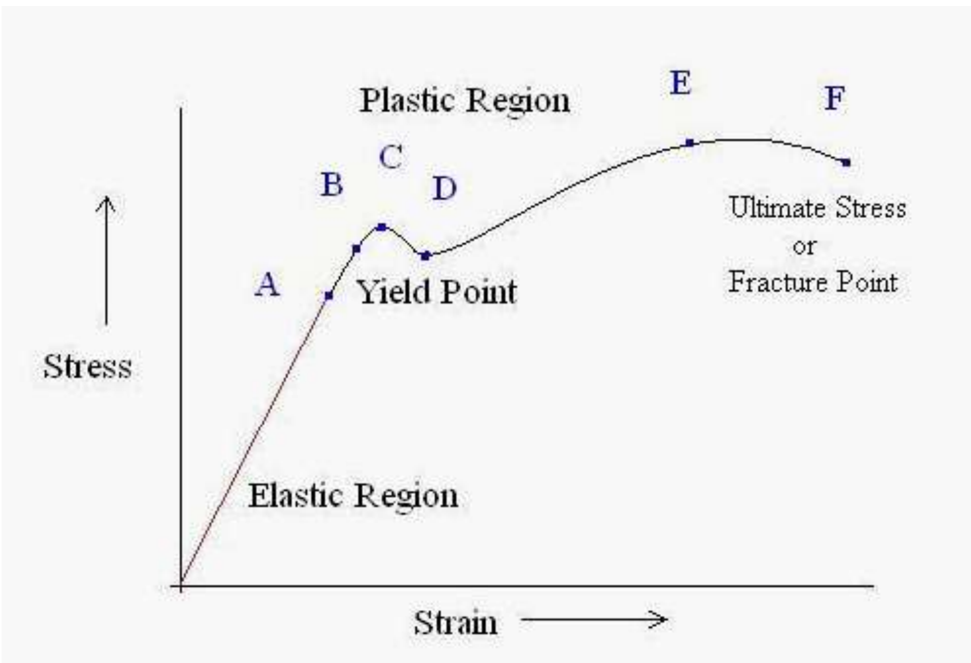
And in more details we can classify a stress as shown in figure below.



σ_{PL} (**Proportional Limit)** - Stress above which stress is not longer proportional to strain.
σ_{EL} (**Elastic Limit)** - The maximum stress that can be applied without resulting in permanent deformation when unloaded.
σ_{YP} (**Yield Point)** - Stress at which there are large increases in strain with little or no increase in stress. Among common structural materials, only steel exhibits this type of response.
σ_{YS} (**Yield Strength)** - The maximum stress that can be applied without exceeding a specified value of permanent strain (typically .2% = .002 in/in).
σ_{U} (**Ultimate Strength)** - The maximum stress the material can withstand (based on the original area).

Tensile Properties:

A typical stress-strain curve is shown in Figure below. If we begin from the origin and follow the graph numbers of points are indicated.



Point A: At origin, there is no initial stress or strain in the test piece. Up to point A Hooke's Law is obeyed according to which stress is directly proportional to strain. That's why the point A is also known as proportional limit. This straight line region is known as elastic region and the material can regain its original shape after removal of load.

Point B: The portion of the curve between AB is not a straight line and strain increases faster than stress at all points on the curve beyond point A. Point B is the point after which any continuous stress results in permanent, or inelastic deformation. Thus, point B is known as the elastic limit or yield point.

Point C & D: Beyond the point B, the material goes to the plastic stage till the point C is reached. At this point the cross- sectional area of the material starts decreasing and the stress decreases to point D. At point D the workpiece changes its length with a little or without any increase in stress up to point E.

Point E: Point E indicates the location of the value of the ultimate stress. The portion DE is called the yielding of the material at constant stress. From point E onwards, the strength of the material increases and requires more stress for deformation, until point F is reached. The **tensile strength** (MPa or psi) is the stress at the maximum on the engineering stress–strain curve**.**

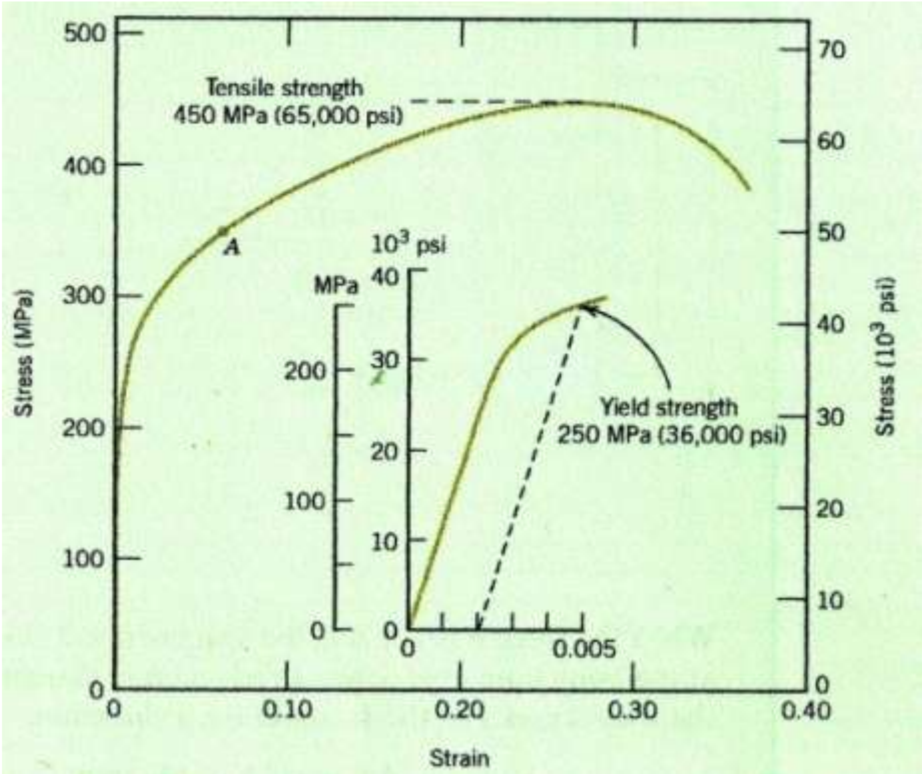
Point F: A material is considered to have completely failed once it reaches the ultimate stress. The point of fracture, or the actual tearing of the material, does not occur until point F. The point F is also called ultimate point or fracture point.

Example: Mechanical Property Determinations from Stress–Strain Plot
From the tensile stress–strain behavior for the brass specimen shown in Figure below, determine the following:
(a) The modulus of elasticity
(b) The yield strength at a strain offset of 0.002
(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm (0.505 in.)
(d) The change in length of a specimen originally 250 mm (10 in.) long that is subjected to a tensile stress of 345 MPa (50,000 psi)

Solution

(a) The modulus of elasticity is the slope of the elastic or initial linear portion of the stress–strain curve. The strain axis has been expanded in the inset, Figure below, to facilitate this computation. The slope of this linear region is the rise over the run, or the change in stress divided by the corresponding change in strain; in mathematical terms,

$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1}$$



Inasmuch as the line segment passes through the origin, it is convenient to take both σ_1 and ϵ_1 as zero. If σ_2 is arbitrarily taken as 150 MPa, then ϵ_2 will have a value of 0.0016. Therefore,

$E = \frac{(150-0)MPa}{0.0016-0} = 93.8\ GPa$ (13.6 x 10^6 psi)
which is very close to the value of 97 GPa (psi) given for brass in tables.

(b) : The 0.002 strain offset line is constructed as shown in the inset; its intersection with the stress–strain curve is at approximately 250 MPa (36,000 psi), which is the yield strength of the brass.

(c) : The maximum load that can be sustained by the specimen is calculated by using Equation 6.1, in which σ is taken to be the tensile strength, from the above Figure, 450 MPa (65,000 psi). Solving for *F*, the maximum load, yields

$$F = \sigma A_O = \sigma \left(\frac{d_o}{2}\right)^2 \pi$$

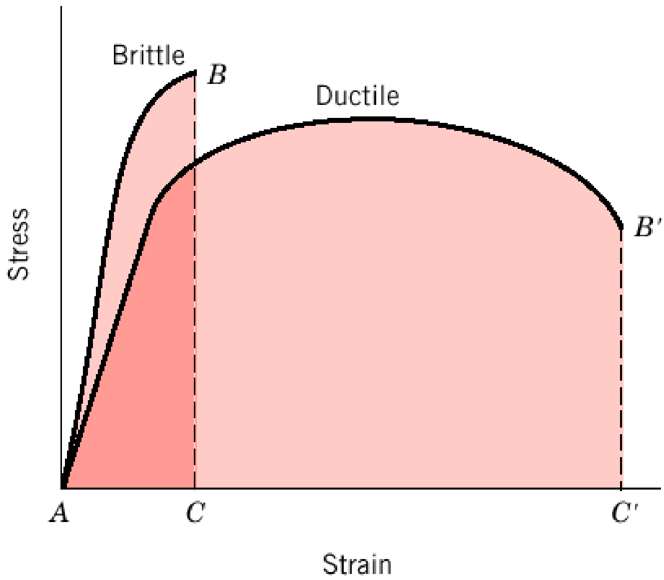
$F = \left(450x10^6\ {N}/{m^2}\right)\left(\frac{12.8x10^{-3}m}{2}\right)^2 \pi$ = 57.9 N (13 Ib_f)

(d) : To compute the change in length, ΔL, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accomplished by locating the stress point on the stress–strain curve, point *A*, and reading the corresponding strain from the strain axis, which is approximately 0.06.
Inasmuch as l_o = 250 mm, we have
$\Delta l = \epsilon l_o$= (0.06)(250 mm)= 15 mm (0.6 in.)

Ductility:

Ductility is another important mechanical property. *It is a measure of the degree of plastic deformation at fracture*. A material that experiences very little or no plastic deformation upon fracture is termed *brittle*. The tensile stress–strain behaviors for both ductile and brittle materials are schematically illustrated in Figure below.

Ductility may be expressed quantitatively as either *percent elongation* or *percent reduction in area*. The percent elongation %EL is the percentage of plastic strain at fracture, or



% EL= $\left(\frac{l_f - l_o}{l_0}\right)$x 100 or

%EL = ε_{max} x 100 %

Where l_f is the fracture length and l_o is the original gauge length

Ductility defined by percent reduction in area

%RA= $\left(\frac{A_0 - A_f}{A_0}\right) X\ 100$

Where A_0 is the original cross-sectional area and A_f is the cross-sectional area at the point of fracture. Both l_f and A_f are measured subsequent to fracture and after the two broken ends have been repositioned back together.

- Percent reduction in area values are independent of both l_o and A_0
- Most metals possess at least a moderate degree of ductility at room temperature;
- some become brittle as the temperature is lowered.
- Knowledge of the ductility of materials is important for at least two reasons.
  - ✓ First, it indicates to a designer the degree to which a structure will deform plastically before fracture.
  - ✓ Second, it specifies the degree of allowable deformation during fabrication operations

- Brittle materials are *approximately* considered to be those having a fracture strain of less than about 5%.
- Table below presents some typical values of mechanical properties of several metals and alloys **at** room-temperature.
- The yield strength and tensile strength vary with prior thermal and mechanical treatment, impurity levels, etc. This variability is related to the behavior of dislocations in the material.
- But elastic moduli are relatively insensitive to these effects.
- The yield and tensile strengths and modulus of elasticity decrease with increasing temperature.
- Ductility increases with temperature.

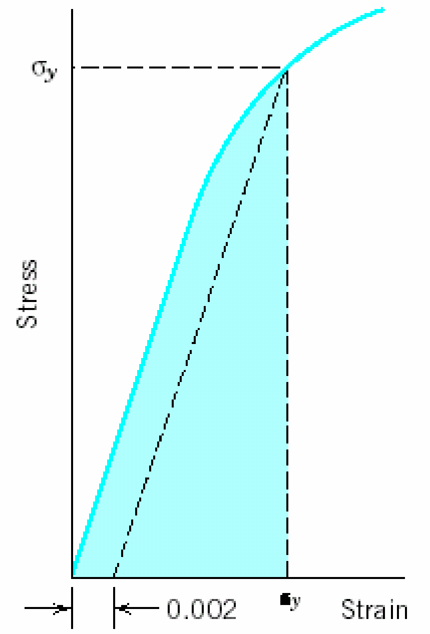
Metal Alloy	Yield Strength MPa (ksi)	Tensile Strength MPa (ksi)	Ductility, %EL [in 50 mm (2 in.)]
Aluminum	35 (5)	90 (13)	40
Copper	69 (10)	200 (29)	45
Brass (70Cu–30Zn)	75 (11)	300 (44)	68
Iron	130 (19)	262 (38)	45
Nickel	138 (20)	480 (70)	40
Steel (1020)	180 (26)	380 (55)	25
Titanium	450 (65)	520 (75)	25
Molybdenum	565 (82)	655 (95)	35

Resilience:

Is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.

Modulus of resilience (Ur): This is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding. (Area under the elastic line)

$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon$$



Ur = ½ σ_y ε_y

Assuming a linear elastic region,

$$U_r = \frac{1}{2}\sigma_y \epsilon_y$$

From Hooks law

$$\epsilon_y = {}^{\sigma}/_{E}$$

$$U_r = {}^{\sigma_y^2}/_{2E}$$

(units should represent energy: Joule/m^3)

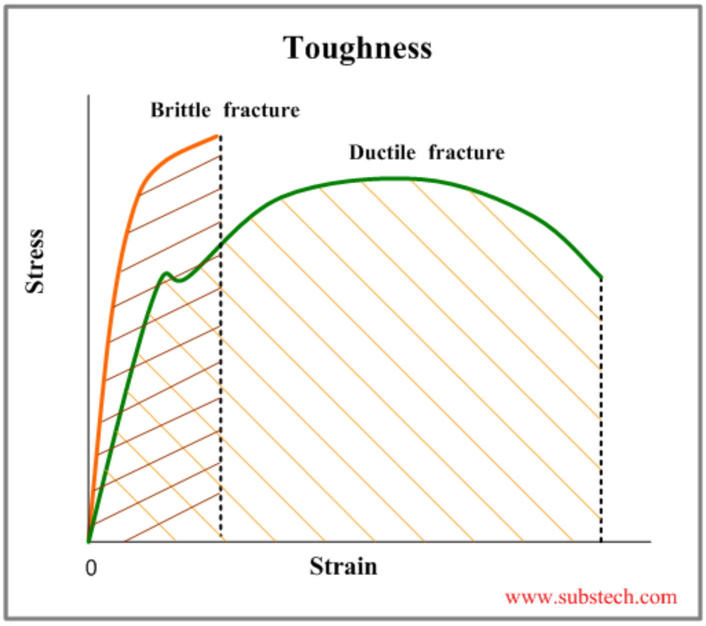
Units: MPa= N/m^2x 10^6N/m^2x m/m =J/m^3

Thus, resilient materials are those having high yield strengths and low moduli of elasticity; such alloys would be used in spring applications.

Toughness:

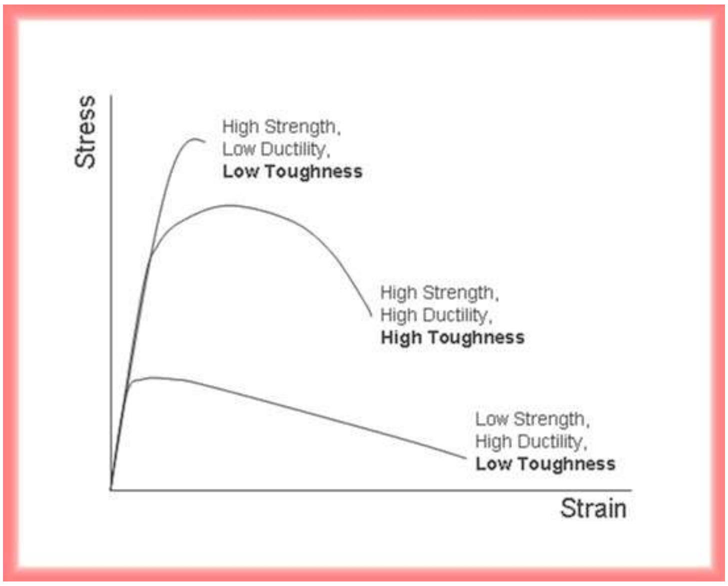
Is a measure of the ability of a material to absorb energy up to fracture.

An approximation for the U_t is the area under the curve. Where U_t is a modulus of toughness



It depends on both strength and ductility of the material in question. A neighbor figure is given to show the relationship between the stress and strain for three materials.

From the figure, it can be concluded that tensile toughness is the area under the stress - strain curve. It is high if a material has high amount of strength and ductility. Materials with low ductility of low strength don't posses ample tensile toughness.



The word toughness is usually used for tensile toughness. In tensile toughness, the strain rate is relatively slow.

True stress and strain:

σ_T= F / A_i , load divided by the instantaneous cross section area

ε_T= ln(li/ lo), li: instantaneous length ,   lo: original length

For plastic deformation (σ>σy) there is conservation of volume: Aolo= Aili   Ao/ Ai = li/lo

Relations between true and engineering stress and strain:

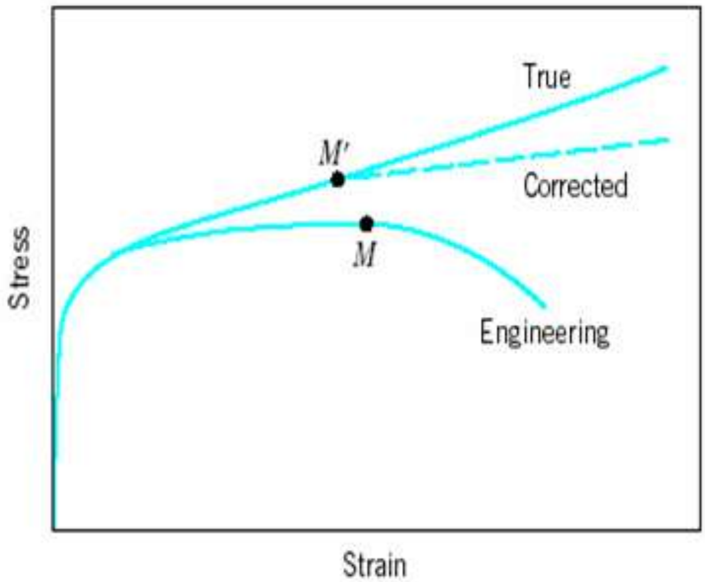
σ_T= F / Ai = F / Ao x Ao/ Ai= σ x Ao/Ai = σ x li / lo

ε= li-lo / lo = li/lo –1

li/lo = 1+ ε

Thus σ_T= σ(1+ ε)This equation is valid from yielding to the on set of necking $\sigma y < \sigma < \sigma u$

ε_T= ln (li/ lo) = ln(1+ε), This equation is valid from yielding to the on set of necking $\sigma y < \sigma < \sigma u$



A comparison of typical tensile engineering stress–strain and true stress–strain behaviors. Necking begins at point M on the engineering curve, which corresponds to M' on the true curve. The "corrected" true stress–strain curve takes into account the complex stress state within the neck region.

For some metals and alloys the region of the true stress –strain curve from the onset of plastic deformation to the point at which necking begins ($\sigma y < \sigma < \sigma u$) may be approximated by:

$\sigma_T = K(\epsilon_T)^n$

n is hardening exponent = 0.15 for some steels

= 0.5 for some copper

ϵ_T is true strain: ln(l_i /l_o)

$\sigma_T$is true stress (F / A)

K is a strength coefficient

K and n are constants that vary from alloy to alloy. and will also depend on the condition of the material (i.e., whether it has been plastically deformed, heat treated, etc.). The parameter n is often has a value less than unity

Taking logarithm of both sides yields a straight line:

$\text{Log } \sigma_T = n \log \varepsilon_T + \log K$
(y=mx+c)
n (strain hardening exponent) defines the slope of the straight line

Example:

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine:

(a) The ductility in terms of percent reduction in area

(b) The true stress at fracture

SOLUTION

(a) Ductility is computed

$$\%\text{RA} = \frac{\left(\dfrac{12.8\text{ mm}}{2}\right)^2\pi - \left(\dfrac{10.7\text{ mm}}{2}\right)^2\pi}{\left(\dfrac{12.8\text{ mm}}{2}\right)^2\pi} \times 100$$

$$= \frac{128.7\text{ mm}^2 - 89.9\text{ mm}^2}{128.7\text{ mm}^2} \times 100 = 30\%$$

(b) The area is taken as the fracture area Af. However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6\text{ N/m}^2)(128.7\text{ mm}^2)\left(\frac{1\text{ m}^2}{10^6\text{ mm}^2}\right) = 59{,}200\text{ N}$$

Thus, the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{59{,}200\text{ N}}{(89.9\text{ mm}^2)\left(\dfrac{1\text{ m}^2}{10^6\text{ mm}^2}\right)}$$

$$= 6.6 \times 10^8\text{ N/m}^2 = 660\text{ MPa (95,700 psi)}$$

Example:

Compute the strain-hardening exponent n for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

$$n = \frac{\log \sigma_T - \log K}{\log \epsilon_T}$$

$$= \frac{\log(415\text{ MPa}) - \log(1035\text{ MPa})}{\log(0.1)} = 0.40$$

Example:

For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strain, prior to necking:

Engineering Stress (MPa)	Engineering Strain
315	0.105
340	0.220

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.28.

For this problem we first need to convert engineering stresses and strains to true stresses and strains so that the constants **K** and **n** may be determined. Since $\boldsymbol{\sigma_T} = \boldsymbol{\sigma}(1 + \boldsymbol{\varepsilon})$ then

$$\sigma_{T_1} = (315 \text{ MPa})(1 + 0.105) = 348 \text{ MPa}$$

$$\sigma_{T_2} = (340 \text{ MPa})(1 + 0.220) = 415 \text{ MPa}$$

Similarly for strains, since $\boldsymbol{\varepsilon_T} = \ln(1 + \boldsymbol{\varepsilon})$ then

$$\varepsilon_{T_1} = \ln(1 + 0.105) = 0.09985$$

$$\varepsilon_{T_2} = \ln(1 + 0.220) = 0.19885$$

$$\log \sigma_T = \log K + n \log \varepsilon_T$$

which allows us to set up two simultaneous equations for the above pairs of true stresses and true strains, with **K** and **n** as unknowns. Thus

$$\log(348) = \log K + n \log(0.09985)$$

$$\log(415) = \log K + n \log(0.19885)$$

Solving for these two expressions yields **K** = 628 MPa and **n** = 0.256.

Now, converting $\boldsymbol{\varepsilon} = 0.28$ to true strain $\quad \varepsilon_T = \ln(1 + 0.28) = 0.247$

The corresponding $\boldsymbol{\sigma_T}$ to give this value of $\boldsymbol{\varepsilon_T}$

$$\sigma_T = K\varepsilon_T^n = (628 \text{ MPa})(0.247)^{0.256} = 439 \text{ MPa}$$

Now converting this $\boldsymbol{\sigma_T}$ to an engineering stress

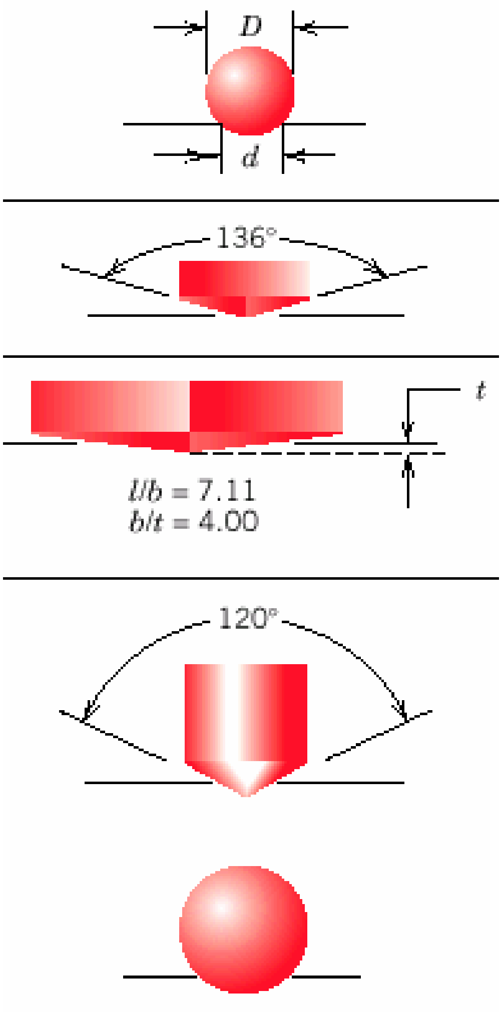
$$\sigma = \frac{\sigma_T}{1 + \varepsilon} = \frac{439 \text{ MPa}}{1 + 0.28} = 343 \text{ MPa}$$

Hardness:

Hardness is a measure of the material's resistance to localized plastic deformation (e.g. dent or scratch) A qualitative Moh's scale, determined by the ability of a material to scratch another material: from 1 (softest = talc) to 10 (hardest = diamond).

Diamond	10
Corundum	9
Topaz	8
Quartz	7
Orthoclase (Feldspar)	6
Apatite	5
Fluorite	4
Calcite	3
Gypsum	2
Talc	1

Different types of quantitative hardness test have been designed (Rockwell, Brinell, Vickers, etc.). Usually a small indenter (sphere, cone, or pyramid) is forced into the surface of a material under conditions of controlled magnitude and rate of loading. The depth or size of indentation is measured. The tests somewhat approximate, but popular because they are easy and non-destructive (except for the small dent).



Quantitative hardness techniques have been developed over the years in which a small indenter is forced into the surface of a material to be tested, under controlled conditions of load and rate of application. The depth or size of the resulting indentation is measured, which in turn is related to a hardness number; the softer the material, the larger and deeper is the indentation, and the lower the hardness index number.
Hardness tests are performed more frequently than any other mechanical test for several reasons:
1. They are simple and inexpensive—ordinarily no special specimen need be prepared, and the testing apparatus is relatively inexpensive.
2. The test is nondestructive—the specimen is neither fractured nor excessively deformed; a small indentation is the only deformation.
3. Other mechanical properties often may be estimated from hardness data, such as tensile strength

Rockwell Hardness Test:

Stanley P. Rockwell invented the Rockwell hardness test. He was a metallurgist for a large ball bearing company and he wanted a fast non-destructive way to determine if the heat treatment process they were doing on the bearing races was successful. The only hardness tests he had available at time were Vickers, Brinell and Scleroscope. The Vickers test was too time consuming, Brinell indents were too big for his parts and the Scleroscope was difficult to use, especially on his small parts.

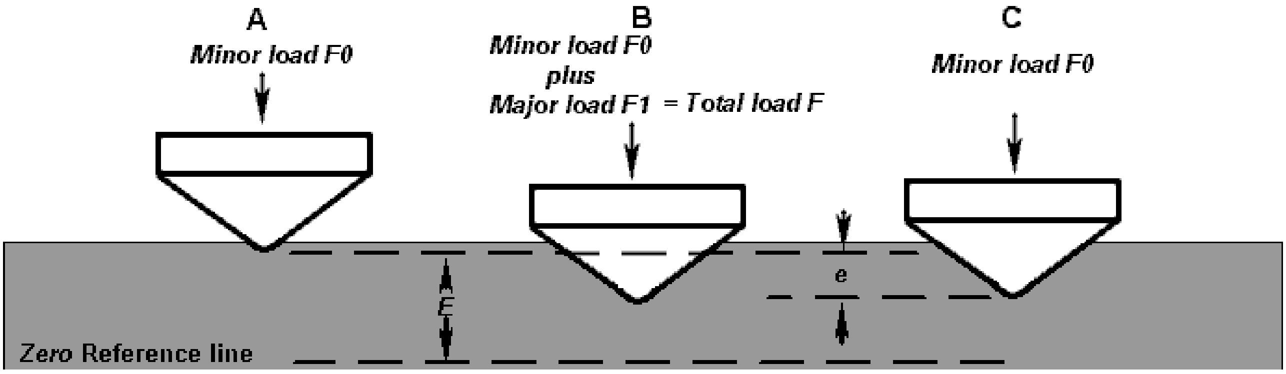
To satisfy his needs he invented the Rockwell test method. This simple sequence of test force application proved to be a major advance in the world of hardness testing. It enabled the user to perform an accurate hardness test on a variety of sized parts in just a few seconds.

The Rockwell tests constitute the most common method used to measure hardness and generally accepted due to
1) Its speed
2) Freedom from personal error (require no special skills).
3) Ability to distinguish small hardnessdifference
4) Small size of indentation.
5) They are so simple to perform.
The Rockwell hardness test method consists of indenting the test material with a diamond cone or hardened steel ball indenter. The indenter is forced into the test material under a preliminary minor load *F0* (Fig. A) usually 10 kgf. When equilibrium has been reached, an indicating device, which follows the movements of the indenter and so responds to changes in depth of penetration of the indenter is set to a datum position. While the preliminary minor load is still applied an additional major load is applied with resulting increase in penetration (Fig. B). When equilibrium has again been reach, the additional major load is removed but the preliminary minor load is still maintained. Removal of the additional major load allows a partial recovery, so reducing the depth of penetration (Fig. C), (The hardness is measured according to the depth of indentation, under a constant load). The permanent increase in depth of penetration, resulting from the application and removal of the additional major load is used to calculate the Rockwell hardness number.

$HR = E - e$

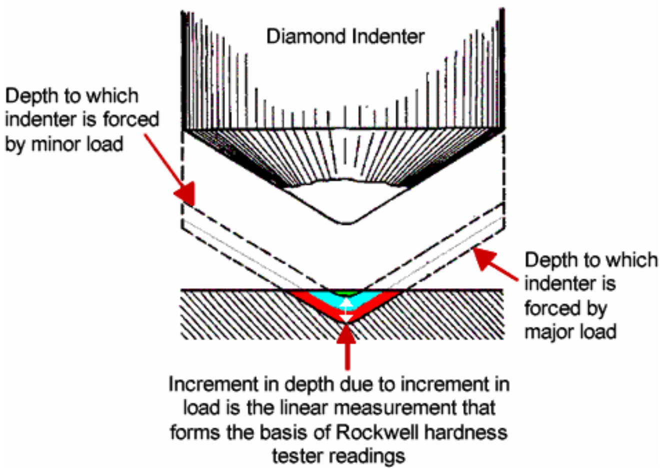
F0 = preliminary minor load in kgf
F1 = additional major load in kgf
F = total load in kgf
e = permanent increase in depth of penetration due to major load F1 measured in units of 0.002 mm
E = a constant depending on form of indenter: 100 units for diamond indenter, 130 units for steel ball indenter

HR = Rockwell hardness number
D = diameter of steel ball



Rockwell Principle

Principal of the Rockwell Test:
• Position the surface area to be measured close to the indenter.
• Applied the minor load and a zero reference position is established
• The major load is applied for a specified time period (dwell time) beyond zero
• The major load is released leaving the minor load applied.



- The Rockwell number represents the difference in depth from the zero reference position as a result of the applied major load.
- Deeper indentation means softer material

Types of the Rockwell Test:

There are two types of Rockwell tests:

1. **Rockwell**: the minor load is 10 kgf, the major load is 60, 100, or 150 kgf.
2. **Superficial Rockwell**: the minor load is 3 kgf and major loads are 15, 30, or 45 kgf.

In both tests, the indenter may be either a diamond cone or steel ball, depending upon the characteristics of the material being tested.

Rockwell hardness scale:

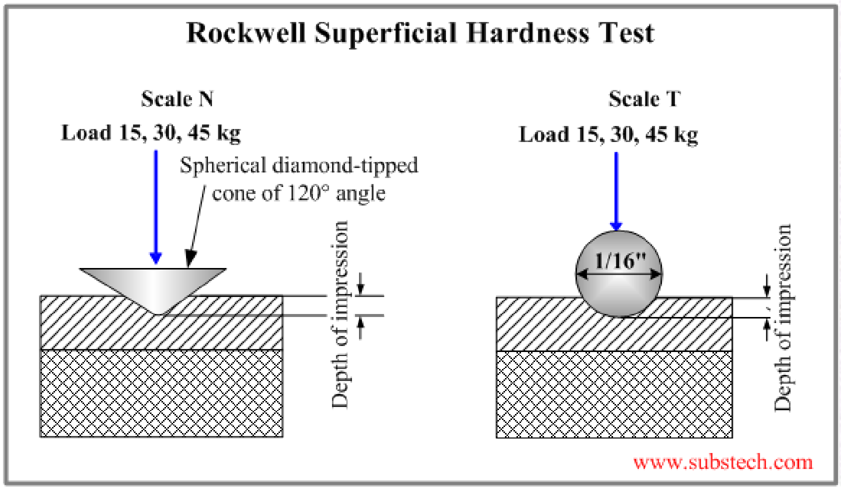
Rockwell hardness values are expressed as a combination of a hardness number and a scale symbol representing the indenter and the minor and major loads. The hardness number is expressed by the symbol HR and the scale designation.

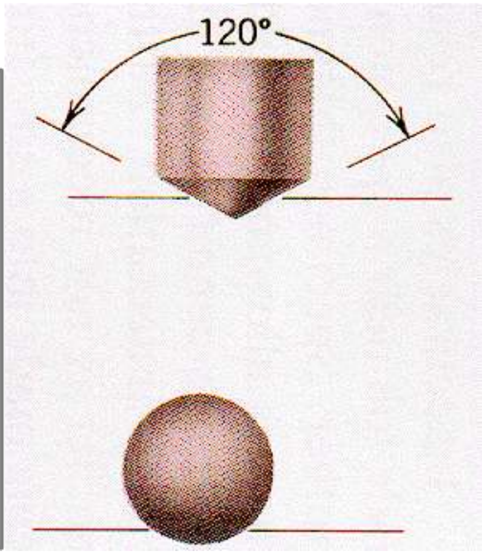
There are 30 different scales. The majority of applications are covered by the Rockwell C and B scales for testing steel, brass, and other metals. However, the increasing use of materials other than steel and brass as well as thin materials necessitates a basic knowledge of the factors that must be considered in choosing the correct scale to ensure an accurate Rockwell test. The choice is not only between the regular hardness test and superficial hardness test, with three different major loads for each, but also between the diamond indenter and the 1/16, 1/8, 1/4 and 1/2 in. diameter steel ball indenters.

If no specification exists or there is doubt about the suitability of the specified scale, an analysis should be made of the following factors that control scale selection:

- Type of material
- Specimen thickness
- Test location
- Scale limitations

• The Hardened steel is tested on the C scale with Rc20-70.
• Softer materials are tested on the B scale with Rb30-100.





Brale indenter,120^{o} diamond cone
1.6-3.2 mm diameter steel ball indenter

On the basis of the magnitude of both major and minor loads, there are two types of tests:

- Rockwell and superficial Rockwell. For Rockwell, the minor load is 10 kg, whereas major loads are 60, 100, and 150 kg.
- Each scale is represented by a letter of the alphabet;
- For superficial tests, 3 kg is the minor load; 15, 30, and 45 kg are the possible major load values.
- These scales are identified by a 15, 30, or 45 (according to load), followed by N, T,W, X, or Y, depending on indenter.
- Superficial tests are frequently performed on thin specimens.

Table of Rockwell Hardness Scales:

Scale	Indenter	Minor Load F0(kgf)	Major Load F1(kgf)	Total Load F(kgf)	Value of *E*
A	Diamond cone	10	50	60	100
B	1/16" steel ball	10	90	100	130
C	Diamond cone	10	140	150	100
D	Diamond cone	10	90	100	100
E	1/8" steel ball	10	90	100	130
F	1/16" steel ball	10	50	60	130
G	1/16" steel ball	10	140	150	130
H	1/8" steel ball	10	50	60	130
K	1/8" steel ball	10	140	150	130
L	1/4" steel ball	10	50	60	130
M	1/4" steel ball	10	90	100	130
P	1/4" steel ball	10	140	150	130
R	1/2" steel ball	10	50	60	130
S	1/2" steel ball	10	90	100	130
V	1/2" steel ball	10	140	150	130

Superficial Rockwell Hardness Scales

Scale	Indenter Type	Minor Load F0(kgf)	Major Load F1(kgf)	Total Load F(kgf)	Value of E
HR 15 N	N Diamond cone	3	12	15	100
HR 30 N	N Diamond cone	3	27	30	100
HR 45 N	N Diamond cone	3	42	45	100
HR 15 T	1/16" steel ball	3	12	15	100
HR 30 T	1/16" steel ball	3	27	30	100
HR 45 T	1/16" steel ball	3	42	45	100
HR 15 W	1/8" steel ball	3	12	15	100
HR 30 W	1/8" steel ball	3	27	30	100
HR 45 W	1/8" steel ball	3	42	45	100
HR 15 X	1/4" steel ball	3	12	15	100
HR 30 X	1/4" steel ball	3	27	30	100
HR 45 X	1/4" steel ball	3	42	45	100
HR 15 Y	1/2" steel ball	3	12	15	100
HR 30 Y	1/2" steel ball	3	27	30	100
HR 45 Y	1/2" steel ball	3	42	45	100

The scale is designated by the symbol HRfollowed by the appropriate scale identification. For example, 80 HRB represents a Rockwell hardness of 80 on the B scale, and 60 HR30W indicates a superficial hardness of 60 on the 30W scale.

Scales properties:
i. All are dimensionless
ii. All have maximum reading of 130
iii. Become inaccurate below 20 or above 100

Rockwell Hardness Scales in brief:

a. ***Rockwell B:*** for unhardened carbon steels, copper, aluminum, malleable cast iron
i. 1/16" dia. steel ball
ii. 100 kg major load
iii. range: 0-100

b. ***Rockwell C:*** for steel, hardened steel, case hardened steel, pearlitic cast iron, titanium, others w/RB > 100
i. 120° diamond cone ("Brale") indenter
ii. 150 kg load
iii. range: 20-70

c. ***Rockwell A:*** for thinner or more brittle specimens of the RC family of materials: cemented carbides, thin case hardened parts, thin gauge steel
i. Brale indenter
ii. 60 kg load
iii. Extended useful range

d. Other standard tests: D, E, F, G, H, K, L, M, P, R, S, V

e. Superficial: N (Brale indenter), T (1/16" dia. steel ball), W(1/8" dia. steel ball), X, Y
i. Same indenters as standard tests
ii. 15, 30, or 45-kg major loads — *e.g.* 30N, 45T

Rockwell hardness instruction:

• Cleaned and well seated indenter and anvil.
• Surface which is clean and dry, smooth and free from oxide.
• Flat surface, which is perpendicular to the indenter.
• Cylindrical surface gives low readings, depending on the curvature.
• Thickness should be 10 times higher that the depth of the indenter.
• The spacing between the indentations should be 3 or 5 times the diameter of the indentation.

• Loading speed should be standardised

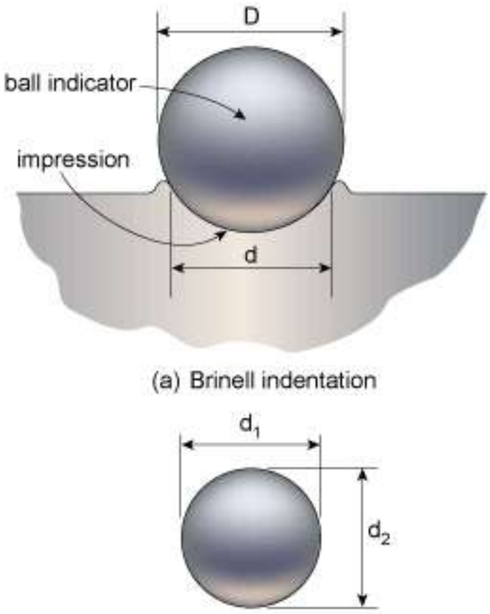
Brinell hardness:

J.A. Brinell introduced the first standardized indentation-hardness test in 1900. The Brinell hardness test consists in indenting the metal surface with a 10-mm diameter steel or tungsten carbide ball at a load range of 500-3000 kg, depending of hardness of particular materials.

The load is applied for a standard time (between 10 - 30 s), and the diameter of the indentation is measured. Giving an average value of two readings of the diameter of the indentation at right angle.

• The Brinell hardness number (BHN or HB) is expressed as the load P divided by surface area of the indentation.

$$\mathrm{BHN} = \frac{2P}{\pi D(D - \sqrt{(D^2 - d^2)})}$$



(a) Brinell indentation

(b) measurement of impression diameter

Where:

P: applied load (kgf)

D: diameter of ball (indenter) (mm)

d: diameter of indentation (mm)

The Brinell hardness number followed by the symbol HB denotes standard test conditions of 3000 kg, 10mm ball and 10-15 seconds loading.

Other testing conditions are suffixed as 'ball/kgs/load duration (secs)' after the number obtained and HB symbol (ie. 50.3 HB 10/500/30) for 10mm ball, 500 kg, 30 seconds load and an impression reading of 3.50mm.

Unit $kgf.mm^{-2}$ =9.8 MPa

Advantage and disadvantages of Brinell hardness test

• Large indentation averages out local heterogeneities of microstructure.

• Different loads are used to cover a wide range of hardness of commercial metals.

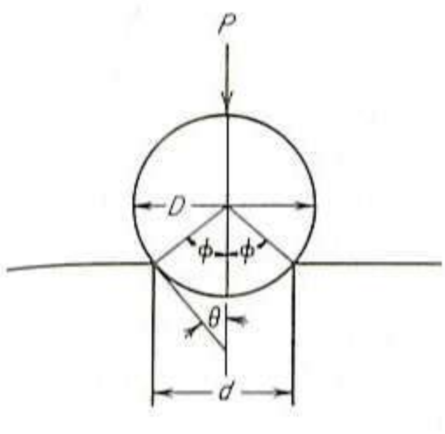
• Brinell hardness test is less influenced by surface scratches and roughness than other hardness tests.

• The test has limitations on small specimens or in critically stressed parts where indentation could be a possible site of failure

Brinell hardness test with nonstandard load or ball diameter:

• From figure below, d = Dsinφ, giving the alternative expression of Brinell hardness number as

$$BHN = \frac{P}{(\pi/2)D^2(1 - COS\varphi)}$$



• In order to obtain the same BHN with a non-standard load or ball diameter, it is necessary to produce a geometrical similar indentations.

• The included angle 2φ should remain constant and the load and the ball diameter must be varied in the ratio

$$\frac{P_1}{D_1^2} = \frac{P_2}{D_2^2} = \frac{P_3}{D_3^2}$$

Meyer hardness

• Meyer suggested that hardness should be expressed in terms of the mean pressure between the surface of the indenter and the indentation, which is equal to the load divided by the projected area of the indentation

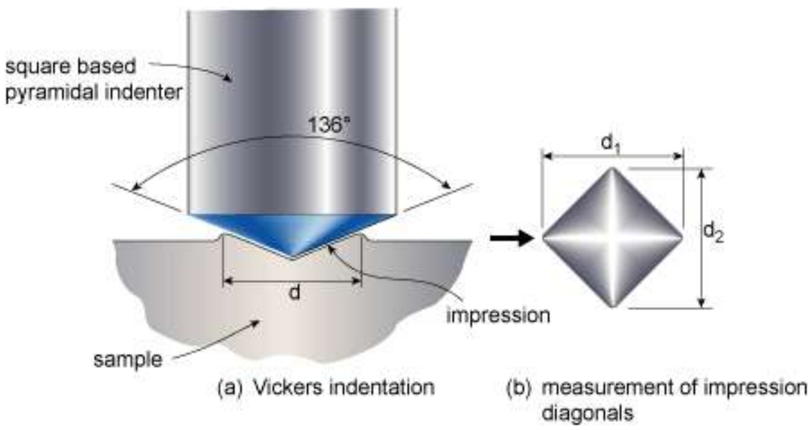
• Meyer hardness is therefore expressed as follows;

Note: - Meyer hardness is less sensitive to the applied load than Brinell hardness.

- Meyer hardness is a more fundamental measure of indentation hardness but it is rarely used for practical hardness measurement

Vickers hardness:

• Vickers hardness test uses a square-base diamond pyramid as the indenter with the included angle between opposite faces of the pyramid of 136°.
• The Vickers hardness number (VHN) is defined as the load divided by the surface area of the indentation



(a) Vickers indentation (b) measurement of impression diagonals

$$VHN = \frac{2P\sin(\theta/2)}{L^2} = \frac{1.854P}{L^2}$$

Where P is the applied load, kg

L is the average length of diagonals, mm

θ is the angle between opposite faces of diamond = 136^o.

Note: not widely used for routine check due to a slower process and requires careful surface preparation.

Note: the unite can be VHN, DPH, Hv

Vickers hardness test uses the loads ranging from 1-120 kgf, applied for between 10 and 15 seconds.

• Provide a fairly wide acceptance for research work because it provides a continuous scale of hardness, for a given load.

• VHN = 5-1,500 can be obtained at the same load level (easy for comparison).

Vickers hardness values of materials:

Materials	Hv
Tin	5
Aluminum	25
Gold	35
Copper	40
Iron	80
Mild steel	230
Full hard steel	1000
Tungsten carbide	2500

Knoop hardness:

1. *Microhardness* only
2. Elongated diamond indenter
   a. 172° 30' between long edges
   b. 130° 0' between short edges
3. Useful for…
   a. … *surfaces*
   b. … *elongated microconstituents*
   c. … *anisotropic properties* — only measure long dimension of indent, which can be reoriented w.r.t. texture (of polycrystalline sample) or crystallographic axes (of a single-crystal sample)
   3. Defined as *load/projected area of indentation* (in contrast to Vickers)

5. KHN ≡ $\frac{P}{A_P} = \frac{P}{CL^2}$

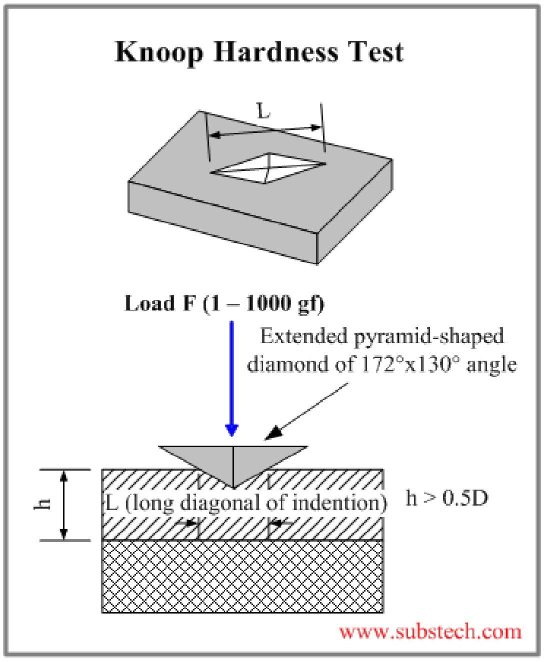
   P: applied load, kg

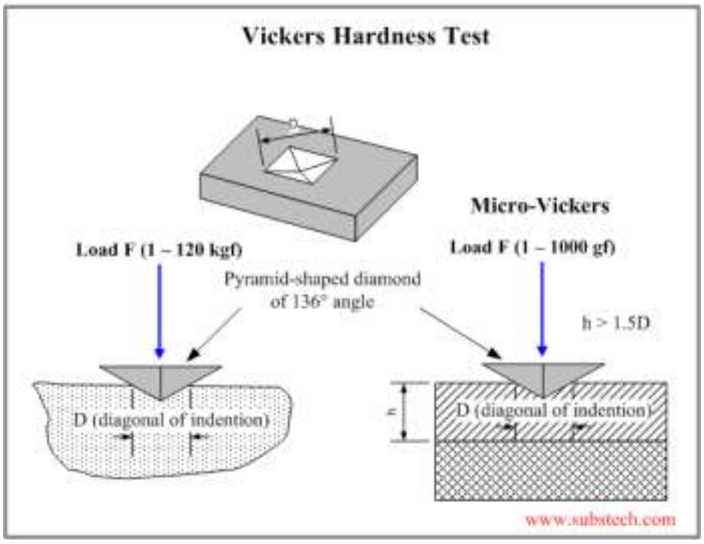
   L: length of long diagonal, mm

   C: constant relating projected area to length of long diagonal, 0.07028 for dimensions given Above
6. Applied loads are much smaller than for Rockwell and Brinell, ranging between 1 and 1000 g
7. Hardness scales for both techniques are approximately equivalent
8. Both are well suited for measuring the hardness of small, selected specimen regions
9. Knoop is used for testing brittle materials such as ceramics

Vickers and Knoop:

1. Slow
2. Sensitive to surface condition
    a. Requires polishing (through diamond or γ-Al_2O_3 step)
    b. Best unetched
3. Requires load to be normal to surface ⇒ plane parallel surfaces
4. Can be done on mounted specimens
5. Subject to error in diagonal measurement
6. Comparison
    a. Knoop — shallower indent ⇒ more surface sensitivity
    b. Vickers — smaller L ⇒ more prone to measurement errors





Shore Scleroscope Hardness Test:

- The Shore Scleroscope hardness is associated with the elasticity of the material.
- The appliance consists of a diamond-tipped hammer, which falls inside a graduated glass tube with the numbers starting from the bottom upward, under the force of its own weight (weighing less than an ounce) from a fixed height, onto the test specimen. The tube is divided into 140 equal parts.
- The hammer is brought to the top of the tube by a vacuum, and then, being released, falls on the part to be tested
- The height of the first rebound is the hardness index of the material.
- The harder the material, the higher the rebound.
- The Shore method is widely used for measuring hardness of large machine components like rolls, gears, dies, etc.
- Advantages of this method are small, mobile and non-marking of the test surface.

The Durometer:

- The Durometer is a popular instrument for measuring the indentation hardness of rubber and rubber-like materials. The most popular testers are the Model A used for measuring softer materials and the Model D for harder materials.
- The operation of the tester is quite simple. The material is subjected to a definite pressure applied by a calibrated spring to an indenter that is either a cone or sphere and an indicating device measures the depth of indentation.

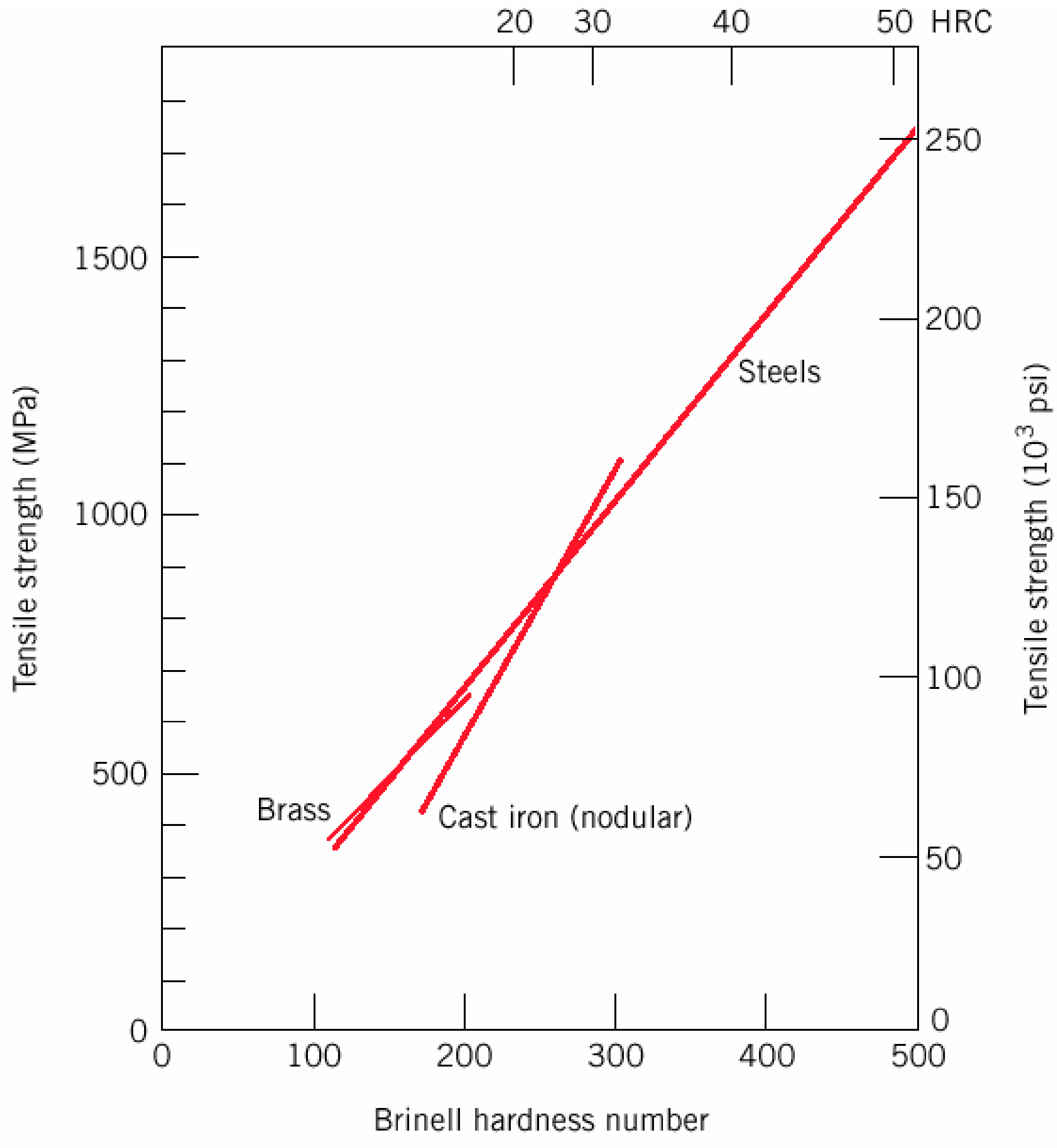
Microhardness:

- Microhardness testing specifies an allowable range of loads for testing with a diamond indenter.
- The resulting indentation is then recorded and converted to a hardness value. Typically loads are very light, ranging from a few grams to one or several kilograms.
- Since the test indentation is very small, microhardness testing is useful for a variety of applications such as testing very thin materials like foils or measuring individual microstructures.
- The procedure for testing is very similar to that of the standard Vickers hardness test, except that it is done on a microscopic scale with higher precision instruments.
- Precision microscopes are used to measure the indentations; these usually have a magnification of around X500 and measure to an accuracy of $\pm$0.5 micrometers.

Correlation Between Hardness and Tensile Strength:

Both tensile strength and hardness are indicators of a metal's resistance to plastic deformation. Consequently, they are roughly proportional, as shown in Figure below, for tensile strength as a function of the HB for cast iron, steel, and brass. The same proportionality relationship does not hold for all metals, as Figure below indicates. As a rule of thumb for most steels, the HB and the tensile strength are related according to:

TS(MPa) = 3.45x HB (For steel alloys, conversion of Brinell hardness to tensile strength)
TS(psi) = 500 x HB



Both tensile strength and hardness may be regarded as degree of resistance to plastic deformation. Hardness is proportional to the tensile strength – but note that the proportionality constant is different for different materials.

Design stress: $\sigma_d = N'\sigma_c$
where σ_c = maximum anticipated stress, N' is the "design factor" > 1.
Want to make sure that $\sigma_d < \sigma_y$
Safe or working stress: $\sigma_w = \sigma_y/N$ where N is "factor of safety" > 1.

EXAMPLE 1-1
Consider the maximum load on the structure is known with an uncertainity of ± 20 %, the load causing failure is known within ± 15 %. If the load causing failure is *nominally* 9 kN, determine the design factor and the maximum allowable load that will ofset the absolute uncertainties.

Solution: To account for its uncertainity the loss of function load must increase to 1/0.85, whereas the maximum allowable load must decrease to1/1.2. Thus to offset the absolute uncertainties the design factor should be: $n_d = \frac{1/0.85}{1/1.2} = 1.4$

Maximum allowable load= $\frac{9}{14}$ = 6.4 kN

It is more common to express the design factor in terms of stress and a relevant strength. Thus:
n_d =loss-of-function strength/allowable stress= $S/_{\sigma}$

EXAMPLE 1-2
A rod with a cross sectional area of A and loaded in tension with an axial force of P=9 kN undergoes a stress of σ = P/A. Using a material strength of 168 N/mm^2 and a *design factor* of 3, determine the minimum diameter of a solid circular rod . Using Table A-15, select a preferred fractional diameter and determine the rod's *factor of safety.*
10
Solution: Since A=πd^2/4 and σ = S/n_d, then,

$$\sigma = \frac{S}{n_d} = \frac{168}{3} = \frac{P}{A} = \frac{9000}{\pi\, d^2/_4}$$

Or $d = \left(\frac{4Pnd}{\pi.S}\right)^{0.5} = (\frac{4(9000)3}{\pi.168})^{0.5} = 14.3\ mm$
from Table A-15 the next higher preferred size is 16 mm. Thus, according to the same equation developed earlier, the factor of safety n is

$$n = \frac{\pi.Sd^2}{4P} = \frac{\pi.(168)\ 16^2}{4(9000)} = 3.75$$

Thus rounding the diameter has increased the actual design factor.

Reliability. Is the statistical measure of probability that a mechanical element will not fail in use . The failure of 6 parts out of every 1000 manufactured parts migth be considered as an acceptable failure rate for a certain class of products. This represents a reliability of

R=1-(6/1000)=0.994

Failure:

Introduction:

The failure of engineering materials is almost always an undesirable event for several reasons; these include human lives that are put in jeopardy, economic losses, and the interference with the availability of products and services. Even though the causes of failure and the behavior of materials may be known, prevention of failures is difficult to guarantee.

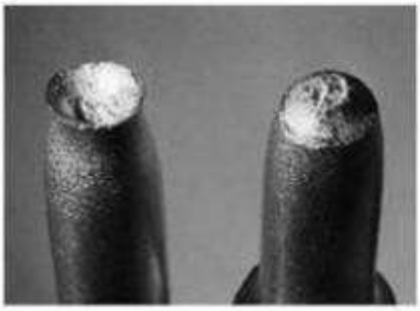
Fundamentals of Fracture:

Fracture is the separation of a single body into pieces by an imposed stress. For engineering materials there are only two possible modes of fracture, ductile and brittle. In general, the main difference between brittle and ductile fracture can be attributed to the amount of plastic deformation that the material undergoes before fracture occurs. Ductile materials show large amounts of plastic deformation while brittle materials show little or no plastic deformation before fracture.

Brittle fracture generally involves rapid propagation of a crack with minimum energy absorption and plastic deformation. In single crystals, brittle fracture occurs by cleavage as the result of tensile stress acting normal to crystallographic planes with low bonding (cleavage planes), creating smooth surfaces (a flat crystal face).

As with plastic deformation, the difference between the theoretical fracture strength and the actual fracture strength is due to structural irregularities. Failure in brittle materials was caused by many fine elliptical, submicroscopic cracks in the metal. The sharpness at the tip of such cracks results in very high stress concentration which may exceed the theoretical fracture strength at this localized area and cause the crack to propagate.

Crack initiation and propagation are essential to fracture. In brittle materials cracks spread very rapidly. Once they are initiated, they will continue to grow and increase in magnitude till fracture. Fractured surfaces of ductile and brittle materials are shown below.



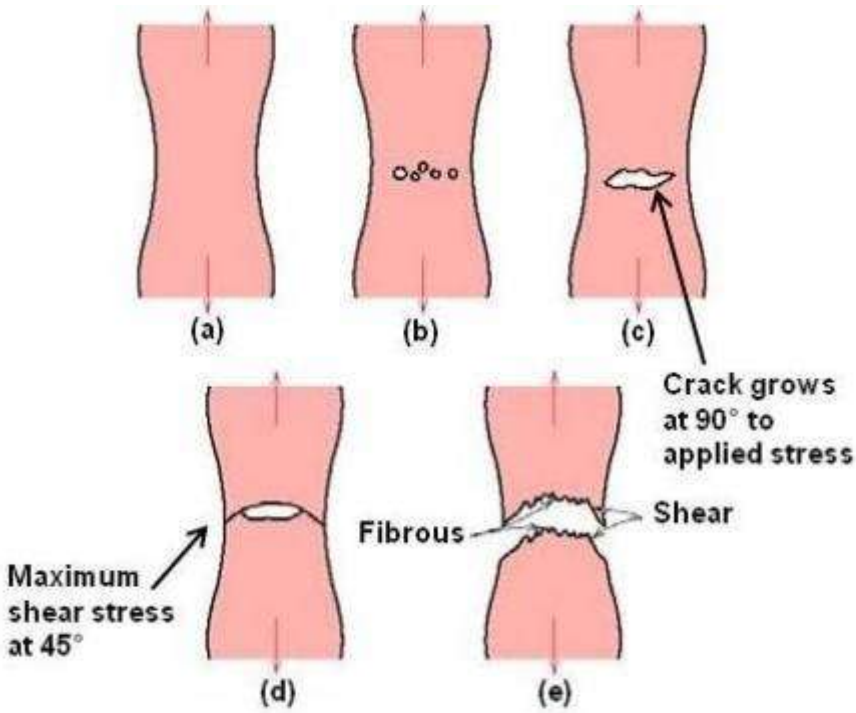
Cup-and-Cone Surfaces
Ductile Materials

Flat Surfaces
Brittle Materials

Fracture Surfaces

Ductile fracture occurs after considerable plastic deformation prior to failure and the crack moves slowly. The crack will usually not extend unless an increased stress is applied. The failure of most polycrystalline ductile materials occurs with a cup-and-cone fracture associated with the formation of a neck in a tensile specimen. In ductile material the fracture begins by the formation of cavities (microvoids) in the center of the necked region. In most commercial metals, these internal cavities probably form at nonmetallic inclusions. This belief is supported by the fact that extremely pure

metals are much more ductile than those of slightly lower purity. Under continued applies stress, the cavities grow and coalesce to form a crack in the center of the sample. The crack proceeds outward toward the surface of the sample in a direction perpendicular to the applied stress. Completion of the fracture occurs vary rapidly along a surface that makes an angle of approximately 45° with the tensile axis. The final stage leaves a circular lip on one half of the sample and a bevel on the surface of the other half. Thus one half has the appearance of a shallow cup, and the other half resembles a cone with a flattened top, giving rise to the term cup-and-cone fracture.

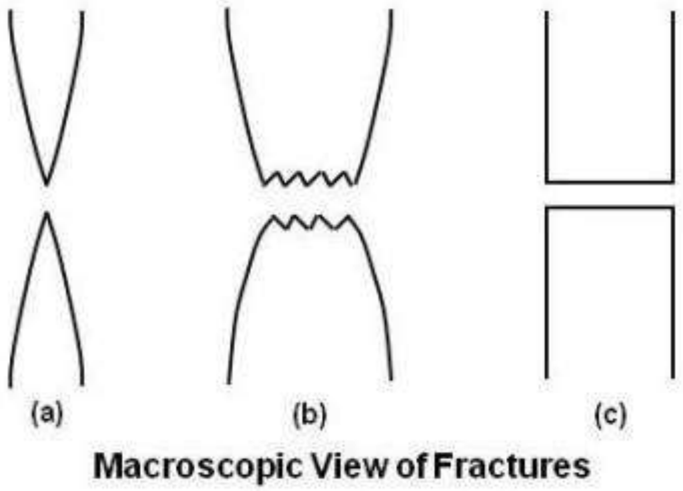


Stages in Ductile Fracture

Various stages during ductile fracture are schematically shown in above figure.

(a) Initial Necking
(b) Cavity formation
(c) Cavity coalescence to form a crack
(d) Crack propagation
(e) Fracture

On both macroscopic and microscopic levels, ductile fracture surfaces have distinct features. Macroscopically, ductile fracture surfaces have larger necking regions and an overall rougher appearance than a brittle fracture surface. Figure given below shows the macroscopic differences between two ductile specimens and one brittle specimen.



(a) Highly ductile fracture in which the specimen necks down to a point
(b) Moderately ductile fracture after some necking
(c) Brittle specimen without any plastic deformation

Materials are normally classified loosely as either `brittle' or `ductile' depending on the characteristic features of the failure. Examples of `brittle' materials include refractory oxides (ceramics) and intermetallics, as well as BCC metals at low temperature (below about ¼ of the melting point). Features of a brittle material are

1. Very little plastic flow occurs in the specimen prior to failure;
2. The two sides of the fracture surface fit together very well after failure.
3. The fracture surface appears faceted – you can make out individual grains and atomic planes.
4. In many materials, fracture occurs along certain crystallographic planes. In other materials, fracture occurs along grain boundaries

Examples of `ductile' materials include FCC metals at all temperatures; BCC metals at high temperatures; polymers at high temperature. Features of a `ductile' fracture are

1. Extensive plastic flow occurs in the material prior to fracture
2. There is usually evidence of considerable necking in the specimen
3. Fracture surfaces don't fit together.
4. The fracture surface has a dimpled appearance – you can see little holes, often with second phase particles inside them.

Brittle fracture:

Brittle fracture is characterized by very low plastic deformation and low energy absorption prior to breaking.

A crack, formed as a result of the brittle fracture, propagates fast and without increase of the stress applied to the material.

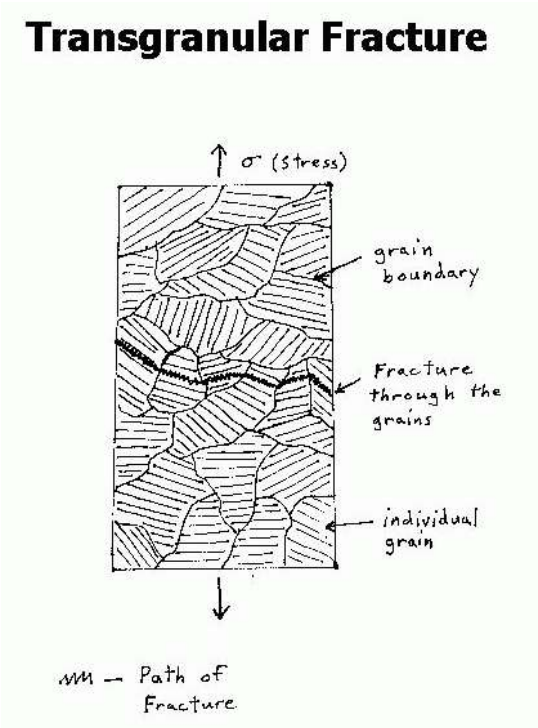
The brittle crack is perpendicular to the stress direction.

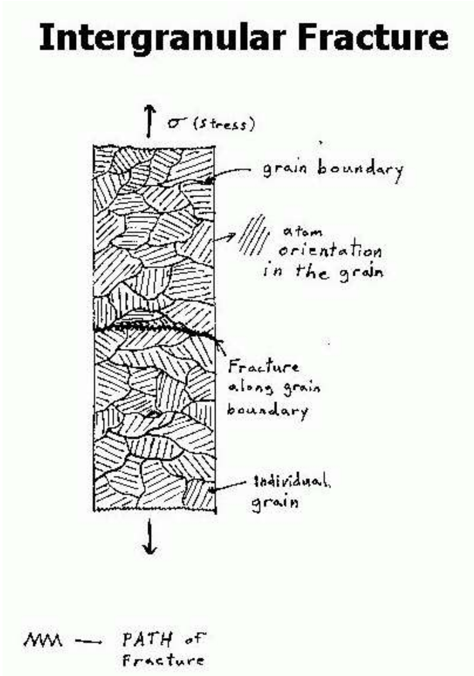
There are two possible mechanisms of the brittle fracture: **transcrystalline (transgranular, cleavage)** or **intercrystalline (intergranular).**

Cleavage cracks pass along crystallographic planes through the grains.

Intercrystalline fracture occurs through the grain boundaries, embrittled by segregated impurities, second phase inclusions and other defects.

The brittle fractures usually possess bright granular appearance.





The process of fracture:

- The specimen elongates, forming a necked region in which cavities form.
- The cavities coalesce in the neck center, forming a crack which propagates toward the specimen surface in a direction perpendicular to the applied stress.
- As the crack approaches the surface, its growth direction shifts to 45º with respect to the tension axis. This redirection allows for the formation of the cup-and-cone configuration and facilitates fracture.

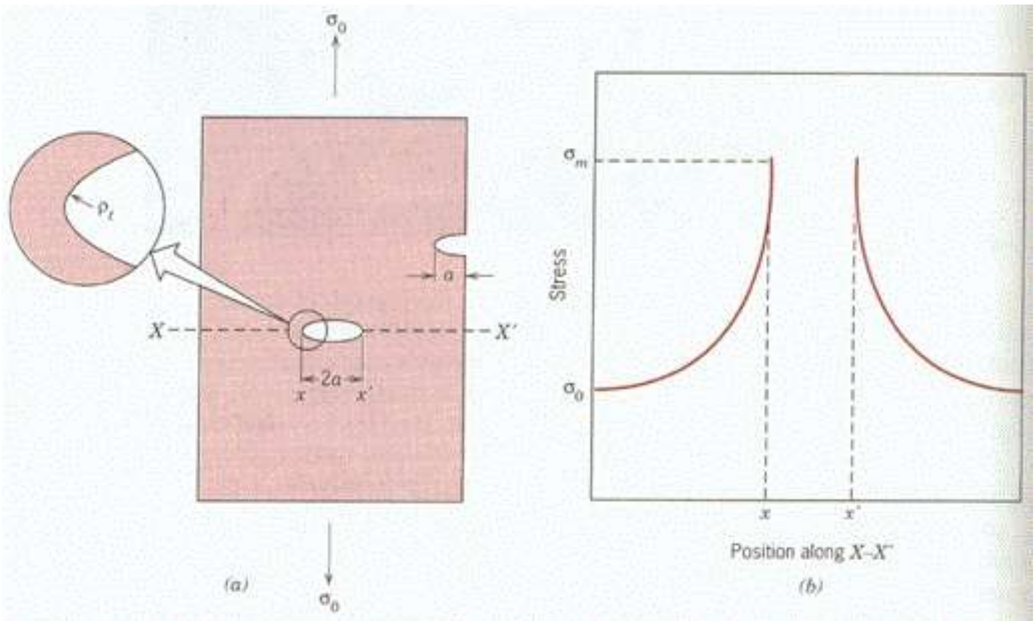
Ductile fracture	Brittle fracture
Plastic deformation	Small/ no plastic deformation
High energy absorption before fracture	Low energy absorption before fracture
Characterized by slow crack propagation	Characterized by rapid crack propagation
Detectable failure	Unexpected failure
Stable crack	Unstable crack
Eg: Metals, polymers	Eg: Ceramics, polymers

Principles of Fracture Mechanics

This subject allows quantification of the relationships between material properties, stress level, the presence of crack-producing flaws, and crack propagation mechanisms.

Stress Concentration

The fracture of a material is dependent upon the forces that exist between the atoms. Because of the forces that exist between the atoms, there is a theoretical strength that is typically estimated to be one-tenth of the elastic modulus of the material. However, the experimentally measured fracture strengths of materials are found to be 10 to 1000 times below this theoretical value. The discrepancy is explained to exist because of the presence of small flaws or cracks found either on the surface or within the material. These flaws cause the stress surrounding the flaw to be amplified where the magnification is dependent upon the orientation and geometry of the flaw. Looking at figure below, one can see a stress profile across a cross section containing an internal, elliptically-shaped crack. One can see that the stress is at a maximum at the crack tip and decreased to the nominal applied stress with increasing distance away from the crack. The stress is concentrated around the crack tip or flaw developing the concept of *stress concentration*. *Stress raisers* are defined as the flaws having the ability to amplify an applied stress in the locale.



(a) The geometry of surface and internal cracks. (b) Schematic stress profile along the line X-X' in (a), demonstrating stress amplification at crack tip positions.

Determination of the Maximum Stress at the Crack Tip

If the crack is assumed to have an elliptical shape and is oriented with its long axis perpendicular to the applied stress, the maximum stress, $\mathbf{s_m}$ can be approximated at the crack tip by this Equation

$$\sigma_m = 2\sigma_o \left(\frac{a}{\rho_t}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Above equation is used to determine the maximum stress surrounding a crack tip.

The magnitude of the nominal applied tensile stress is **s$_o$**; the radius of the curvature of the crack tip is **r**; and ***a*** represents the length of a surface crack, or half the length of an internal crack.

Determination of Stress Concentration Factor

The ratio of the maximum stress and the nominal applied tensile stress is denoted as the stress concentration factor, K_t, where K_t can be calculated by Equation 2. The *stress concentration factor* is a simple measure of the degree to which an external stress is amplified at the tip of a small crack.

$$K_t = \frac{\sigma_m}{\sigma_o} = 2\left(\frac{a}{\rho_t}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Eqn. 2: Determination of the stress concentration factor.

Using principles of fracture mechanics, it is possible to show that the critical stress required for crack propagation in a brittle material is described by the expression

$$\sigma_C = \left(\frac{2E\gamma_S}{\pi a}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

E = modulus of elasticity
γ_S = specific surface energy
a = one half the length of an internal crack

Importance of the equation (3)
• This equation is significant because it relates the size of the imperfection (2a) to the tensile strength of the material.
• It predicts that small imperfections are less damaging than large imperfections, as observed experimentally.

All brittle materials contain a population of small cracks and flaws that have a variety of sizes, geometries, and orientations.When the magnitude of a tensile stress at the tip of one of these flaws exceeds the value of this critical stress, a crack forms and then propagates, which results in fracture. Very small and virtually defect-free metallic and ceramic whiskers have been grown with fracture strengths that approach their theoretical values.

Example١ :

A relatively large plate of a glass is subjected to a tensile stress of 40 MPa. If the specific surface energy and modulus of elasticity for this glass are 0.3 J/m2 and 69 GPa, respectively, determine the maximum length of a surface flaw that is possible without fracture.

Solution

To solve this problem it is necessary to employ Equation.3. Rearrangement of this expression such that *a* is the dependent variable, and realizing that σ = 40 MPa, γ_S = 0.3 J/m^2, and *E* =69 GPa leads to

$$\boldsymbol{a = \frac{2E\gamma_S}{\pi\sigma^2} = \frac{(2)\left(69\times10^9\ {}^{N}/_{m^2}\right)\left(0.3\ {}^{N}/_{m}\right)}{\pi\left(40\times10^6\ {}^{N}/_{m^2}\right)^2}} \quad \mathbf{= 8.2x10^{-6}\ m = 0.0082\ mm = 8.2\ \mu m}$$

Example 2:

What is the magnitude of the maximum stress that exists at the tip of an internal crack having a radius of curvature of 1.9×10^{-4} mm and a crack length of 3.8×10^{-2} mm when a tensile stress of 140 MPa (20,000 psi) is applied?

$\sigma_m = 2\sigma_o \left(\frac{a}{\rho_t}\right)^{1/2}$ = (2)(140 MPa) (3.8×10^{-2}mm / 2 / 1.9×10^{-4} mm)$^{0.5}$ = 2800 MPa (400,000 psi)

Example 3:

An MgO component must not fail when a tensile stress of 13.5 MPa (1960 psi) is applied. Determine the maximum allowable surface crack length if the surface energy of MgO is 1.0 J/m^2, and E of MgO is 225×10^9 N/ m^2.

$$\boldsymbol{a = \frac{2E\gamma_S}{\pi\sigma^2}}$$

a = (2)(225x10^9 N/m^2)(1.0 N/m) / (3.14)(13.5x10^6 N/m^2)^2 = 7.9x10^{-4}m

Fracture toughness:

Fracture toughness, K_c, is the resistance of a material to failure from fracture starting from a preexisting crack. This definition can be mathematically expressed by the following expression:

$\boldsymbol{K_C = Y\sigma_C\sqrt{\pi a}}$ **……………………. (4)**

Where *Y* is a dimensionless factor dependent on: the geometry of the crack and material, the loading configuration (i.e. if the sample is subject to tension or bending), and the ratio of crack length to specimen width, *b*. is the amount of load (stress) applied to the specimen, and *a* is the crack length.

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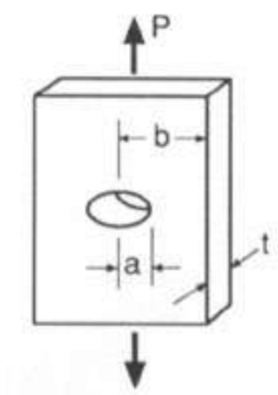
<td>
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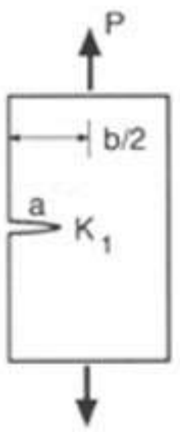
<td>
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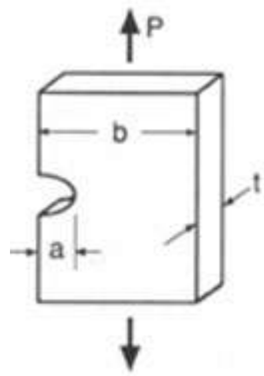
<td>
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<tr>
<td>Figure 1 A specimen with an interior crack. Note that the entire crack length is equal to 2a.</td>
<td>Figure 2 A specimen with a through-thickness crack.</td>
<td>Figure 3 A specimen with a half circle surface crack.</td>
</tr>
</table>

Figure 1 shows that *a* is not always the total length of the crack, but is sometimes half the crack length, as for an interior crack. The values for *Y* vary with respect to the shape and location of the crack. Some useful values of Y for short cracks subjected to a tension load are as follows:

Y = 1.00 For an interior crack similar to the crack shown in Figure 1

Y = 1.12 For a through-thickness surface crack as shown in Figure 2

Y = 0.73 For a half-circular surface crack as shown in Figure 3

Fracture toughness, K_c, has the English customary units of psi $in^{1/2}$, and the SI units of MPA $m^{1/2}$.

What is the plane strain fracture toughness, K_{Ic}?

For thin samples, the value K_c decreases with increasing sample thickness, *b*, as shown by Figure 4.

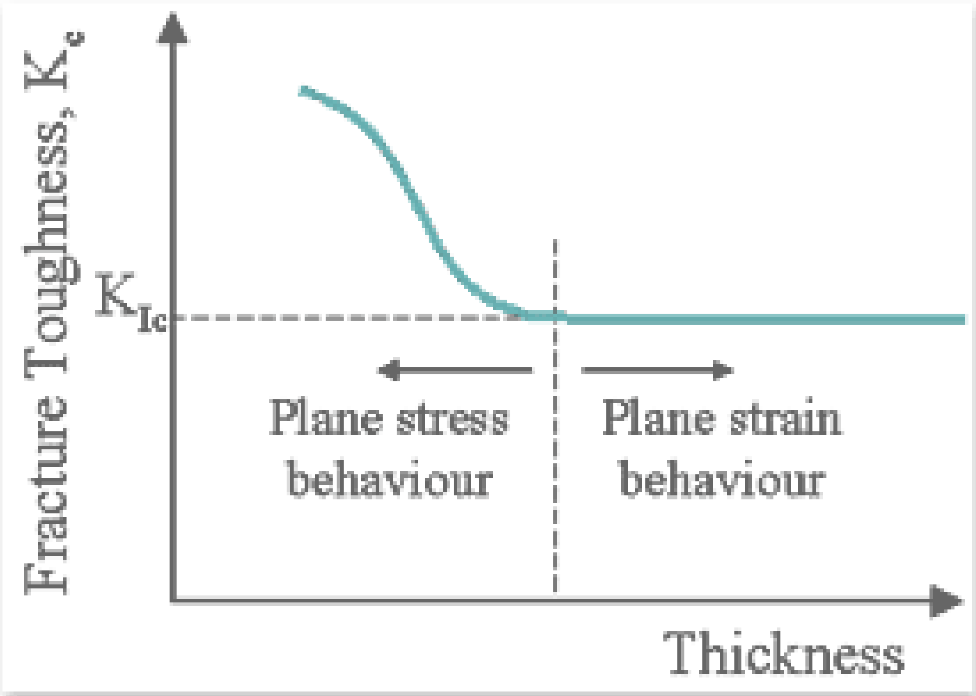


Figure 4. A fracture toughness vs. thickness graph. Note the location of K_{Ic}

Ultimately, K_c becomes independent of *b*, at this point the sample is said to be under the conditions of plane strain. This fixed value of K_c becomes known as the plane strain fracture toughness, K_{Ic}. K_{Ic} is mathematically defined by:

$$K_{Ic} = Y\sigma\sqrt{\pi a} \quad \text{..........................} \quad (5)$$

This value for the fracture toughness is the value normally specified because it is never greater than or equal to K_c. The I subscript for K_{Ic}, stands for mode I, or tensile mode, crack displacement as shown in Figure 5(a).

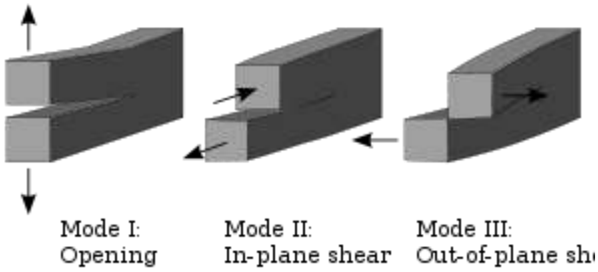


Figure 5. The three modes of crack surface displacement.(a) Mode I, tensile mode; (b) mode II, sliding mode; and (c)mode III, tearing mode. Source

In general, K_{Ic} is low for brittle materials and high for ductile materials. This trend is supported by the K_{Ic} values in Table 1 (3,4).

1. If a support beam of 4340 Steel (tempered at 260 C) has an interior crack of length 5 mm, how much stress, σ, can be applied to it before it is expected to fracture?

K_{Ic} = Y σ SQRT{π a}
50.0 MPa $m^{1/2}$ = 1.00 σ SQRT{π{5*10^{-3}}} , σ = 564 MPa

Table 1 Room-Temperature Plane Strain Fracture Toughness Values

Material	K_{Ic}	
	MPA $m^{1/2}$	psi $in^{1/2}$
Metals		
2024-T351 Aluminum	36	33,000
4340 Steel (tempered @ 260 C)	50.0	45,800
Titanium Alloy (Ti-6Al-4V)	44-66	40,000-60,000
Ceramics		
Aluminum Oxide	3.0-5.3	2,700-4,800
Soda-lime glass	0.7-0.8	640-730
Concrete	0.2-1.4	180-1,270

Polymers		
Polymethyl methacrylate (PMMA)	1.0	900
Polystyrene (PS)	0.8-1.1	730-1,000

Fatigue:

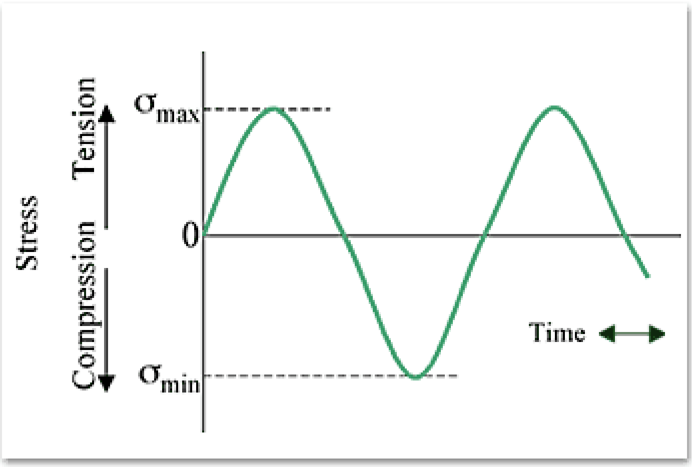
Fatigue is failure that occurs in structures that undergo repeated cyclic stress, for example bridges and connecting rods.

In **fatigue** it is possible for failure to occur at stresses much lower than the yield strength.

Fatigue is responsible for about 90% of all metallic component failures, and it is also possible for polymers and ceramics to fail by fatigue.

It is catastrophic in that it occurs without warning because it is a brittle fracture (with little or no **plastic deformation**) that occurs in materials that are normally ductile.

There are three different modes of fatigue loading, shown below:

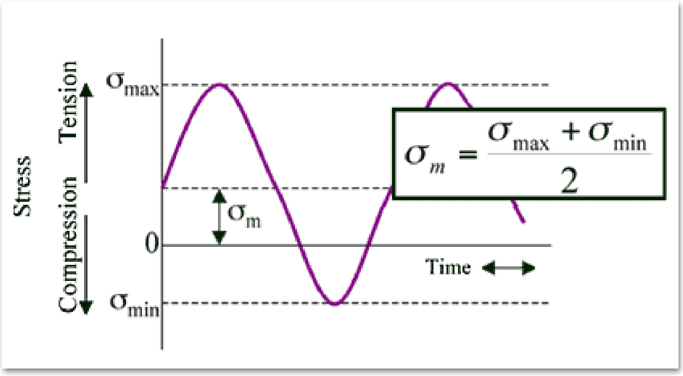


Reversed stress cycle

Repeated stress cycle

Random stress cycle

There are a number of fatigue parameters used to characterise the fluctuating stress cycle, shown below:



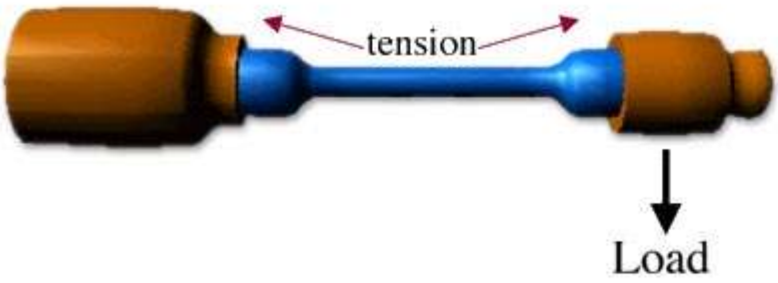
Mean stress σ_m

Stress range $\Delta\sigma$

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Stress ratio R

Fatigue 2



A common method of testing **fatigue** resistance is the Wohler rotating rod test. One end of the specimen is mounted in a rotating chuck and a load suspended from the other end. The specimen experiences cyclic forces, from tension to compression in a sinusoidal cycle, as it rotates.

After a number of cycles, the specimen may fail.

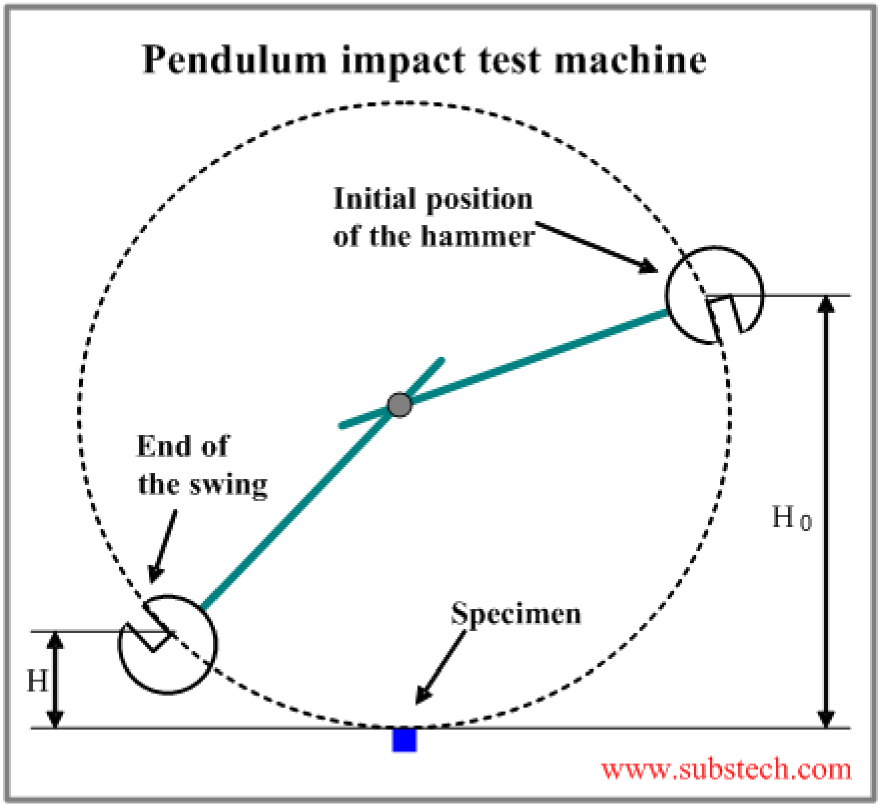
Generally a number of specimens are tested at different applied stresses and the number of cycles to failure is recorded.

Fatigue failures are caused by slow crack growth through the material. The failure process involves four stages

1. Crack initiation
2. Micro-crack growth (with crack length less than the materials grain size) (Stage I)
3. Macro crack growth (crack length between 0.1mm and 10mm) (Stage II)
4. Failure by fast fracture.

Impact test:

Normalized tests, like the Charpy and Izod tests measure the impact energy required to fracture a notched specimen with a hammer mounted on a pendulum. The energy is measured by the change in potential energy (height) of the pendulum. This energy is called notch toughness.



Impact test is used for measuring toughness of materials and their capacity of resisting shock.

In this test the pendulum is swing up to its starting position (height **H**) and then it is allowed to strike the notched specimen, fixed in a vice. The pendulum fractures the specimen, spending a part of its energy. After the fracture the pendulum swings up to a height **H**.

The **impact toughness** of the specimen is calculated by the formula:

a = A/ S

Where

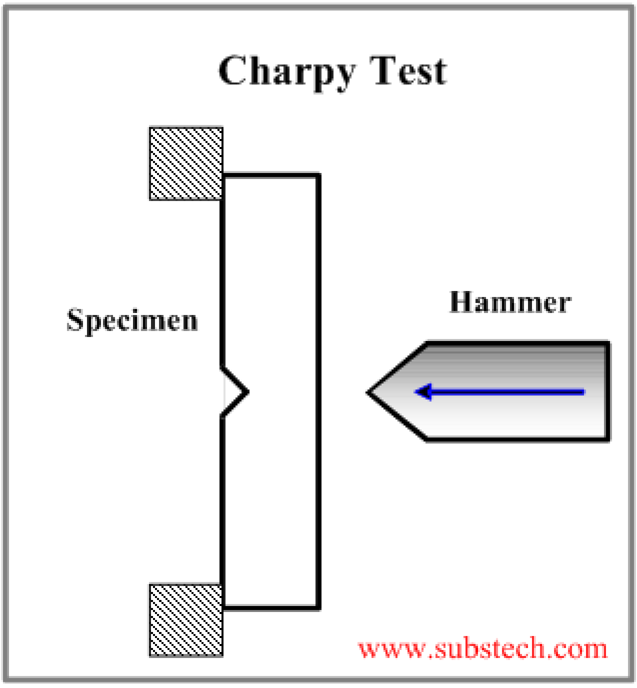
a-impact toughness,

A – the work, required for breaking the specimen ($\mathbf{A = M*g*H_0 – M*g*H}$),

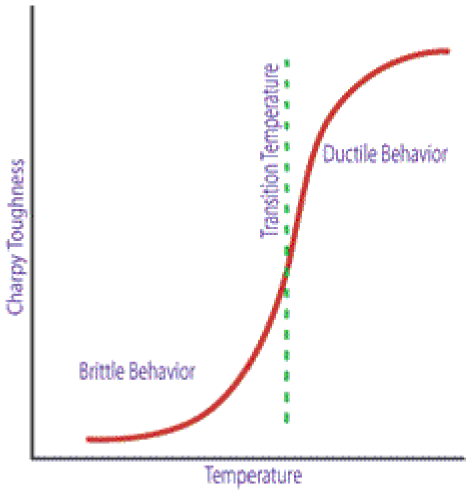
M - the pendulum mass,

S - cross-section area of the specimen at the notch.

One of the most popular impact tests is the **Charpy Test**, schematically presented in the figure below:



The hammer striking energy in the Charpy test is 220 ft*lbf (300 J).



Since toughness is greatly affected by temperature, a Charpy or Izod test is often repeated numerous times with each specimen tested at a different temperature. This produces a graph of impact toughness for the material as a function of temperature. An impact toughness versus temperature graph for a steel is shown in the image. It can be seen that at low temperatures the material is more brittle and impact toughness is low. At high temperatures the material is more ductile and impact toughness is higher. The transition temperature is the boundary between brittle and ductile behavior and this temperature is often an extremely important consideration in the selection of a material.

Fatigue life is measured by subjecting the material to cyclic loading. Usually the loading is uniaxial tension, although other cycles are used too (e.g. contact fatigue applications). The cycle can be stress controlled, or strain controlled. A cycle of uniaxial load is characterized by

- The stress amplitude $(\sigma_{\max} - \sigma_{\min})/2$
- The mean stress $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2$
- The stress ratio $R = \sigma_{\min} / \sigma_{\max}$

The crack growth rate is a function of the stress level, the crack size and material properties. The relationship is expressed in terms of the stress intensity factor *K*:

$$\frac{da}{dN} = A(\Delta K)^m$$

where A and m are constants for the material, dependent on environment and stress.

Also, ΔK is the stress intensity factor range and therefore can be written as:

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} = Y(\sigma_{max} - \sigma_{min})\sqrt{\pi a}$$

Typical fatigue crack growth behaviour is shown in the graph. There are three distinct regions.

<u>Region 1</u>
Non-propagating cracks and slow crack growth

<u>Region 2</u>
Steady crack growth: power-law relationship (shown above)

<u>Region 3</u>
Rapid unstable crack growth $K \sim K_{Ic}$

To predict fatigue life, we take the initial crack length, a_0, and the critical crack length a_c

$$K_c = Y\sigma_{max}\sqrt{\pi a_c} \rightarrow a_c = \frac{K_c^2}{\pi Y^2 \sigma_m^2}$$

and use the previous formulae to determine N_f, the number of cycles to failure due to stage II crack growth.

We start with: $$\frac{da}{dN} = A(\Delta K)^m$$

Rearranged: $$dN = \frac{da}{A(\Delta K)^m}$$

Integrated: $$N_f = \int_0^{N_f} dN = \int_{a_0}^{a_c} \frac{da}{A(\Delta K)^m}$$

Substitute for ΔK: $$N_f = \int_{a_0}^{a_c} \frac{da}{A\left(Y\Delta\sigma\sqrt{\pi a}\right)^m}$$

We now have an expression for the fatigue life of a component.

However, it should be remembered that this expression is only valid in the crack propagation stage and does not include crack initiation or rapid fracture and therefore the fatigue life calculated should be taken as an estimate of fatigue life. This expression is more accurate when the crack initiation stage is small (under high stresses).

This expression also assumes that $\Delta\sigma$ is constant - which is not true in many applications.

There are a number of factors that affect the fatigue life of a component:

1. Stress Level

Fatigue life is highly dependent on $\Delta\sigma$ and R
$R = \frac{\sigma_{min}}{\sigma_{max}}$

2. Surface Effects

Surface finish is important because in fatigue, cracks usually start at the surface.

Design: Notches, discontinuities, grooves, holes, threads increase the stress concentration, and the sharper the discontinuity the more severe the stress concentration. Therefore to design against fatigue, avoid irregularities and use rounded fillets where possible.

Surface treatment: Machining introduces scratches and grooves, therefore polishing a machined surface will increase fatigue life. Fatigue life can be improved by introducing a compressive **residual stress** on the surface layer (shot peening and case hardening).

3. Environment

Thermal fatigue: Fluctuating temperatures can cause thermal stresses due to thermal expansion of the components.

Corrosion fatigue: If the component is exposed to a corrosive environment, pits caused by corrosion can act as initiation sites and corrosion can also increase the crack growth rate.

Problem	**Solution**
A component is made of a steel for which K_c=54 MPa $m^{1/2}$. Non-destructive testing showed that the component contains cracks of up to 0.2 mm in length. Tests have shown that the crack growth rate is given by: $\frac{da}{dN} = A(\Delta K)^n$ where A=4x10^{-13} (MPa m) and n=4. The component is subjected to an alternating stress of $\Delta\sigma$=180 MPa, with a mean stress of $\Delta\sigma/2$. Calculate the number of cycles to failure, then take the stress concentration factor Y=1.	Catastrophic failure will occur when: $\sigma_{\max}\sqrt{\pi a} = 54MPa.m^{1/2}$ $\sqrt{\pi a} = \frac{54MPa.m^{1/2}}{180MPa}$ $a = 0.029m$ --- $\frac{da}{dN} = A(\Delta K)^n$ $\frac{da}{dN} = 4\times10^{-13}MPa^{-4}m^{-1}\left(\Delta\sigma\sqrt{\pi a}\right)^4$ $\frac{da}{dN} = 4\times10^{-13}MPa^{-4}m^{-1}(180MPa)^4\pi^2a^2$ $\frac{da}{dN} = 4.14\times10^{-3}a^2m^{-1}$ --- Now integrate from a_0= 0.1 to a_c= 29 mm: $N_f = \int_{a_0}^{a_c}\frac{da}{4.14\times10^{-3}a^2m^{-1}}$ $N_f = \int_{1\times10^{-4}m}^{0.029m}\frac{da}{4.14\times10^{-3}a^2m^{-1}}$ $N_f = \frac{1}{4.14\times10^{-3}m^{-1}}\left(\frac{1}{1\times10^{-4}m} - \frac{1}{0.029m}\right)$ $N_f = 2.4\times10^6$ Therefore the fatigue life of the component is 2.4 x 10^6 cycles to failure.

Creep:

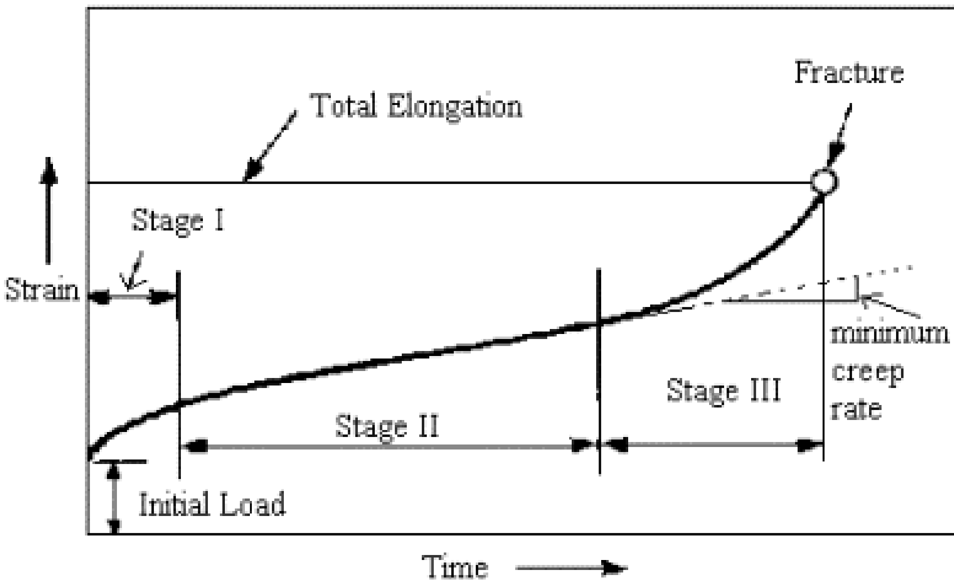
What is Creep?

Creep is a time varying, permanent strain due to long term application of constant or near constant stress level. Creep is observed in most engineering materials especially metals at elevated temperatures, high polymer plastics, concrete, solid propellant in rocket motors turbine rotors in jet engines and steam generators that experience centrifugal stresses.

creep is normally an undesirable phenomenon and is often the limiting factor in the lifetime of a part. It is observed in all materials types; for metals it becomes important only for temperatures greater than about 0.4*Tm*(*Tm* = absolute melting temperature).
Amorphous polymers, which include plastics and rubbers, are especially sensitive to creep deformation.

A typical creep test consists of subjecting a specimen to a constant load or stress while maintaining the temperature constant; deformation or strain is measured and plotted as a function of elapsed time. Most tests are the constant load type, which yield information of an engineering nature; constant stress tests are employed to provide a better understanding of the mechanisms of creep.

In a creep test a constant load is applied to a tensile specimen maintained at a constant temperature. Strain is then measured over a period of time. The slope of the curve, identified in the above figure, is the strain rate of the test during stage II or the creep rate of the material.



Primary creep, Stage I, is a period of decreasing creep rate. Primary creep is a period of primarily transient creep. During this period deformation takes place and the resistance to creep increases until stage II. Secondary creep, Stage II, is a period of roughly constant creep rate. Stage II is referred to

as steady state creep. Tertiary creep, Stage III, occurs when there is a reduction in cross sectional area due to necking or effective reduction in area due to internal void formation.

General creep equation:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{C\sigma^m}{d^b} e^{\frac{-Q}{kT}}$$

where ε is the creep strain, *C* is a constant dependent on the material and the particular creep mechanism, *m* and *b* are exponents dependent on the creep mechanism, *Q* is the activation energy of the creep mechanism, σ is the applied stress, *d* is the grain size of the material, *k* is Boltzmann's constant, and *T* is the absolute temperature.