المرحلة الأولى

الكهربائية والميكانيكية والصوت

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الجامعة التكنولوجية

قسم العلوم التطبيقية

فرع علم المواد

Textbooks

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Some Physical Constants*

Speed of light	c	2.998×10^{8} m/s
Gravitational constant	G	6.673×10^{-11} N · m²/kg²
Avogadro constant	N_A	6.022×10^{23} mol⁻¹
Universal gas constant	R	8.314 J/mol · K
Mass–energy relation	c^2	8.988×10^{16} J/kg
		931.49 MeV/u
Permittivity constant	ϵ_0	8.854×10^{-12} F/m
Permeability constant	μ_0	1.257×10^{-6} H/m
Planck constant	h	6.626×10^{-34} J · s
		4.136×10^{-15} eV·s
Boltzmann constant	k	1.381×10^{-23} J/K
		8.617×10^{-5} eV/K
Elementary charge	e	1.602×10^{-19} C
Electron mass	m_e	9.109×10^{-31} kg
Proton mass	m_p	1.673×10^{-27} kg
Neutron mass	m_n	1.675×10^{-27} kg
Deuteron mass	m_d	3.344×10^{-27} kg
Bohr radius	a	5.292×10^{-11} m
Bohr magneton	μ_B	9.274×10^{-24} J/T
		5.788×10^{-5} eV/T
Rydberg constant	R	$1.097\,373 \times 10^{7}$ m⁻¹

Some Conversion Factors

Mass and Density

1 kg = 1000 g = 6.02×10^{26} u

1 slug = 14.59 kg

1 u = 1.661×10^{-27} kg

1 kg/m³ = 10^{-3} g/cm³

Length and Volume

1 m = 100 cm = 39.4 in. = 3.28 ft

1 mi = 1.61 km = 5280 ft

1 in. = 2.54 cm

1 nm = 10^{-9} m = 10 Å

1 pm = 10^{-12} m = 1000 fm

1 light-year = 9.461×10^{15} m

1 m³ = 1000 L = 35.3 ft³ = 264 gal

Time

1 d = 86 400 s

1 y = $365\frac{1}{4}$ d = 3.16×10^{7} s

Angular Measure

1 rad = 57.3° = 0.159 rev

π rad = 180° = $\frac{1}{2}$ rev

Speed

1 m/s = 3.28 ft/s = 2.24 mi/h

1 km/h = 0.621 mi/h = 0.278 m/s

Force and Pressure

1 N = 10^{5} dyne = 0.225 lb

1 lb = 4.45 N

1 ton = 2000 lb

1 Pa = 1 N/m² = 10 dyne/cm²
= 1.45×10^{-4} lb/in.²

1 atm = 1.01×10^{5} Pa = 14.7 lb/in.²
= 76.0 cm Hg

Energy and Power

1 J = 10^{7} erg = 0.2389 cal = 0.738 ft · lb

1 kW · h = 3.6×10^{6} J

1 cal = 4.1868 J

1 eV = 1.602×10^{-19} J

1 horsepower = 746 W = 550 ft · lb/s

Magnetism

1 T = 1 Wb/m² = 10^{4} gauss

chapter 1

Physics and Measurement

Physics

Physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict تتنباء the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

Classical physics, which means all of the physics developed before 1900, includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, and electromagnetism.

STANDARDS OF LENGTH, MASS, AND TIME

The system established is metric system, and it is called the SI system of units. The abbreviation SI comes from the system's French name "System International."

In this system, the units of length, mass, and time are the meter, kilogram, and second, respectively. Other SI standards established by the committee are those for temperature (the kelvin), electric current (the ampere), luminous intensity (the candela), and the amount of substance (the mole)

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes milli and nano-denote various powers of ten.

TABLE 1 SI Units

Power	Prefix	Abbreviation
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deka	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Electric Fields

We begin this chapter by describing some of the basic properties of electric forces. We then discuss Coulomb's law, which is the fundamental law governing the force between any two charged particles. Next, we introduce the concept of an electric field associated with a charge distribution and describe its effect on other charged particles. We then show how to use Coulomb's law to calculate the electric field for a given charge distribution. We conclude the chapter with a discussion of the motion of a charged particle in a uniform electric field.

Charges

There are two kinds of electric charges, which were given the names positive and negative, the like charges are generating **repulsive** force, while the unlike charges are generating **attractive** electric forces.

Electrical **conductors** are materials in which electric charges move freely, whereas electrical **insulators** are materials in which electric charges cannot move freely. **Semiconductors** are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors.

The electric charge is always conserved(محفوظة) (when rubber is rubbed (يفرك) with fur, electrons are transferred from the fur to the rubber, giving the rubber a net negative charge and the fur a net positive charge.

Robert Millikan (1868 – 1953) discovered that electric charge always occurs as some integral multiple of a fundamental amount of charge e. In modern terms, the electric charge (q) is said to be **quantized** (قيمه ثابته) ,where (q) is the standard symbol used for charge. That is, electric charge exists as discrete "packets," and (q = Ne) where(N) is integer,(-e) is electron charge. And the proton has a charge of equal magnitude but opposite sign (+e), some particles, such as the neutron.

<u>**COULOMB'S LAW**</u>

Charles Coulomb (1736 – 1806) measured the magnitudes of the electric forces between charged. He confirmed that the electric force between two small charged spheres is proportional to the inverse square of their separation distance (r).

$$F_e \propto 1/r^2$$

Coulomb's experiments showed that the electric force between two stationary charged particles

• is inversely proportional to the square of the separation (r) between the particles and directed along the line joining them;

• is proportional to the product of the charges (q1) and (q2) on the two particles;

• is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

Coulomb's law as an equation giving the magnitude of the electric force (sometimes called the Coulomb force) between two point charges:

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

Where

k_e is a constant called the Coulomb constant.The (SI) unit of charge is the coulomb (C)

$$k_e = 8.987\,5 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

Where

The constant $\boldsymbol{\varepsilon_o}$ is known as the permittivity of free space and has the value

$$\boldsymbol{\varepsilon_o} = 8.85 \times 10^{-12} C^2/N.m^2$$

The smallest of charge on an electron or proton, which has an absolute value of

$$|e| = 1.602\,19 \times 10^{-19}\ \mathrm{C}$$

$_o{}^o{}_o$ 1 C=6.24×10^{18}e

Electrons or protons.

The charges and masses of the electron, proton, and neutron are given in Table (2).

TABLE 23.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

Example

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k_e \frac{|e|^2}{r^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \text{N}}$$

$$F_g = G\frac{m_e m_p}{r^2}$$

$$= \left(6.7 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)$$

$$= \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \text{N}}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces.

When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. Thus, the law expressed in vector form for the electric force exertedالمبذولـة by a charge $q1$ on a second charge $q2$, written F12 , The equation (1-3)is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

where is a unit vector directed from $q1$ to $q2$, as shown in Figure 6a. Because the electric force obeys تخضع Newton's third law, the electric force exerted by $q2$ on $q1$ is equal in magnitude to the force exerted by $q1$ on $q2$ and in the opposite direction; that is,

F12= - F21

Finally, from Equation (1-3), we see that if ($q1$) and ($q2$) have the same sign, as in Figure 6a, the product $q1q2$ is positive and the force is repulsive. If $q1$ and $q2$ are of opposite sign, as shown in Figure 23.6b, the product $q1q2$ is negative and the force is attractive. Noting the sign of the product $q1q2$ is an easy way of determining the direction of forces acting on the charges.

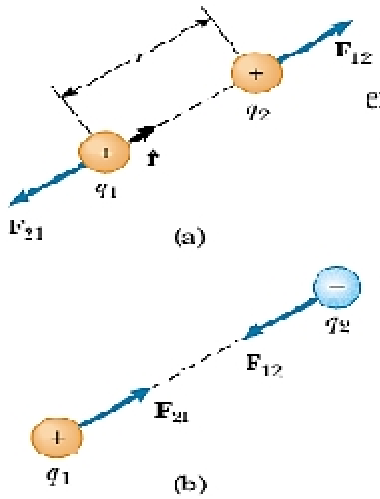


Figure 6 Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\mathbf{F}_{21}$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{12}$ exerted by q_1 on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.

<u>EXAMPLE</u>

Consider three point charges located at the corners of a right triangle as shown in Figure. Where q1=q3= 5µC, q2= -2µC. and a=0.1m. Find the resultant force exerted on q3.

Solution First, note the direction of the individual forces exerted by q1 and q2 on q3 . The force F23 exerted by q2 on q3 is attractive because q2 and

q3 have opposite signs. The force F13 exerted by q1 on q3 is repulsive because both charges are positive. The magnitude of F23 is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$
$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right) \frac{(2.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{(0.10\ \mathrm{m})^2}$$
$$= 9.0\ \mathrm{N}$$

Note that because $q3$ and $q2$ have opposite signs, F23 is to the left, as shown in Figure.
The magnitude of the force exerted by $q1$ on $q3$ is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$
$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right) \frac{(5.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{2(0.10\ \mathrm{m})^2}$$
$$= 11\ \mathrm{N}$$

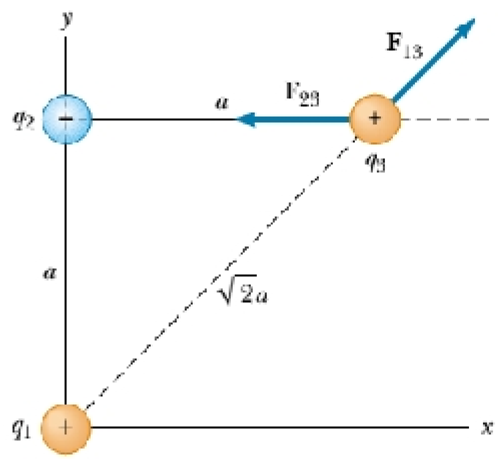
The force F13 is repulsive and makes an angle of 45° with the x axis. Therefore, the x and y components of F13 are equal, with magnitude given by F13 cos 45°=7.9 N.
The force F23 is in the negative x direction. Hence, the x and y components of the resultant force acting on q3 are

$$F_{3x} = F_{13x} + F_{23} = 7.9\ \mathrm{N} - 9.0\ \mathrm{N} = -1.1\ \mathrm{N}$$

$$F_{3y} = F_{13y} = 7.9\ \mathrm{N}$$

We can also express the resultant force acting on $q3$ in unit vector form as

$$\mathbf{F}_3 = (-1.1\mathbf{i} + 7.9\mathbf{j})\ \mathrm{N}$$



The Electric Field

The electric field is a *vector field;* it consists of a distribution of *vectors,* one for each point in the region around a charged object, such as a charged rod. In principle, we can define the electric field at some point near the charged object, such as point P in Fig. 22-1*a*, as follows: We first place a *positive* charge q_0, called a *test charge,* at the point. We then measure the electrostatic force $\vec{F}$ that acts on the test charge. Finally, we define the electric field $\vec{E}$ at point P due to the charged object as

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).} \tag{22-1}$$

Thus, the magnitude of the electric field $\vec{E}$ at point P is $E = F/q_0$, and the direction of $\vec{E}$ is that of the force $\vec{F}$ that acts on the *positive* test charge. As shown in Fig. 22-1*b*, we represent the electric field at P with a vector whose tail is at P. To define the electric field within some region, we must similarly define it at all points in the region.

The SI unit for the electric field is the newton per coulomb (N/C).

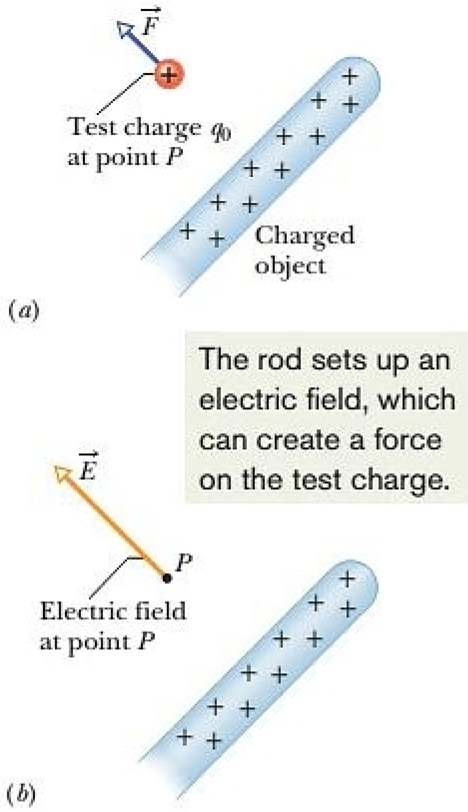


Fig. 22-1 (*a*) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (*b*) The electric field $\vec{E}$ at point P produced by the charged object.

Electric field lines extend away"تمتد بعيدا" from positive charge (where they originate(تنشأ)) and toward negative charge (where they terminate(تنتهي)). The fig. 22-5 is expressing, the pattern for two charges that are equal in magnitude but of opposite sign, a configuration that we call an **electric dipole**. Although we do not often use field lines quantitatively, they are very useful to visualize (يصور ، يتخيل) what is going on.

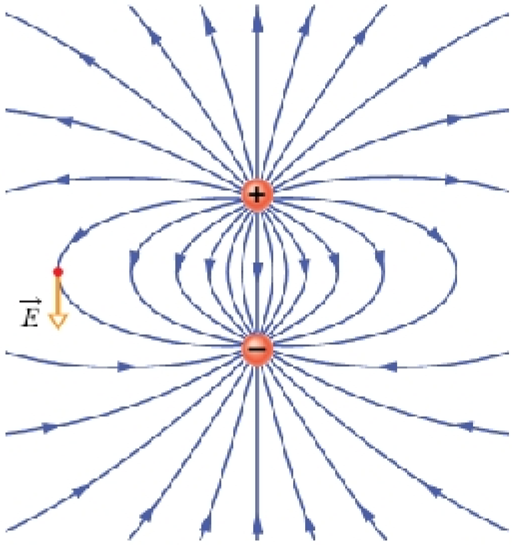


Fig. 22-5 Field lines for a positive point charge and a nearby negative point charge that are equal in magnitude. The charges attract each other. The pattern of field lines and the electric field it represents have rotational symmetry about an axis passing through both charges in the plane of the page. The electric field vector at one point is shown; the vector is tangent to the field line through the point.

The electrostatic force acting on q_0 is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,\hat{\mathrm{r}}$$

The direction of $\vec{F}$ is directly away from the point charge if q is positive, and directly toward the point charge if q is negative. The electric field vector is, from Eq.

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{\mathrm{r}} \qquad \text{(point charge)}$$

<u>Electric Potential</u>

When a test charge q_0 is placed in an electric field E created by some other charged object, the electric force acting on the test charge is q_0E.

The force $q_0E$is conservative because the individual forces described by Coulomb's law are conservative. When the test charge is moved in the field

by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. For an infinitesimal displacement ds, the work done by the electric field on the charge is

$$\mathbf{F} \cdot d\mathbf{s} = q_0 \mathbf{E} \cdot d\mathbf{s}$$

As this amount of work is done by the field, the potential energy of the charge – field system is decreased by an amount

$$dU = -q_0 \mathbf{E} \cdot d\mathbf{s}$$

For a finite

Displacement of the charge from a point Ato a point B, the change in potential energy of the system

$$\Delta U = U_B - U_A$$

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The **potential energy per unit charge** U/q_0 is independent of the value of q_0 and has a unique value at every point in an electric field. This quantity U/q_0 is called the electric **potential** (or simply the potential) V. Thus, the **electric potential** at any point in an electric field is

$$V = \frac{U}{q_0}$$

The potential difference is between any two points A and Bin an electric field is defined as the change in potential energy of the system divided by the test charge q_0:

$$\Delta V = V_B - V_A$$

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Potential difference should not be confused with difference in potential energy. The potential difference is proportional to the change in potential energy

$$\Delta U = q_0 \Delta V$$

Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field. However, when we speak of potential energy, we are referring to the charge – field system.

The electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.

The SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

$$1\ \mathrm{V} \equiv 1\ \frac{\mathrm{J}}{\mathrm{C}}$$

The electron volt (eV), which is defined as the energy an electron (or proton) gains or losses by moving through a potential difference of 1 V.

$$1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{C \cdot V} = 1.60 \times 10^{-19}\ \mathrm{J}$$

*A positive charge loses electric potential energy when it moves in the direction of the electric field.

* When the charged particle gains kinetic energy, it loses an equal amount of potential energy.

*A negative charge gains electric potential energy when it moves in the direction of the electric field.

EXAMPLE 25.2 **Motion of a Proton in a Uniform Electric Field**

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m and is directed along the positive x axis (Fig. 25.5). The proton undergoes a displacement of 0.50 m in the direction of **E**. (a) Find the change in electric potential between points A and B.

Solution Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential.

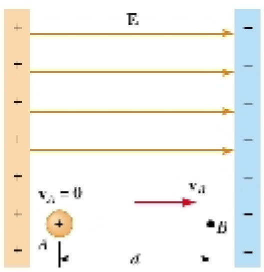


Figure 25.5 A proton accelerates from A to B in the direction of the electric field.

From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4\ \mathrm{V/m})(0.50\ \mathrm{m})$$
$$= \boxed{-4.0 \times 10^4\ \mathrm{V}}$$

(b) Find the change in potential energy of the proton for this displacement.

Solution

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\ \mathrm{C})(-4.0 \times 10^4\ \mathrm{V})$$
$$= \boxed{-6.4 \times 10^{-15}\ \mathrm{J}}$$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time loses electric potential energy (because energy is conserved).

Exercise Use the concept of conservation of energy to find the speed of the proton at point B.

Answer 2.77×10^6 m/s.

• If a charge Q is uniformly distributed throughout a volume V, the volume charge density ρ is defined by

$$\rho \equiv \frac{Q}{V}$$

Where ρ has units of coulombs per cubic meter (C/m3).

• If a charge Q is uniformly distributed on a surface of area A, the surface charge density б (lowercase Greek sigma) is defined by

$$\rho \equiv \frac{Q}{V}$$

Where б has units of coulombs per square meter (C/m^2).

• If a charge Q is uniformly distributed along a line of length , the linear charge density λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

Where λ has units of coulombs per meter (C/m).

• If the charge is nonuniformly distributed over a volume, surface, or line, we have to express the charge densities as

$$\rho = \frac{dQ}{dV} \qquad \sigma = \frac{dQ}{dA} \qquad \lambda = \frac{dQ}{d\ell}$$

Where dQ is the amount of charge in a small volume, surface, or length element.

<u>Example</u>

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12 \text{ nC}, \qquad q_3 = +31 \text{ nC},$$
$$q_2 = -24 \text{ nC}, \qquad q_4 = +17 \text{ nC}.$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four point charges.

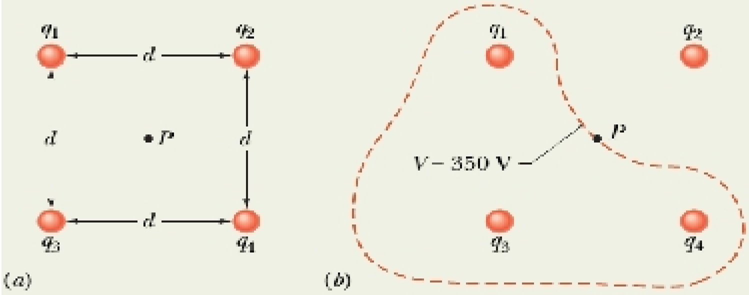


Fig. 24-8 (a) Four point charges are held fixed at the corners of a square. (b) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point P. (The curve is drawn only roughly.)

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9} \text{ C}$$
$$= 36 \times 10^{-9} \text{ C}.$$

Thus, $$V = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}}$$
$$\approx 350 \text{ V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 24-8a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P. The curve in Fig. 24-8b shows the intersection of the plane of the figure with the equipotential surface that contains point P. Any point along that curve has the same potential as point P.

Flux of an Electric Field

The word “flux” comes from the Latin (لاتيني) word meaning “to flow.” That meaning makes sense if we talk about the flow of air volume through the loop.

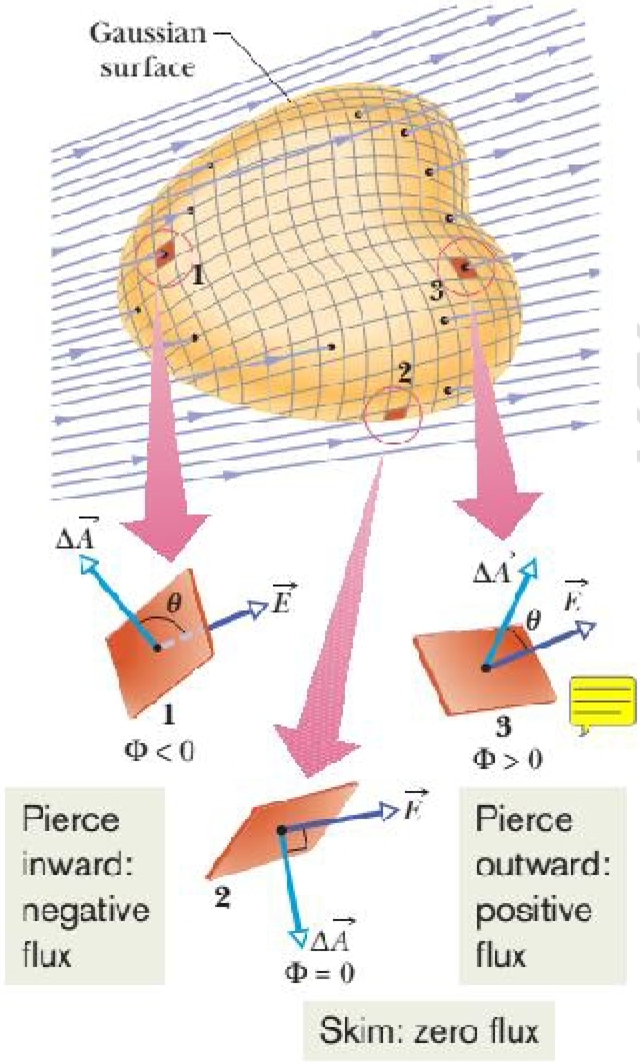


Fig. 23-3 A Gaussian surface of arbitrary shape immersed in an electric field. The surface is divided into small squares of area ΔA. The electric field vectors $\vec{E}$ and the area vectors $\Delta\vec{A}$ for three representative squares, marked 1, 2, and 3, are shown.

Which shows an (asymmetric) **<u>Gaussian surface immersed in a nonuniform electric field</u>**. Let us divide the surface into small squares of area ΔA

The number of field lines passing through a surface can be written:

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}.$$

The exact definition of the flux of the electric field through a closed surface is found by allowing the area of the squares shown in Fig. 23-3 to become smaller and smaller, approaching a differential limit dA. The area vectors then approach a differential limit .The sum of Eq. 23-3 then becomes an integral:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

Sample Problem

Flux through a closed cylinder, uniform field

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

KEY IDEA

We can find the flux Φ through the Gaussian surface by integrating the scalar product $\vec{E} \cdot d\vec{A}$ over that surface.

Calculations: We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a, the cylindrical surface b, and the right cap c. Thus, from Eq. 23-4,

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}. \qquad (23\text{-}5)$$

For all points on the left cap, the angle θ between $\vec{E}$ and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180°)\, dA = -E \int dA = -EA,$$

where $\int dA$ gives the cap's area $A\ (= \pi R^2)$. Similarly, for the

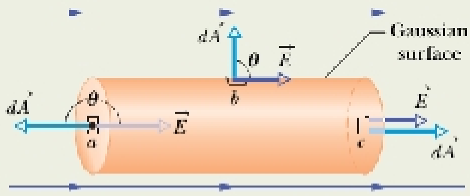


Fig. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

Sample Problem

Flux through a closed cube, nonuniform field

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

KEY IDEA

We can find the flux Φ through the surface by integrating the scalar product $\vec{E} \cdot d\vec{A}$ over each face.

Right face: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for any area element (small section) on the right face of the cube must point in the positive direction of the x axis. An example of such an element is shown in Figs. 23-5*b* and *c*, but we would have an identical vector for any other choice of an area element on that face. The most convenient way to express the vector is in unit-vector notation,

$$d\vec{A} = dA\hat{i}.$$

From Eq. 23-4, the flux Φ_r through the right face is then

$$\begin{aligned}\Phi_r &= \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}] \\ &= \int (3.0x\, dA + 0) = 3.0 \int x\, dA.\end{aligned}$$

We are about to integrate over the right face, but we note that x has the same value everywhere on that face—namely, $x = 3.0$ m. This means we can substitute that constant value for x. This can be a confusing argument. Although x is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the x axis, every point on the face has the same x coordinate. (The y and z coordinates do not matter in our integral.) Thus, we have

$$\Phi_r = 3.0 \int (3.0)\, dA = 9.0 \int dA.$$

The integral $\int dA$ merely gives us the area $A = 4.0\ \text{m}^2$ of the right face; so

$$\Phi_r = (9.0\ \text{N/C})(4.0\ \text{m}^2) = 36\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-5*d*). (2) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0$ m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Top face: The differential area vector $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-5*e*). The flux Φ_t through the top face is then

$$\begin{aligned}\Phi_t &= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)\hat{j} \cdot \hat{j}] \\ &= \int (0 + 4.0\, dA) = 4.0 \int dA \\ &= 16\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}\end{aligned}$$

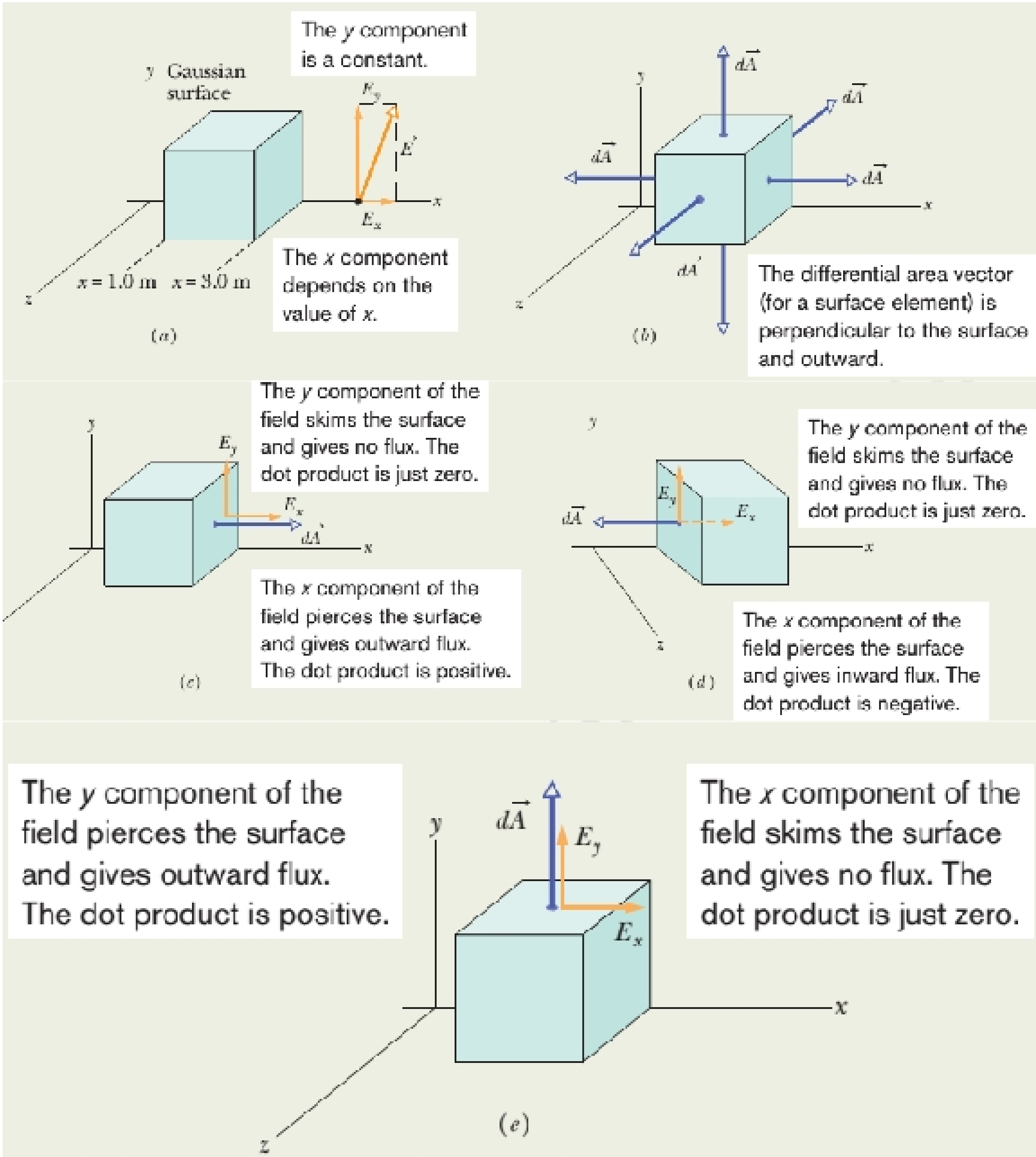


Fig. 23-5 (a) A Gaussian cube with one edge on the x axis lies within a nonuniform electric field that depends on the value of x.(b)Each differential area element has an outward vector that is perpendicular to the area. (c)Right face: the x component of the field pierces the area and produces positive (outward) flux. The y component does not pierce the area and thus does not produce any flux. (d) Left face: the x component of the field produces negative (inward) flux. (e) Top face: the y component of the field produces positive (outward) flux.

Gauss' Law

Gauss' law relates the **net flux** of an electric field through a closed surface(a Gaussian surface) to the net charge q_{enc} that is enclosed (محاط)by that surface. It tells us that

$$\varepsilon_0 \Phi = q_{\text{enc}} \qquad \text{(Gauss' law).}$$

By substituting Eq. 23-4, the definition of flux, we can also write Gauss' law as

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad \text{(Gauss' law).}$$

Let us apply these ideas to Fig. 23-6, which shows two point charges, equal in magnitude but opposite in sign, and the field lines describing the electric fields the charges set up in the surrounding space. Four Gaussian surfaces are also shown, in cross section. Let us consider each in turn.

Surface S1. The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires. (That is, in Eq. 23-6, if is positive, q_{enc} must be also.)

Surface S2. The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

Surface S3. This surface encloses no charge, and thus q_{enc} =0. Gauss' law (Eq.23-6) requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

Surface S4. This surface encloses no net charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S4as entering it.

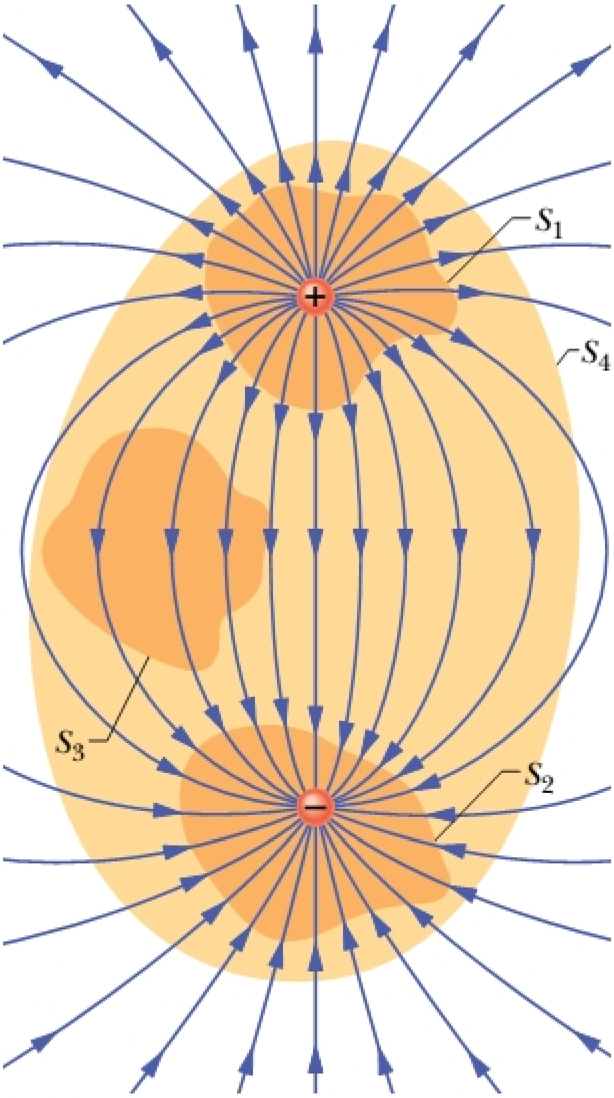


Fig. 23-6 two point charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S1encloses the positive charge. Surface S2encloses the negative charge. Surface S3encloses no charge. Surface S4encloses both charges and thus no net charge.

SUMMARY

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the surface is

$$\Phi_E = EA \cos \theta$$

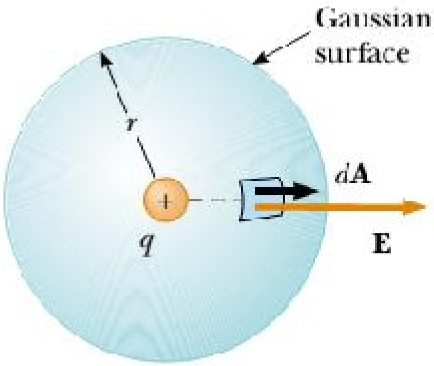
In general, the electric flux through a surface is

$$\Phi_E = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$

You need to be able to apply Equations and in a variety of situations, particularly those in which symmetry simplifies the calculation.

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge inside the surface divided by ϵ_0:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$



EXAMPLE Flux Through a Cube

Consider a uniform electric field $\mathbf{E}$ oriented in the x direction. Find the net electric flux through the surface of a cube of edges ℓ, oriented as shown in Figure 24.5.

Solution The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (③, ④, and the unnumbered ones) is zero because $\mathbf{E}$ is perpendicular to $d\mathbf{A}$ on these faces.

The net flux through faces ① and ② is

$$\Phi_E = \int_1 \mathbf{E} \cdot d\mathbf{A} + \int_2 \mathbf{E} \cdot d\mathbf{A}$$

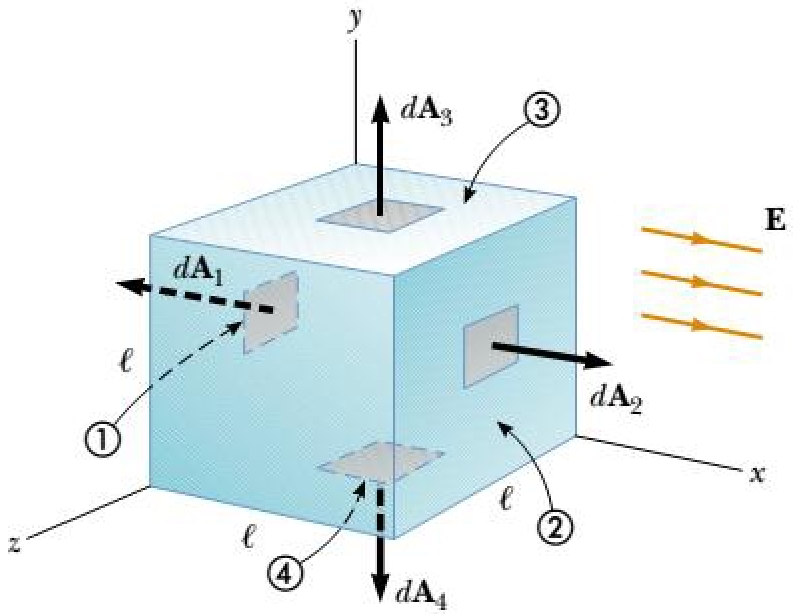


Figure A closed surface in the shape of a cube in a uniform electric field oriented parallel to the *x* axis. The net flux through the closed surface is zero. Side ④ is the bottom of the cube, and side ① is opposite side ②.

For ①, $\mathbf{E}$ is constant and directed inward but $d\mathbf{A}_1$ is directed outward ($\theta = 180°$); thus, the flux through this face is

$$\int_1 \mathbf{E} \cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA = -E \int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

For ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}_2 (\theta = 0°)$; hence, the flux through this face is

$$\int_2 \mathbf{E} \cdot d\mathbf{A} = \int_2 E(\cos 0°)\, dA = E \int_2 dA = +EA = E\ell^2$$

Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

END

EXAMPLE Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of + 1.00 μC at its center?

Solution The magnitude of the electric field 1.00 m from this charge is given by Equation 23.4,

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9 \,\mathrm{N \cdot m^2/C^2}) \frac{1.00 \times 10^{-6}\,\mathrm{C}}{(1.00\,\mathrm{m})^2}$$
$$= 8.99 \times 10^3 \,\mathrm{N/C}$$

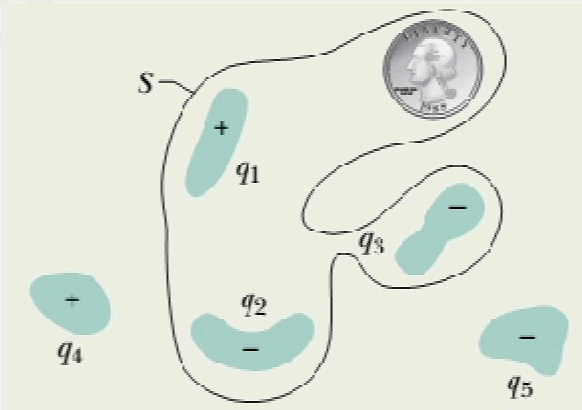
The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\,\mathrm{m^2}$) is thus

$$\Phi_E = EA = (8.99 \times 10^3\,\mathrm{N/C})(12.6\,\mathrm{m^2})$$
$$= \boxed{1.13 \times 10^5\,\mathrm{N \cdot m^2/C}}$$

END

Example **Relating the net enclosed charge and the net flux**

The Figure. shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC, and $q_3 = -3.1$ nC?



KEY IDEA

The *net* flux Φ through the surface depends on the *net* charge q_{enc} enclosed by surface S.

Calculation: The coin does not contribute to Φ because it is neutral and thus contains equal amounts of positive and negative charge. We could include those equal amounts, but they would simply sum to be zero when we calculate the *net* charge enclosed by the surface. So, let's not bother. Charges q_4 and q_5 do not contribute because they are outside surface S. They certainly send electric field lines through the surface, but as much enters as leaves and no net flux is contributed. Thus, q_{enc} is only the sum $q_1 + q_2 + q_3$ and Eq. 23-6 gives us

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}\,\mathrm{C} - 5.9 \times 10^{-9}\,\mathrm{C} - 3.1 \times 10^{-9}\,\mathrm{C}}{8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}}$$

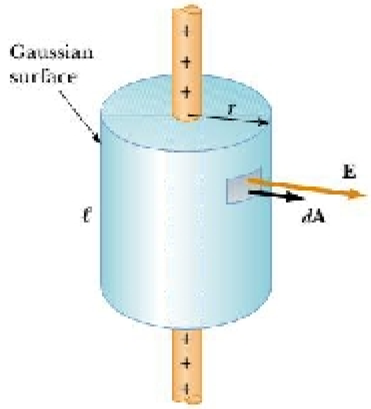
$$= -670\,\mathrm{N \cdot m^2/C}. \qquad \text{(Answer)}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

*

λ:charge per unit length

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

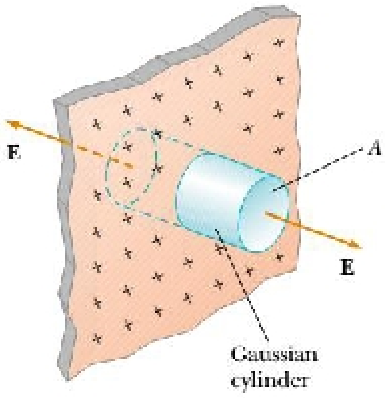


*

Noting that the total charge inside the surface is $q_{\text{in}} = \sigma A$, we use Gauss's law and find that

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \boxed{\frac{\sigma}{2\epsilon_0}}$$



DEFINITION OF CAPACITANCE

The **capacitance** C of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C \equiv \frac{Q}{\Delta V} \quad \textbf{(26.1)}$$

From following Equation, we see that capacitance has SI units of coulombs per volt. The SI unit of capacitance is the farad (F), which was named in honor of Michael Faraday:

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

*<u>**Isolated spherical capacitors**</u>

We can calculate the capacitance of an isolated spherical conductor of radius (R) and charge (Q).

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

*<u>**Parallel-Plate Capacitors**</u>

The surface charge density on either plate is $\sigma = Q/A$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

EXAMPLE

A parallel-plate capacitor has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 26.3, we find that

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\left(\frac{2.00 \times 10^{-4}\ \text{m}^2}{1.00 \times 10^{-3}\ \text{m}}\right)$$

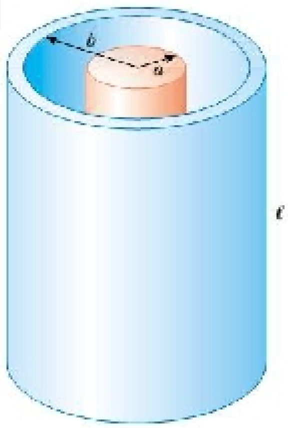
$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$

Exercise What is the capacitance for a plate separation of 3.00 mm?

Answer 0.590 pF.

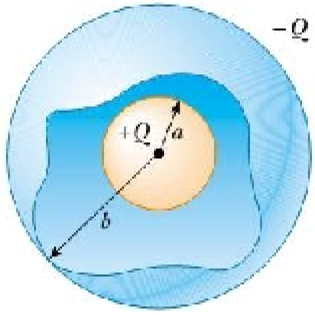
*__The Cylindrical Capacitor__

$$C = \boxed{\frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}}$$

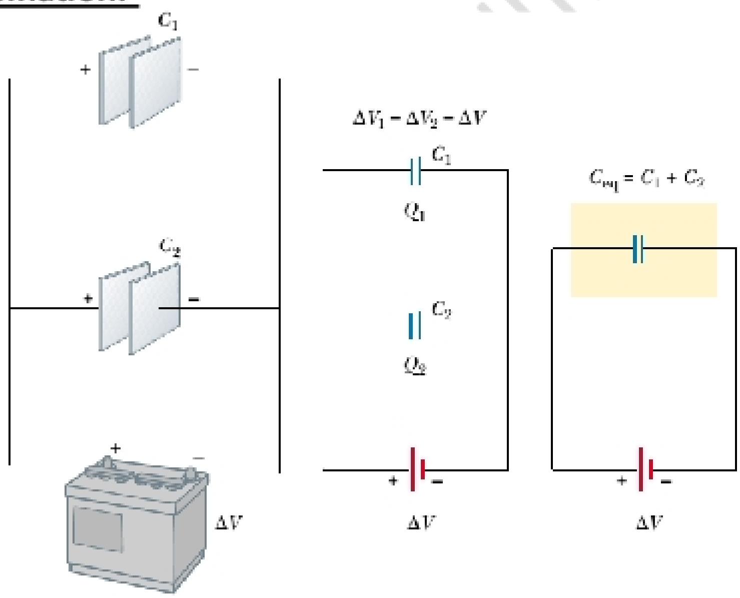


* The Spherical Capacitor

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b - a)}$$



Parallel Combination:-



The total charge Q stored by the two capacitors is

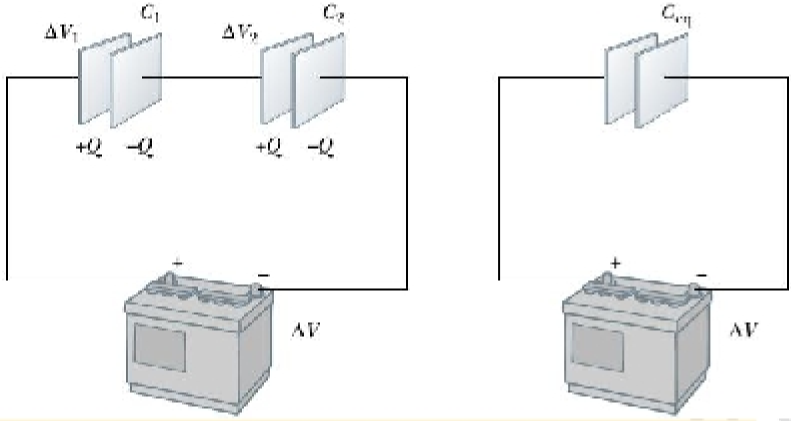
$$Q = Q_1 + Q_2$$

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

$$\Delta V_1 = \Delta V_2 = \Delta V$$

Series Combination



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \left(\begin{array}{l}\text{series} \\ \text{combination}\end{array}\right)$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$Q = Q_1 = Q_2$$

EXAMPLE

Find the equivalent capacitance between (a) and (b) for the combination of capacitors shown in Figure. All capacitances are in microfarads.

Solution:-

We reduce thecombination step by step as indicated in the figure. The (1.0)µF and (3.0)µF capacitors are in parallel and combine according to the expression

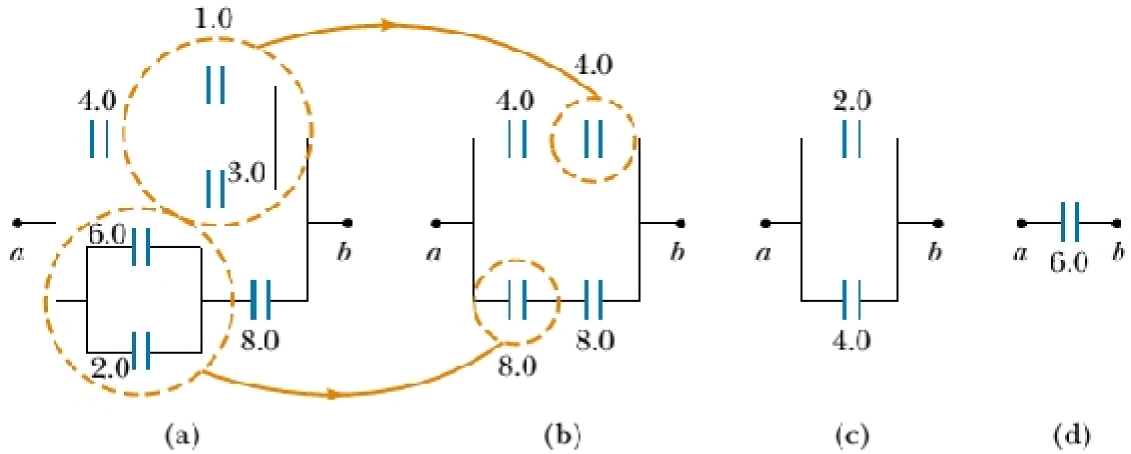
$C_{eq} = C_1 + C_2 = 4.0\ \mu F$

2.0-μF and 6.0-μF capacitors also are in parallel and have an equivalent capacitance of 8.0 μF. Thus, the upper branch in Figure 26.10b consists of two 4.0-μF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = \frac{1}{1/2.0\ \mu F} = 2.0\ \mu F$$

The lower branch in Figure 26.10b consists of two 8.0-μF capacitors in series, which combine to yield an equivalent capacitance of 4.0 μF. Finally, the 2.0-μF and 4.0-μF capacitors in Figure 26.10c are in parallel and thus have an equivalent capacitance of 6.0 μF.



Exercise Consider three capacitors having capacitances of 3.0 μF, 6.0 μF, and 12 μF. Find their equivalent capacitance when they are connected (a) in parallel and (b) in series.

Answer (a) 21 μF; (b) 1.7 μF.

Sample Problem

Charging the plates in a parallel-plate capacitor

In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m^3. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ F)(12\ V) = 3.0 \times 10^{-6}\ C.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ C}{1.602 \times 10^{-19}\ C} = 1.873 \times 10^{13}\ \text{electrons}.$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ m^2)(8.49 \times 10^{28}\ \text{electrons}/m^3)}$$
$$= 1.1 \times 10^{-12}\ m = 1.1\ \text{pm}. \qquad \text{(Answer)}$$

In common speech, we would say that the battery charges the capacitor by supplying the charged particles. But what the battery really does is set up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

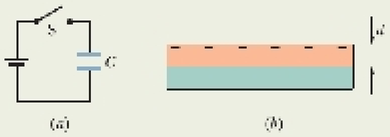


Fig. 25-7 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

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Energy Stored in an Electric Field

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be q'/C. If an extra increment of charge dq' is then transferred the increment of work required will be, from Eq. 24-7,

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

This work is stored as potential energy U in the capacitor, so that

$$U = \frac{q^2}{2C} \qquad \text{(potential energy).}$$

Or

$$U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

Energy Density

The energy density u—that is, the potential energy per unit volume between the plates —should also be uniform. We can find u by dividing the total potential energy by the volume (Ad) of the space between the plates, we obtain

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}$$

$(C = \varepsilon_0 A/d)$, this result becomes

$$u = \tfrac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

E=V/d

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density)}$$

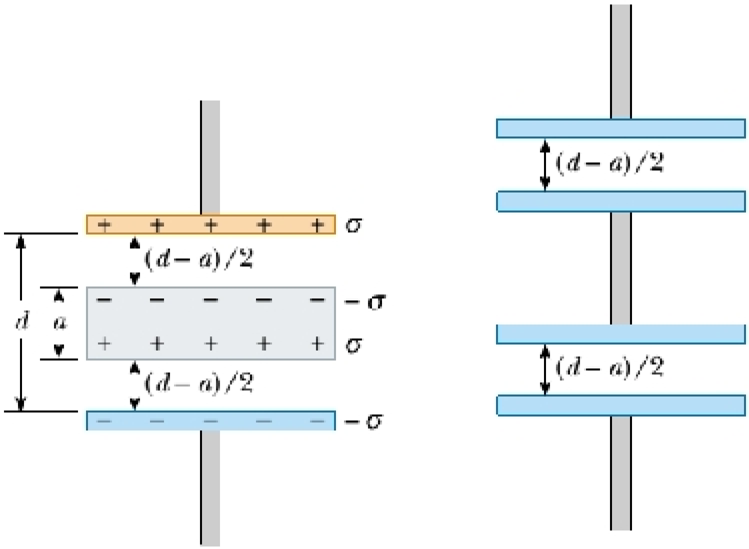
Example

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates. (a) Find the capacitance of the device.

Solution. We can solve this problem by noting that any charge that appears on one plate of the capacitor must induce a charge of equal magnitude but opposite sign on the near side of the slab, as shown in Figure a. the net charge on the slab remains zero, and the electric field inside the slab is zero. Hence, the capacitor is equivalent to two capacitors in series, each having a plate separation as shown in Figure b. Using the rule for adding two capacitors in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}} + \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}}$$

$$C = \frac{\epsilon_0 A}{d - a}$$



(a) (b)

(b) Show that the capacitance is unaffected if the metallic slab is infinitesimally thin

Solution In the result for part (a), we let $a \rightarrow 0$:

$$C = \lim_{a\to 0} \frac{\epsilon_0 A}{d - a} = \frac{\epsilon_0 A}{d}$$

which is the original capacitance.

(c) Show that the answer to part (a) does not depend on where the slab is inserted.

Solution let us imagine that the slab in Figure, is moved upward, so that the distance between the upper edge of the slab and the upper plate is b. Then, the distance between the lower edge of the slab and the lower plate is d-b-a. As in part (a), we find the total capacitance of the series combination:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{b}} + \frac{1}{\frac{\epsilon_0 A}{d - b - a}}$$

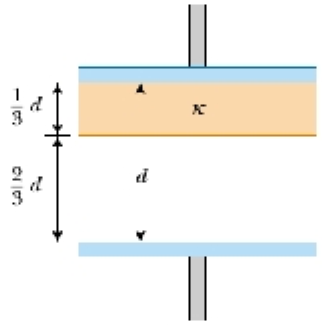
$$= \frac{b}{\epsilon_0 A} + \frac{d - b - a}{\epsilon_0 A} = \frac{d - a}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d - a}$$

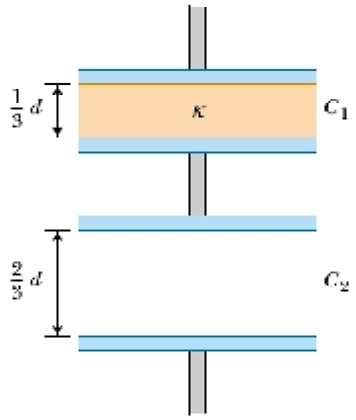
This is the same result as in part (a). It is independent of the value of b, so it does not matter where the slab is located.

Example

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant K and thickness is inserted between the plates?



(a)



(b)

Solution In Example 26.9, we found that we could insert a metallic slab between the plates of a capacitor and consider the combination as two capacitors in series. The resulting capacitance was independent of the location of the slab. Furthermore, if the thickness of the slab approaches zero, then the capacitance of the system approaches the capacitance when the slab is absent. From this, we conclude that we can insert an infinitesimally thin metallic slab anywhere between the plates of a capacitor without affecting the capacitance. Thus, let us imagine sliding an infinitesimally thin metallic slab along the bottom face of the dielectric shown in Figure 26.27a. We can then consider this system to be the series combination of the two capacitors shown in Figure 26.27b: one having a plate separation $d/3$ and filled with a dielectric, and the other having a plate separation $2d/3$ and air between its plates.

From Equations 26.15 and 26.3, the two capacitances are

$$C_1 = \frac{\kappa\epsilon_0 A}{d/3} \qquad \text{and} \qquad C_2 = \frac{\epsilon_0 A}{2d/3}$$

Using Equation 26.10 for two capacitors combined in series, we have

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa\epsilon_0 A} + \frac{2d/3}{\epsilon_0 A}$$

$$= \frac{d}{3\epsilon_0 A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\epsilon_0 A}\left(\frac{1 + 2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa + 1}\right)\frac{\epsilon_0 A}{d}$$

Because the capacitance without the dielectric is $C_0 = \epsilon_0 A/d$, we see that

$$\boxed{C = \left(\frac{3\kappa}{2\kappa + 1}\right) C_0}$$

Example

Sample Problem

Work and energy when a dielectric is inserted into a capacitor

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5 \times 10^{-12}\ \text{F})(12.5\ \text{V})^2$$
$$= 1.055 \times 10^{-9}\ \text{J} = 1055\ \text{pJ} \approx 1100\ \text{pJ}. \quad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\ \text{pJ}}{6.50}$$
$$= 162\ \text{pJ} \approx 160\ \text{pJ}. \quad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\ \text{pJ} = 893\ \text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

Sample Problem

Dielectric partially filling the gap in a capacitor

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24$ cm, $V_0 = 85.5$ V, $b = 0.780$ cm, and $\kappa = 2.61$.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$
$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \quad \text{(Answer)}$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0V_0 = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \qquad \text{(Answer)}$$

Because the battery was disconnected before the slab was inserted, the free charge is unchanged.

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

KEY IDEA

We need to apply Gauss' law, in the form of Eq. 25-36, to Gaussian surface I in Fig. 25-17.

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0\, dA \cos 0° = E_0\, dA.$$

Equation 25-36 then becomes

$$\varepsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0 \kappa E_0 A = q,$$

or

$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\varepsilon_0 \kappa A} = \frac{7.02 \times 10^{-10}\ \mathrm{C}}{(8.85 \times 10^{-12}\ \mathrm{F/m})(1)(115 \times 10^{-4}\ \mathrm{m}^2)}$$

$$= 6900\ \mathrm{V/m} = 6.90\ \mathrm{kV/m}. \qquad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

(d) What is the electric field E_1 in the dielectric slab?

KEY IDEA

Now we apply Gauss' law in the form of Eq. 25-36 to Gaussian surface II in Fig. 25-17.

Calculations: That surface encloses free charge $-q$ and induced charge $+q'$, but we ignore the latter when we use Eq. 25-36. We find

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q. \qquad (25\text{-}37)$$

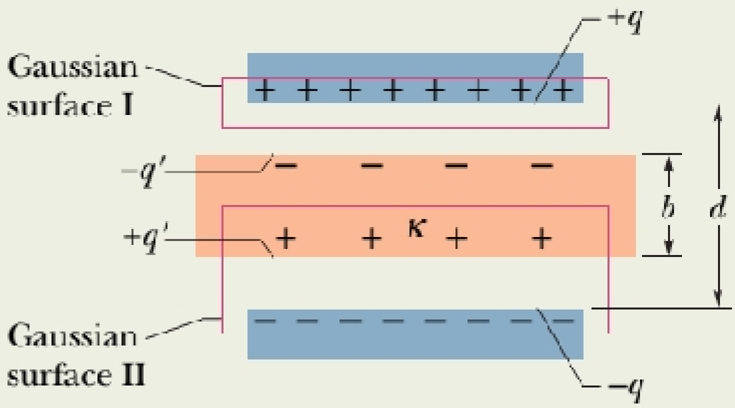


Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector $\vec{E}_1$ is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90 \text{ kV/m}}{2.61}$$

$$= 2.64 \text{ kV/m.} \qquad \text{(Answer)}$$

(e) What is the potential difference V between the plates after the slab has been introduced?

KEY IDEA

We find V by integrating along a straight line directly from the bottom plate to the top plate.

Calculation: Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 25-6 then yields

$$\begin{aligned} V &= \int_{-}^{+} E\, ds = E_0(d-b) + E_1 b \\ &= (6900\ \text{V/m})(0.0124\ \text{m} - 0.00780\ \text{m}) \\ &\quad + (2640\ \text{V/m})(0.00780\ \text{m}) \\ &= 52.3\ \text{V}. \end{aligned} \qquad \text{(Answer)}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place between the plates of the capacitor?

KEY IDEA

The capacitance C is related to the free charge q and the potential difference V via Eq. 25-1.

Calculation: Taking q from (b) and V from (e), we have

$$\begin{aligned} C &= \frac{q}{V} = \frac{7.02 \times 10^{-10}\ \text{C}}{52.3\ \text{V}} \\ &= 1.34 \times 10^{-11}\ \text{F} = 13.4\ \text{pF}. \end{aligned} \qquad \text{(Answer)}$$

This is greater than the original capacitance of 8.21 pF.

EXAMPLE

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant K and thickness is inserted between the plates Figure?

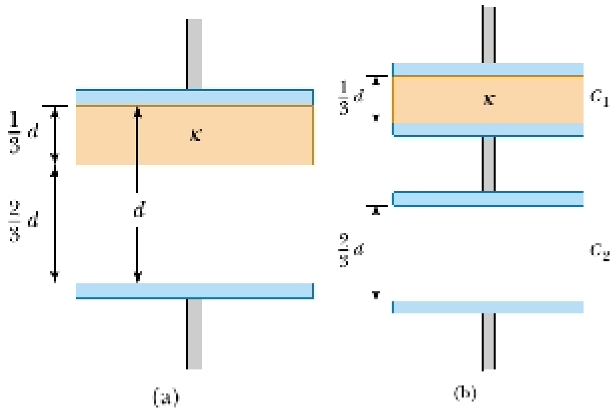


Figure (a) A parallel-plate capacitor of plate separation d partially filled with a dielectric of thickness d/3. (b) The equivalent circuit of the capacitor consists of two capacitors connected in series.

$$C_1 = \frac{\kappa\epsilon_0 A}{d/3} \qquad \text{and} \qquad C_2 = \frac{\epsilon_0 A}{2d/3}$$

Using Equation 26.10 for two capacitors combined in series, we have

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa\epsilon_0 A} + \frac{2d/3}{\epsilon_0 A}$$

$$= \frac{d}{3\epsilon_0 A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\epsilon_0 A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\epsilon_0 A}{d}$$

Because the capacitance without the dielectric is $C_0 = \epsilon_0 A/d$, we see that

$$\boxed{C = \left(\frac{3\kappa}{2\kappa+1}\right)C_0}$$

Current and Resistance

The current is the rate at which charge flows through this surface.If $\Delta Q$is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through A per unit time.

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t}$$

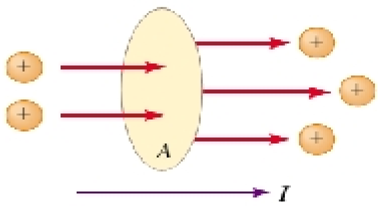
Instantaneous current I:

$$I \equiv \frac{dQ}{dt}$$

The SI unit of current is the **ampere** (A):

$$1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}}$$

The direction of the current is opposite the direction of flow of electrons.



Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

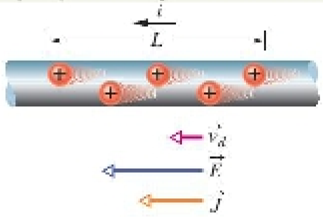
We can find the charge that passes through the plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i\, dt,$$

Drift Speed

When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to drift with a drift speed v_d in the direction opposite that of the applied electric field that causes the current.

Current is said to be due to positive charges that are propelled by the electric field.



The figure. Positive charge carriers drift at speed v_d in the direction of the applied electric field $\vec{E}$. the direction of the current density $\vec{J}$ and the sense of the current arrow are drawn in that same direction.

The number of charge carriers in a length L of the wire is (nAL), where n is the number of carriers per unit volume. The total charge of the carriers in the length L, each with charge e, is then

$$q = (nAL)e$$

Because the carriers all move along the wire with speed v_d, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$\vec{J} = (ne)\vec{v}_d$$

(ne) is the carrier charge density.

Example **Drift Speed in a Copper Wire**

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ m^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ g/cm^3$.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ g}{8.95\ g/cm^3} = 7.09\ cm^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ cm^3}(1.00 \times 10^6\ cm^3/m^3)$$
$$= 8.49 \times 10^{28}\ \text{electrons}/m^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$
$$= \frac{10.0\ C/s}{(8.49 \times 10^{28}\ m^{-3})(1.60 \times 10^{-19}\ C)(3.31 \times 10^{-6}\ m^2)}$$
$$= \boxed{2.22 \times 10^{-4}\ m/s}$$

Current density

The flow of charge through a cross section of the conductor at a particular point. To describe this flow, we can use the current density $\vec{J}$, which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude J is equal to the current per unit area through that element. We can write the amount of current through the element as $\vec{J} \cdot d\vec{A}$, where $d\vec{A}$ is the area vector of the element, perpendicular to the element. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}.$$

If parallel

$$i = \int J\, dA = J \int dA = JA$$

$$J = \frac{i}{A}$$

Sample Problem

Current density, uniform and nonuniform

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5\ \mathrm{A/m^2}$. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6*a*)?

KEY IDEA

Because the current density is uniform across the cross section, the current density J, the current i, and the cross-sectional area A are related by Eq. 26-5 ($J = i/A$).

Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020\ \mathrm{m})^2 = 9.424 \times 10^{-6}\ \mathrm{m}^2.$$

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5\ \text{A/m}^2)(9.424 \times 10^{-6}\ \text{m}^2)$$
$$= 1.9\ \text{A}. \qquad \text{(Answer)}$$

Resistance and Resistivity

The characteristic of the conductor that enters here is its electrical resistance. We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current I that results. The resistance R is then

$$R = \frac{V}{i}$$

$$1\ \text{ohm} = 1\ \Omega = 1\ \text{volt per ampere} = 1\ \text{V/A}.$$

We have done several times in other connections, we often wish to take a general view and deal not with particular objects but with materials. Here we do so by focusing not on the potential difference V across a particular resistor but on the electric field at a point in a resistive material. The resistivity ρ of the material:

$$\rho = \frac{E}{J}$$

$$\frac{\text{unit}\ (E)}{\text{unit}\ (J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\ \text{m} = \Omega \cdot \text{m}$$

$$E = V/L \quad \text{and} \quad J = i/A$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}$$

Then, the relation between Resistance and Resistivity is:-

$$R = \rho \frac{L}{A}$$

Current, Resistance, and Ohm's Law

A CURRENT (I) of electricity exists in a region when a net electric charge is transported from one point to another in that region. Suppose the charge is moving through a wire. If a charge q is transported through a given cross section of the wire in a time t, then the current through the wire is

$$I = \frac{q}{t}$$

Here, q is in coulombs, t is in seconds, and I is in *amperes* (1 A = 1 C/s). By custom the direction of the current is taken to be in the direction of flow of positive charge. Thus, a flow of electrons to the right corresponds to a current to the left.

★If no internal energy losses occur in the battery, then the potential difference between its terminals is called the *electromotive force* (emf) of the battery.

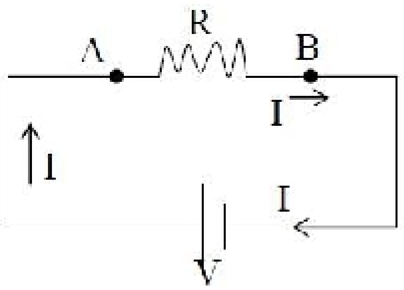
THE RESISTANCE (R) of a wire or other object is a measure of the potential difference (V) that must be impressed across the object to cause a current of one ampere to flow through it:

$$R = \frac{V}{I}$$

The unit of resistance is the *ohm*, for which the symbol Ω (Greek omega) is used. 1 Ω = 1 V/A.

OHM'S LAW

The relation $V = IR$ can be applied to any resistor, where V is the potential difference (p.d.) between the two ends of the resistor, I is the current through the resistor, and R is the resistance of the resistor under those conditions.



RESISTIVITY: The resistance R of a wire of length L and cross-sectional area A is

$$R = \rho\frac{L}{A}$$

where ρ is a constant called the *resistivity*. The resistivity is a characteristic of the material from which the wire is made. For L in m, A in m^2, and R in Ω, the units of ρ are Ω·m.

RESISTANCE VARIES WITH TEMPERATURE: If a wire has a resistance R_0 at a temperature T_0, then its resistance R at a temperature T is

$$R = R_0 + \alpha R_0(T - T_0)$$

where α is the *temperature coefficient of resistance* of the material of the wire. Usually α varies with temperature and so this relation is applicable only over a small temperature range. The units of α are K^{-1} or $°C^{-1}$.

A similar relation applies to the variation of resistivity with temperature. If ρ_0 and ρ are the resistivities at T_0 and T, respectively, then

$$\rho = \rho_0 + \alpha\rho_0(T - T_0)$$

(EX.1) Number 10 wire has a diameter of 2.59 mm. How many meters of number 10 aluminum wire are needed to give a resistance of 1.0 Ω? ρ for aluminum is $2.8 \times 10^{-8}\ \Omega \cdot m$.

From $R = \rho L/A$, we have

$$L = \frac{RA}{\rho} = \frac{(1.0\ \Omega)(\pi)(2.59 \times 10^{-3}\ m)^2/4}{2.8 \times 10^{-8}\ \Omega \cdot m} = 0.19\ km$$

(EX.2) The resistance of a coil of copper wire is 3.35 Ω at 0°C. What is its resistance at 50 °C? For copper, $\alpha = 4.3 \times 10^{-3}\ °C^{-1}$.

$$R = R_0 + \alpha R_0(T - T_0) = 3.35\ \Omega + (4.3 \times 10^{-3}\ °C^{-1})(3.35\ \Omega)(50\ °C) = 4.1\ \Omega$$

(EX.3) A wire that has a resistance of 5.0 Ω is passed through an extruder so as to make it into a new wire three times as long as the original. What is the new resistance?

We shall use $R = \rho L/A$ to find the resistance of the new wire. To find ρ, we use the original data for the wire:

$$5.0\ \Omega = \rho L_0/A_0 \qquad \text{or} \qquad \rho = (A_0/L_0)(5.0\ \Omega)$$

We were told that $L = 3L_0$. To find A in terms of A_0, we note that the volume of the wire cannot change. Hence,

$$V_0 = L_0 A_0 \qquad \text{and} \qquad V_0 = LA$$

from which

$$LA = L_0 A_0 \qquad \text{or} \qquad A = \left(\frac{L_0}{L}\right)(A_0) = \frac{A_0}{3}$$

Therefore,

$$R = \frac{\rho L}{A} = \frac{(A_0/L_0)(5.0\ \Omega)(3L_0)}{A_0/3} = 9(5.0\ \Omega) = 45\ \Omega$$

(H.W) How many electrons flow through a light bulb each second if the current through the light bulb is 0.75 A?

Ans. : 4.7×10^{18}

Example **The Resistance of a Conductor**

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

Solution From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

Example **The Resistance of Nichrome Wire**

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi (0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega \cdot \text{m}$. Thus, we can find the resistance per unit length by

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, gives

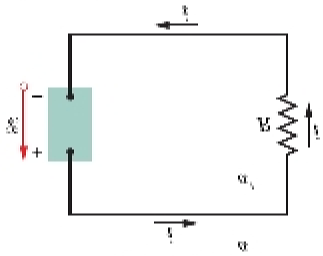
$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

Work, Energy, and Emf

Figure 27-1 shows an emf device (consider it to be a battery) that is part of a simple circuit containing a single resistance R (the symbol for resistance and a resistor is -ʌʌʌ-). The emf device keeps one of its terminals (called the positive terminal and often labeled +) at a higher electric potential than the other terminal (called the negative terminal and labeled -).
The device must do an amount of work dW on the charge dq to force it to move in this way. We define the emf of the emf device in terms of this work:

$$\mathscr{E} = \frac{dW}{dq}$$

In words, the emf of an emf device is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal. The SI unit for emf is the joule per coulomb; in Chapter 24 we defined that unit as the volt



Electrical Power

THE ELECTRICAL WORK (in joules) required to transfer a charge q (in coulombs) through a potential difference V (in volts) is given by

$$W = qV$$

When q and V are given their proper signs (i.e., voltage rises positive, and drops negative), the work will have its proper sign. Thus, to carry a positive charge through a potential rise, a positive amount of work must be done on the charge.

THE ELECTRICAL POWER (in watts) delivered by an energy source as it carries a charge q (in coulombs) through a potential rise V (in volts) in a time t (in seconds) is

$$\text{Power finished} = \frac{\text{work}}{\text{time}}$$

$$\mathbf{P} = \frac{Vq}{t}$$

Because $q/t = I$, this can be rewritten as $\quad \mathbf{P} = VI \quad$ where I is in amperes.

THE POWER LOSS IN A RESISTOR is found by replacing V in VI by IR, or by replacing I in VI by V/R, to obtain

$$\mathbf{P} = VI = I^2R = \frac{V^2}{R}$$

(EX.1) An electric motor takes 5.0 A from a 110 V line. Determine the power input and the energy, in J and kW·h, supplied to the motor in 2.0 h.

$$\text{Power} = \mathbf{P} = VI = (110\text{ V})(5.0\text{ A}) = 0.55\text{ kW}$$
$$\text{Energy} = \mathbf{P}t = (550\text{ W})(7200\text{ s}) = 4.0\text{ MJ}$$
$$= (0.55\text{ kW})(2.0\text{ h}) = 1.1\text{ kW·h}$$

(EX.2) The lights on a car are inadvertently left on. They dissipate 95.0 W. About how long will it take for the fully charged 12.0 V car battery to run down if the battery is rated at 150 ampere-hours (A·h)?

As an approximation, assume the battery maintains 12.0 V until it goes dead. Its 150 A·h rating means it can supply the energy equivalent of a 150 A current that flows for 1.00 h (3600 s). Therefore, the total energy the battery can supply is

$$\text{Total output energy} = (\text{power})(\text{time}) = (VI)t = (12.0\text{ V} \times 150\text{ A})(3600\text{ s}) = 6.48 \times 10^6\text{ J}$$

The energy consumed by the lights in a time t is

$$\text{Energy dissipated} = (95\text{ W})(t)$$

Equating these two energies and solving for t, we find $t = 6.82 \times 10^4\text{ s} = 18.9\text{ h}$.

(H.W) An electric iron of resistance 20 Ω takes a current of 5.0 A. Calculate the thermal energy, in joules, developed in 30 s.

Ans. : 15000 J

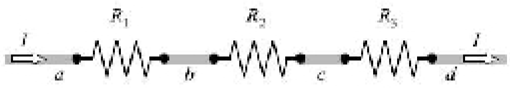
Equivalent Resistance

RESISTORS IN SERIES: A typical case is shown in Fig. (*a*). For several resistors in series, their equivalent resistance R_{eq} is given by

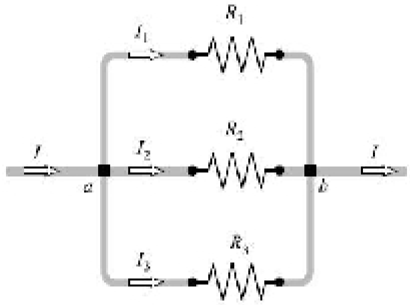
$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad \text{(series combination)}$$

Observe that resistances in series combine like capacitances in parallel. It is assumed that all connection wire is effectively resistanceless.

In a series combination, the current through each resistance is the same as that through all the others. The potential drop (p.d.) across the combination is equal to the sum of the individual potential drops. *The equivalent resistance in series is always greater than the largest of the individual resistances.*



(*a*) Resistors in series



(*b*) Resistors in parallel

RESISTORS IN PARALLEL: A typical case is shown in Fig. (*b*), where points *a* and *b* are nodes. Their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad \text{(parallel combination)}$$

The equivalent resistance in parallel is always less than the smallest of the individual resistances. Connecting additional resistances in parallel decreases R_{eq} for the combination. Observe that resistances in parallel combine like capacitances in series.

The potential drop V across each resistor in a parallel combination is the same as the potential drop across each of the others. The current through the nth resistor is $I_n = V/R_n$ and the total current entering the combination is equal to the sum of the individual branch currents.

(EX.1) Derive the formula for the equivalent resistance R_{eq} of resistors R_1, R_2, and R_3 (*a*) in series and (*b*) in parallel, as shown in Fig. (*a*) and (*b*).

(*a*) For the series network,

$$V_{ad} = V_{ab} + V_{bc} + V_{cd} = IR_1 + IR_2 + IR_3$$

since the current I is the same in all three resistors. Dividing by I gives

$$\frac{V_{ad}}{I} = R_1 + R_2 + R_3 \qquad \text{or} \qquad R_{eq} = R_1 + R_2 + R_3$$

since V_{ad}/I is by definition the equivalent resistance R_{eq} of the network.

(*b*) The p.d. is the same for all three resistors, whence

$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

Since the line current I is the sum of the branch currents,

$$I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

Dividing by V_{ab} gives

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{or} \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

since V_{ab}/I is by definition the equivalent resistance R_{eq} of the network.

(EX.2) A 120-V house circuit has the following light bulbs turned on: 40.0 W, 60.0 W, and 75.0 W. Find the equivalent resistance of these lights.

House circuits are so constructed that each device is connected in parallel with the others. From $P = VI = V^2/R$, we have for the first bulb

$$R_1 = \frac{V^2}{P_1} = \frac{(120\ \text{V})^2}{40.0\ \text{W}} = 360\ \Omega$$

Similarly, $R_2 = 240\ \Omega$ and $R_3 = 192\ \Omega$. Because they are in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{360\ \Omega} + \frac{1}{240\ \Omega} + \frac{1}{192\ \Omega} \qquad \text{or} \qquad R_{eq} = 82.3\ \Omega$$

As a check, we note that the total power drawn from the line is 40.0 W + 60.0 W + 75.0 W = 175.0 W. Then, using $P = V^2/R$, we have

$$R_{eq} = \frac{V^2}{\text{total power}} = \frac{(120\ \text{V})^2}{175.0\ \text{W}} = 82.3\ \Omega$$

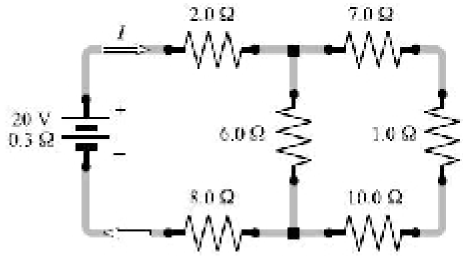
(H.W) Several 40-Ω resistors are to be connected so that 15 A flows from a 120-V source. How can this be done?

Ans: n=5

(EX.3) For each the shown in Fig. , determine the current I through the battery.

The 7.0-Ω, 1.0-Ω, and 10.0-Ω resistors are in series; their joint resistance is 18.0 Ω. Then 18.0 Ω is in parallel with 6.0 Ω; their combined resistance R_1 is given by

$$\frac{1}{R_1}=\frac{1}{18.0\ \Omega}+\frac{1}{6.0\ \Omega} \qquad \text{or} \qquad R_1 = 4.5\ \Omega$$



Hence, the equivalent resistance of the entire circuit is

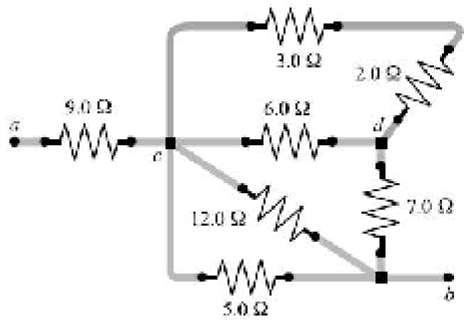
$$R_{eq} = 4.5\ \Omega + 2.0\ \Omega + 8.0\ \Omega + 0.3\ \Omega = 14.8\ \Omega$$

and the battery current is

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{20\ \text{V}}{14.8\ \Omega} = 1.4\ \text{A}$$

(H.W) Find the equivalent resistance between points a and b for the combination shown in Fig.

Ans: 11.6 Ω.



KIRCHHOFF'S RULES

As we saw in the preceding section, we can analyze simple circuits using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop. The procedure for analyzing more complex circuits is greatly simplified if we use two principles called **Kirchhoff's rules:**

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

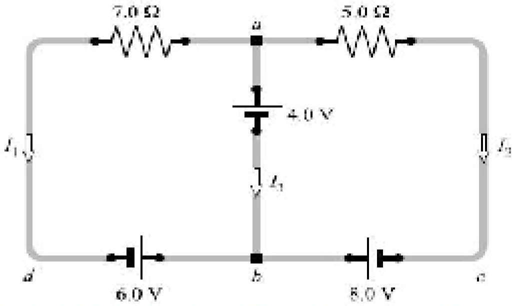
2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0$$

In justifying our claim that Kirchhoff's second rule is a statement of conservation of energy, we imagined carrying a charge around a loop. When applying this rule, we imagine *traveling* around the loop and consider changes in *electric potential,* rather than the changes in *potential energy* described in the previous paragraph. You should note the following sign conventions when using the second rule:

- Because charges move from the high-potential end of a resistor to the low-potential end, if a resistor is traversed in the direction of the current, the change in potential ΔV across the resistor is $-IR$ (Fig. 28.12a).
- If a resistor is traversed in the direction *opposite* the current, the change in potential ΔV across the resistor is $+IR$ (Fig. 28.12b).
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from $-$ to $+$), the change in potential ΔV is $+\mathcal{E}$ (Fig. 28.12c). The emf of the battery increases the electric potential as we move through it in this direction.
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from $+$ to $-$), the change in potential ΔV is $-\mathcal{E}$ (Fig. 28.12d). In this case the emf of the battery reduces the electric potential as we move through it.

(EX.1) Find the currents in the circuit shown in Fig.



This circuit cannot be reduced further because it contains no resistors in simple series or parallel combinations. We therefore revert to Kirchhoff's rules. If the currents had not been labeled and shown by arrows, we would do that first. No special care need be taken in assigning the current directions, since those chosen incorrectly will simply give negative numerical values.

We apply the node rule to node b in Fig. :

$$\text{Current into } b = \text{current out of } b$$

$$I_1 + I_2 + I_3 = 0 \qquad (1)$$

Next we apply the loop rule to loop $adba$. In volts,

$$-7.0\,I_1 + 6.0 + 4.0 = 0 \qquad \text{or} \qquad I_1 = \frac{10.0}{7.0}\,\text{A}$$

(Why must the term $7.0\,I_1$ have a negative sign?) We then apply the loop rule to loop $abca$. In volts,

$$-4.0 - 8.0 + 5.0\,I_2 = 0 \qquad \text{or} \qquad I_2 = \frac{12.0}{5.0}\,\text{A}$$

(Why must the signs be as written?)

Now we return to Eq. (1) to find

$$I_3 = -I_1 - I_2 = -\frac{10.0}{7.0} - \frac{12.0}{5.0} = \frac{-50-84}{35} = -3.8\ \text{A}$$

The minus sign tells us that I_3 is opposite in direction to that shown in the figure.

(EX.2) Each of the cells shown in Fig. has an emf of 1.50 V and a 0.075 0-Ω internal resistance. Find I_1, I_2, and I_3.

Applying the node rule to point a gives

$$I_1 = I_2 + I_3 \qquad (1)$$

Applying the loop rule to loop $abcea$ gives, in volts,

$$-(0.0750)I_2 + 1.50 - (0.075\,0)I_2 + 1.50 - 3.00\,I_1 = 0$$

or

$$3.00\,I_1 + 0.150\,I_2 = 3.00 \qquad (2)$$

Also, for loop $adcea$,

$$-(0.075\,0)I_3 + 1.50 - (0.075\,0)I_3 + 1.50 - 3.00\,I_1 = 0$$

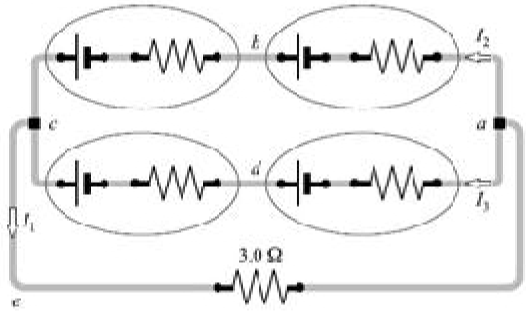
or

$$3.00\,I_1 + 0.150\,I_3 = 3.00 \qquad (3)$$

We solve Eq. (2) for $3.00\,I_1$ and substitute in Eq. (3) to get

$$3.00 - 0.150\,I_3 + 0.150\,I_2 = 3.00 \qquad \text{or} \qquad I_2 = I_3$$

as we might have guessed from the symmetry of the problem. Then Eq. (1) yields $I_1 = 2I_2$

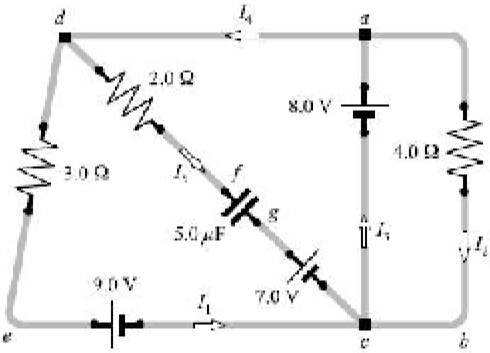


and substituting this in Eq. (2) gives

$$6.00\,I_2 + 0.150\,I_2 = 3.00 \qquad \text{or} \qquad I_2 = 0.488\ \text{A}$$

Then, $I_3 = I_2 = 0.488$ A and $I_1 = 2I_2 = 0.976$ A.

(EX.3) The currents are steady in the circuit of Fig. . Find I_1, I_2, I_3, I_4, I_5, and the charge on the capacitor.



The capacitor passes no current when charged, and so $I_5 = 0$. Consider loop *acba*. The loop rule gives

$$-8.0 + 4.0\,I_2 = 0 \qquad \text{or} \qquad I_2 = 2.0\text{ A}$$

Using loop *adeca* gives

$$-3.0I_1 - 9.0 + 8.0 = 0 \qquad \text{or} \qquad I_1 = -0.33\text{ A}$$

Applying the node rule at point c results in

$$I_1 + I_5 + I_2 = I_3 \qquad \text{or} \qquad I_3 = 1.67\text{ A} = 1.7\text{ A}$$

and at point a, in

$$I_3 = I_4 + I_2 \qquad \text{or} \qquad I_4 = -0.33\text{ A}$$

(We should have realized this at once, because $I_5 = 0$ and so $I_4 = I_1$.)

To find the charge on the capacitor, we need the voltage V_{fg} across it. Applying the loop rule to loop *dfgced* gives

$$-2.0I_5 + V_{fg} - 7.0 + 9.0 + 3.0I_1 = 0 \qquad \text{or} \qquad 0 + V_{fg} - 7.0 + 9.0 - 1.0 = 0$$

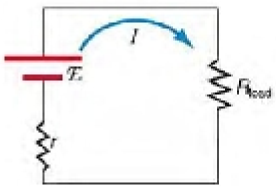
from which $V_{fg} = -1.0$ V. The minus sign tells us that plate g is negative. The capacitor's charge is

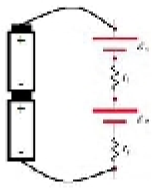
$$Q = CV = (5.0\,\mu\text{F})(1.0\text{ V}) = 5.0\,\mu\text{C}$$

* Multiple Voltage Sources:

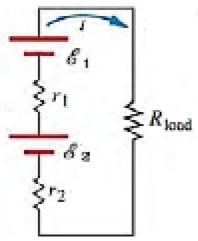
There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. Series connections of voltage sources are common for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

$$I = \frac{\text{emf}}{R_{\text{load}} + r}$$

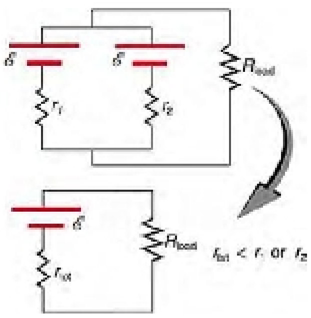




$$I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$$



There are two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.



$$I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$$

Potential Difference Between Two Points

The internal resistance reduces the potential difference between the terminals.

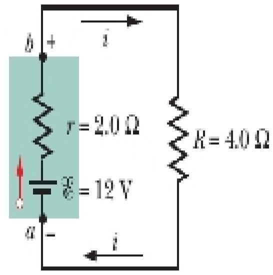


Fig. Points a and b, which are at the terminals of a real battery, differ in potential.

We often want to find the potential difference between two points in a circuit. For example, in Fig. , what is the potential difference $V_b - V_a$ between points a and b? To find out, let's start at point a (at potential V_a) and move through the battery to point b (at potential V_b) while keeping track of the potential changes we encounter. When we pass through the battery's emf, the potential increases by $\mathscr{E}$. When we pass through the battery's internal resistance r, we move in the direction of the current and thus the potential decreases by ir. We are then at the potential of point b and we have

$$V_a + \mathscr{E} - ir = V_b,$$

or

$$V_b - V_a = \mathscr{E} - ir.$$

To evaluate this expression, we need the current i. Note that the circuit is the same as in Fig. 27-4a, for which Eq. 27-4 gives the current as

$$i = \frac{\mathscr{E}}{R + r}.$$

Substituting this equation into Eq. 27-8 gives us

$$V_b - V_a = \mathscr{E} - \frac{\mathscr{E}}{R + r} r$$

$$= \frac{\mathscr{E}}{R + r} R.$$

Now substituting the data given in Fig. 27-6, we have

$$V_b - V_a = \frac{12\text{ V}}{4.0\ \Omega + 2.0\ \Omega} 4.0\ \Omega = 8.0\text{ V}.$$

Suppose, instead, we move from a to b counterclockwise, passing through resistor R rather than through the battery. Because we move opposite the current, the potential increases by iR. Thus,

$$V_a + iR = V_b$$

or

$$V_b - V_a = iR.$$

the same result, $V_b - V_a = 8.0$ V. In general,

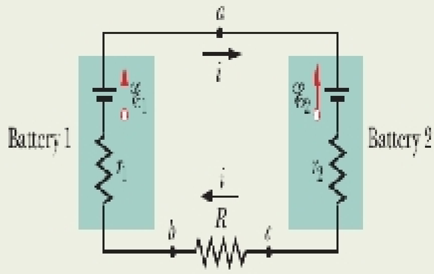
Sample Problem

Single-loop circuit with two real batteries

The emfs and resistances in the circuit of Fig. 27-8*a* have the following values:

$$\mathscr{E}_1 = 4.4\ \text{V}, \quad \mathscr{E}_2 = 2.1\ \text{V},$$

$$r_1 = 2.3\ \Omega, \quad r_2 = 1.8\ \Omega, \quad R = 5.5\ \Omega.$$



(a) What is the current i in the circuit?

KEY IDEA

We can get an expression involving the current i in this single-loop circuit by applying the loop rule.

Calculations: Although knowing the direction of i is not necessary, we can easily determine it from the emfs of the two batteries. Because $\mathscr{E}_1$ is greater than $\mathscr{E}_2$, battery 1 controls the direction of i, so the direction is clockwise. (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) Let us then apply the loop rule by going counterclockwise—against the current—and starting at point a. We find

$$-\mathscr{E}_1 + ir_1 + iR + ir_2 + \mathscr{E}_2 = 0.$$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than a. Also, take the time to compare this equation term by term with Fig. 27-8*b*, which shows the potential changes graphically (with the potential at point a arbitrarily taken to be zero).

Solving the above loop equation for the current i, we obtain

$$i = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R + r_1 + r_2} = \frac{4.4\ \text{V} - 2.1\ \text{V}}{5.5\ \Omega + 2.3\ \Omega + 1.8\ \Omega}$$

$$= 0.2396\ \text{A} \approx 240\ \text{mA}. \qquad \text{(Answer)}$$

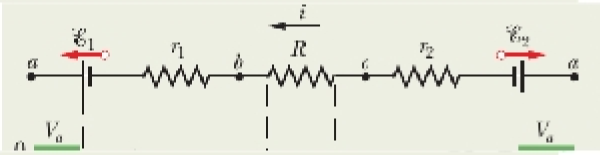
(b) What is the potential difference between the terminals of battery 1 in Fig. 27-8*a*?

KEY IDEA

We need to sum the potential differences between points *a* and *b*.

Calculations: Let us start at point *b* (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point *a* (effectively the positive terminal), keeping track of potential changes. We find that

$$V_b - ir_1 + \mathscr{E}_1 = V_a,$$



which gives us

$$\begin{aligned} V_a - V_b &= -ir_1 + \mathscr{E}_1 \\ &= -(0.2396\ \text{A})(2.3\ \Omega) + 4.4\ \text{V} \\ &= +3.84\ \text{V} \approx 3.8\ \text{V}, \qquad \text{(Answer)} \end{aligned}$$

Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

Sample Problem

Resistors in parallel and in series

Figure 27-11*a* shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega,\quad R_2 = 20\ \Omega,\quad \mathscr{E} = 12\ \text{V},$$
$$R_3 = 30\ \Omega,\quad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?

KEY IDEA

Noting that the current through the battery must also be the current through R_1, we see that we might find the current by applying the loop rule to a loop that includes R_1 because the current would be included in the potential difference across R_1.

Incorrect method: Either the left-hand loop or the big loop should do. Noting that the emf arrow of the battery points upward, so the current the battery supplies is clockwise, we might apply the loop rule to the left-hand loop, clockwise from point *a*. With *i* being the current through the battery, we would get

$$+\mathscr{E} - iR_1 - iR_2 - iR_4 = 0 \qquad \text{(incorrect).}$$

However, this equation is incorrect because it assumes that R_1, R_2, and R_4 all have the same current *i*. Resistances R_1 and R_4 do have the same current, because the current passing through R_4 must pass through the battery and then through R_1 with no change in value. However, that current splits at junction point *b*—only part passes through R_2, the rest through R_3.

Dead-end method: To distinguish the several currents in the circuit, we must label them individually as in Fig. 27-11*b*. Then, circling clockwise from *a*, we can write the loop rule for the left-hand loop as

$$+\mathscr{E} - i_1R_1 - i_2R_2 - i_1R_4 = 0.$$

Unfortunately, this equation contains two unknowns, i_1 and i_2; we would need at least one more equation to find them.

Successful method: A much easier option is to simplify the circuit of Fig. 27-11*b* by finding equivalent resistances. Note carefully that R_1 and R_2 are *not* in series and thus cannot be replaced with an equivalent resistance. However, R_2 and R_3 are in parallel, so we can use either Eq. 27-24 or Eq. 27-25 to find their equivalent resistance R_{23}. From the latter,

$$R_{23} = \frac{R_2R_3}{R_2 + R_3} = \frac{(20\ \Omega)(30\ \Omega)}{50\ \Omega} = 12\ \Omega.$$

We can now redraw the circuit as in Fig. 27-11*c*; note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23}. For this simple one-loop circuit, the loop rule (applied clockwise from point *a* as in Fig. 27-11*d*) yields

$$+\mathscr{E} - i_1R_1 - i_1R_{23} - i_1R_4 = 0.$$

Substituting the given data, we find

$$12\ \text{V} - i_1(20\ \Omega) - i_1(12\ \Omega) - i_1(8.0\ \Omega) = 0,$$

which gives us

$$i_1 = \frac{12\ \text{V}}{40\ \Omega} = 0.30\ \text{A}. \qquad \text{(Answer)}$$

(b) What is the current i_2 through R_2?

KEY IDEAS

(1) We must now work backward from the equivalent circuit of Fig. 27-11*d*, where R_{23} has replaced R_2 and R_3. (2) Because R_2 and R_3 are in parallel, they both have the same potential difference across them as R_{23}.

Working backward: We know that the current through R_{23} is $i_1 = 0.30$ A. Thus, we can use Eq. 26-8 ($R = V/i$) and Fig. 27-11*e* to find the potential difference V_{23} across R_{23}. Setting $R_{23} = 12\ \Omega$ from (a), we write Eq. 26-8 as

$$V_{23} = i_1R_{23} = (0.30\ \text{A})(12\ \Omega) = 3.6\ \text{V}.$$

The potential difference across R_2 is thus also 3.6 V (Fig. 27-11*f*), so the current i_2 in R_2 must be, by Eq. 26-8 and Fig. 27-11*g*,

$$i_2 = \frac{V_2}{R_2} = \frac{3.6\ \text{V}}{20\ \Omega} = 0.18\ \text{A}. \qquad \text{(Answer)}$$

(c) What is the current i_3 through R_3?

KEY IDEAS

We can answer by using either of two techniques: (1) Apply Eq. 26-8 as we just did. (2) Use the junction rule, which tells us that at point *b* in Fig. 27-11*b*, the incoming current i_1 and the outgoing currents i_2 and i_3 are related by

$$i_1 = i_2 + i_3.$$

Calculation: Rearranging this junction-rule result yields the result displayed in Fig. 27-11*g*:

$$i_3 = i_1 - i_2 = 0.30\ \text{A} - 0.18\ \text{A}$$
$$= 0.12\ \text{A}. \qquad \text{(Answer)}$$

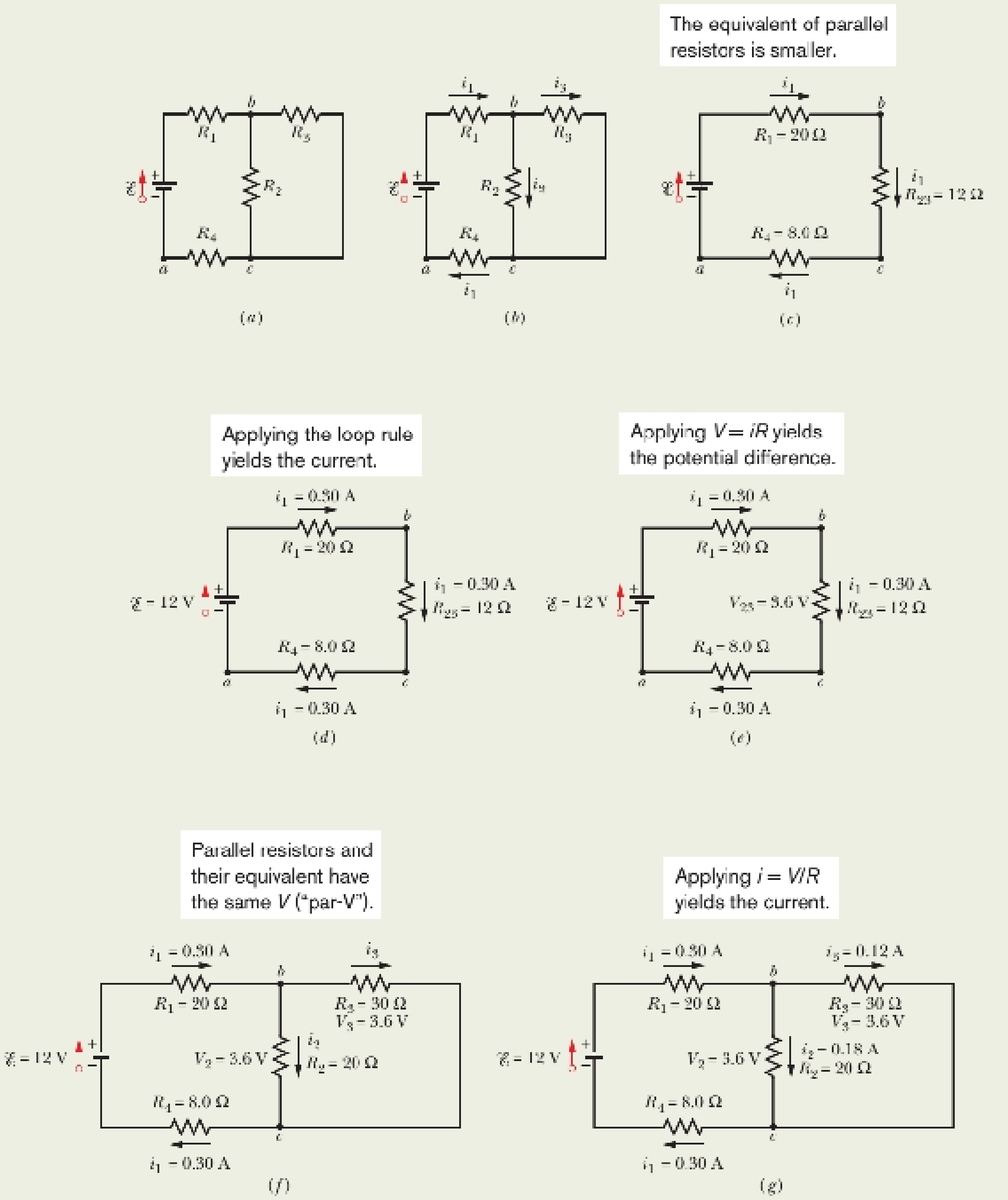


Fig. 27-11 (*a*) A circuit with an ideal battery. (*b*) Label the currents. (*c*) Replacing the parallel resistors with their equivalent. (*d*) – (g) Working backward to find the currents through the parallel resistors.

Sample Problem

Many real batteries in series and in parallel in an electric fish

Electric fish are able to generate current with biological cells called *electroplaques*, which are physiological emf devices. The electroplaques in the type of electric fish known as a South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. The arrangement is suggested in Fig. 27-12*a*; each electroplaque has an emf $\mathscr{E}$ of 0.15 V and an internal resistance r of 0.25 Ω. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the animal's head and the other near its tail.

(a) If the water surrounding the eel has resistance $R_w = 800\ \Omega$, how much current can the eel produce in the water?

KEY IDEA

We can simplify the circuit of Fig. 27-12*a* by replacing combinations of emfs and internal resistances with equivalent emfs and resistances.

Calculations: We first consider a single row. The total emf $\mathscr{E}_{row}$ along a row of 5000 electroplaques is the sum of the emfs:

$$\mathscr{E}_{row} = 5000\mathscr{E} = (5000)(0.15\ \text{V}) = 750\ \text{V}.$$

The total resistance R_{row} along a row is the sum of the internal resistances of the 5000 electroplaques:

$$R_{row} = 5000r = (5000)(0.25\ \Omega) = 1250\ \Omega.$$

We can now represent each of the 140 identical rows as having a single emf $\mathscr{E}_{row}$ and a single resistance R_{row} (Fig. 27-12*b*).

In Fig. 27-12*b*, the emf between point *a* and point *b* on any row is $\mathscr{E}_{row} = 750$ V. Because the rows are identical and because they are all connected together at the left in Fig. 27-12*b*, all points *b* in that figure are at the same electric potential. Thus, we can consider them to be connected so that there is only a single point *b*. The emf between point *a* and this single point *b* is $\mathscr{E}_{row} = 750$ V, so we can draw the circuit as shown in Fig. 27-12*c*.

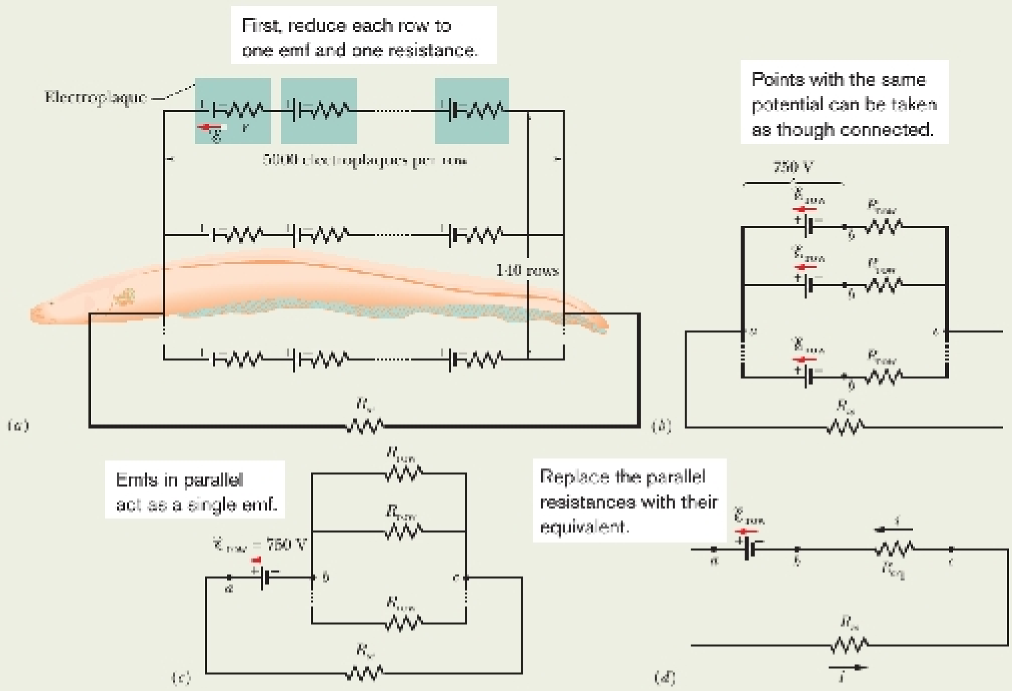


Fig. 27-12 (*a*) A model of the electric circuit of an eel in water. Each electroplaque of the eel has an emf $\mathscr{E}$ and internal resistance r. Along each of 140 rows extending from the head to the tail of the eel, there are 5000 electroplaques. The surrounding water has resistance R_w. (*b*) The emf $\mathscr{E}_{row}$ and resistance R_{row} of each row. (*c*) The emf between points *a* and *b* is $\mathscr{E}_{row}$. Between points *b* and *c* are 140 parallel resistances R_{row}. (*d*) The simplified circuit, with R_{eq} replacing the parallel combination.

Between points b and c in Fig. 27-12c are 140 resistances $R_{row} = 1250\ \Omega$, all in parallel. The equivalent resistance R_{eq} of this combination is given by Eq. 27-24 as

$$\frac{1}{R_{eq}} = \sum_{j=1}^{140} \frac{1}{R_j} = 140\frac{1}{R_{row}},$$

or
$$R_{eq} = \frac{R_{row}}{140} = \frac{1250\ \Omega}{140} = 8.93\ \Omega.$$

Replacing the parallel combination with R_{eq}, we obtain the simplified circuit of Fig. 27-12d. Applying the loop rule to this circuit counterclockwise from point b, we have

$$\mathscr{E}_{row} - iR_w - iR_{eq} = 0.$$

Solving for i and substituting the known data, we find

$$i = \frac{\mathscr{E}_{row}}{R_w + R_{eq}} = \frac{750\ \text{V}}{800\ \Omega + 8.93\ \Omega}$$
$$= 0.927\ \text{A} \approx 0.93\ \text{A}. \qquad \text{(Answer)}$$

If the head or tail of the eel is near a fish, some of this current could pass along a narrow path through the fish, stunning or killing it.

(b) How much current i_{row} travels through each row of Fig. 27-12a?

KEY IDEA

Because the rows are identical, the current into and out of the eel is evenly divided among them.

Calculation: Thus, we write

$$i_{row} = \frac{i}{140} = \frac{0.927\ \text{A}}{140} = 6.6 \times 10^{-3}\ \text{A}. \qquad \text{(Answer)}$$

Thus, the current through each row is small, about two orders of magnitude smaller than the current through the water. This tends to spread the current through the eel's body, so that the eel need not stun or kill itself when it stuns or kills a fish.

Sample Problem

Multiloop circuit and simultaneous loop equations

Figure 27-13 shows a circuit whose elements have the following values:

$$\mathscr{E}_1 = 3.0\ \text{V}, \quad \mathscr{E}_2 = 6.0\ \text{V},$$
$$R_1 = 2.0\ \Omega, \quad R_2 = 4.0\ \Omega.$$

The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.

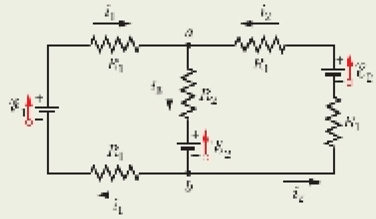


Fig. 27-13 A multiloop circuit with three ideal batteries and five resistances.

KEY IDEAS

It is not worthwhile to try to simplify this circuit, because no two resistors are in parallel, and the resistors that are in series (those in the right branch or those in the left branch) present no problem. So, our plan is to apply the junction and loop rules.

Junction rule: Using arbitrarily chosen directions for the currents as shown in Fig. 27-13, we apply the junction rule at point a by writing

$$i_3 = i_1 + i_2. \qquad (27\text{-}26)$$

An application of the junction rule at junction b gives only the same equation, so we next apply the loop rule to any two of the three loops of the circuit.

Left-hand loop: We first arbitrarily choose the left-hand loop, arbitrarily start at point b, and arbitrarily traverse the loop in the clockwise direction, obtaining

$$-i_1R_1 + \mathscr{E}_1 - i_1R_1 - (i_1 + i_2)R_2 - \mathscr{E}_2 = 0,$$

where we have used $(i_1 + i_2)$ instead of i_3 in the middle branch. Substituting the given data and simplifying yield

$$i_1(8.0\ \Omega) + i_2(4.0\ \Omega) = -3.0\ \text{V}. \qquad (27\text{-}27)$$

Right-hand loop: For our second application of the loop rule, we arbitrarily choose to traverse the right-hand loop counterclockwise from point b, finding

$$-i_2R_1 + \mathscr{E}_2 - i_2R_1 - (i_1 + i_2)R_2 - \mathscr{E}_2 = 0.$$

Substituting the given data and simplifying yield

$$i_1(4.0\ \Omega) - i_2(8.0\ \Omega) = 0. \qquad (27\text{-}28)$$

Combining equations: We now have a system of two equations (Eqs. 27-27 and 27-28) in two unknowns (i_1 and i_2) to solve either "by hand" (which is easy enough here) or with a "math package." (One solution technique is Cramer's rule, given in Appendix E.) We find

$$i_1 = -0.50\ \text{A}. \qquad (27\text{-}29)$$

(The minus sign signals that our arbitrary choice of direction for i_1 in Fig. 27-13 is wrong, but we must wait to correct it.) Substituting $i_1 = -0.50$ A into Eq. 27-28 and solving for i_2 then give us

$$i_2 = 0.25\ \text{A}. \qquad \text{(Answer)}$$

With Eq. 27-26 we then find that

$$i_3 = i_1 + i_2 = -0.50\ \text{A} + 0.25\ \text{A}$$
$$= -0.25\ \text{A}.$$

The positive answer we obtained for i_2 signals that our choice of direction for that current is correct. However, the negative answers for i_1 and i_3 indicate that our choices for those currents are wrong. Thus, as a *last step* here, we correct the answers by reversing the arrows for i_1 and i_3 in Fig. 27-13 and then writing

$$i_1 = 0.50\ \text{A} \quad \text{and} \quad i_3 = 0.25\ \text{A}. \qquad \text{(Answer)}$$

Caution: Always make any such correction as the last step and not before calculating *all* the currents.

A MODEL FOR ELECTRICAL CONDUCTION

In the absence of an electric field, there is no net displacement after many collisions (Figure a). An electric field E modifies the random motion and causes the electrons to drift in a direction opposite that of E. The slight curvature in the paths shown in (Figure b) results from the acceleration of the electrons between collisions, which is caused by the applied field.

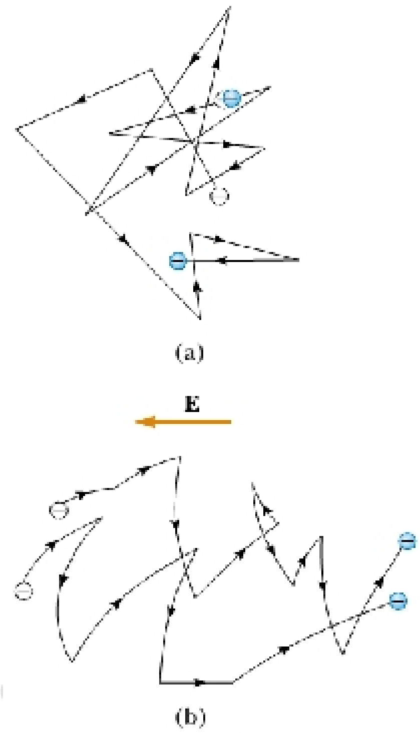


Figure 27.9 (a) A schematic diagram of the random motion of two charge carriers in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of the charge carriers in a conductor in the presence of an electric field. Note that the random motion is modified by the field, and the charge carriers have a drift velocity.

We are now in a position to derive an expression for the drift velocity. When a free electron of mass m_e and charge q $(= -e)$ is subjected to an electric field $\mathbf{E}$, it experiences a force $\mathbf{F} = q\mathbf{E}$. Because $\Sigma\mathbf{F} = m_e\mathbf{a}$, we conclude that the acceleration of the electron is

$$\mathbf{a} = \frac{q\mathbf{E}}{m_e}$$

The term is the velocity added by the field during one trip between atoms. If the electron starts with zero velocity, then the average value of the second term of equation up is $(q\mathbf{E}/m_e)\tau$, where is the average time interval

between successive collisions. Because the average value of v_f is equal to the drift velocity, we have

$$\overline{\mathbf{v}}_f = \mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\tau$$

We find that the magnitude of the current density is

$$J = nqv_d = \frac{nq^2E}{m_e}\tau$$

Comparing this expression with Ohm's law, $J = \sigma E$,

$$\sigma = \frac{nq^2\tau}{m_e}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

The average time between collisions is related to the average distance between collisions (that is, the mean free path) and the average speed through the expression.

$$\tau = \frac{\ell}{\overline{v}}$$

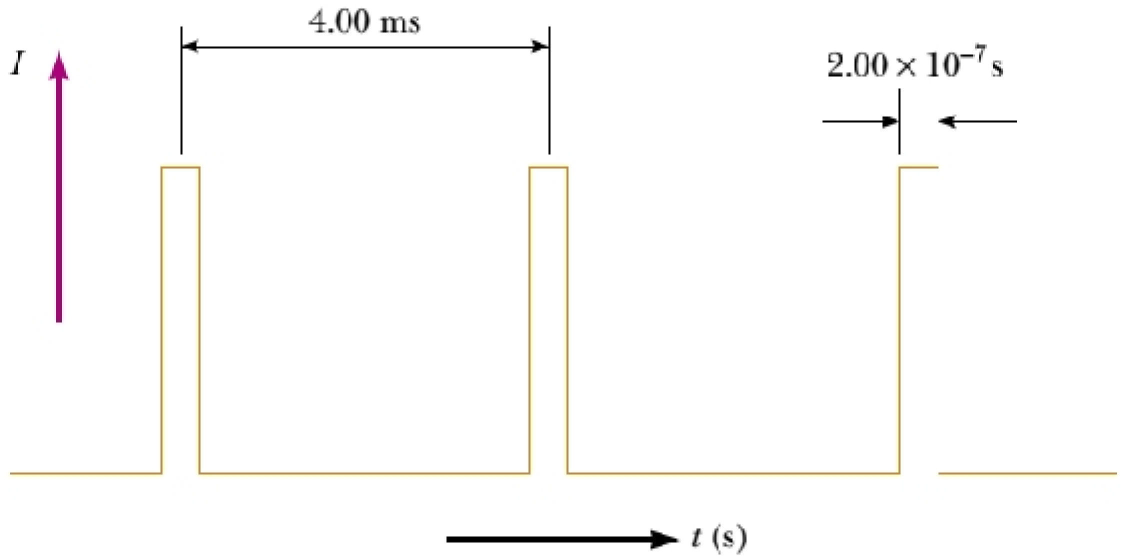
Example **Current in an Electron Beam**

In a certain particle accelerator, electrons emerge with an energy of 40.0 MeV ($1\text{ MeV} = 1.60 \times 10^{-13}\text{ J}$). The electrons emerge not in a steady stream but rather in pulses at the rate of 250 pulses/s. This corresponds to a time between pulses of 4.00 ms (Fig. 27.18). Each pulse has a duration of 200 ns, and the electrons in the pulse constitute a current of 250 mA. The current is zero between pulses. (a) How many electrons are delivered by the accelerator per pulse?

Solution We use Equation 27.2 in the form $dQ = I\,dt$ and integrate to find the charge per pulse. While the pulse is on, the current is constant; thus,

$$Q_{\text{pulse}} = I\int dt = I\Delta t = (250 \times 10^{-3}\text{ A})(200 \times 10^{-9}\text{ s})$$
$$= 5.00 \times 10^{-8}\text{ C}$$

Dividing this quantity of charge per pulse by the electronic charge gives the number of electrons per pulse:



$$\text{Electrons per pulse} = \frac{5.00 \times 10^{-8}\ \text{C/pulse}}{1.60 \times 10^{-19}\ \text{C/electron}}$$

$$= \boxed{3.13 \times 10^{11}\ \text{electrons/pulse}}$$

(b) What is the average current per pulse delivered by the accelerator?

Solution Average current is given by Equation 27.1, $I_{av} = \Delta Q/\Delta t$. Because the time interval between pulses is 4.00 ms, and because we know the charge per pulse from part (a), we obtain

$$I_{av} = \frac{Q_{pulse}}{\Delta t} = \frac{5.00 \times 10^{-8}\ \text{C}}{4.00 \times 10^{-3}\ \text{s}} = \boxed{12.5\ \mu\text{A}}$$

This represents only 0.005% of the peak current, which is 250 mA.

(c) What is the maximum power delivered by the electron beam?

Solution By definition, power is energy delivered per unit time. Thus, the maximum power is equal to the energy delivered by a pulse divided by the pulse duration:

$$\mathcal{P} = \frac{E}{\Delta t}$$

$$= \frac{(3.13 \times 10^{11}\ \text{electrons/pulse})(40.0\ \text{MeV/electron})}{2.00 \times 10^{-7}\ \text{s/pulse}}$$

$$= (6.26 \times 10^{19}\ \text{MeV/s})(1.60 \times 10^{-13}\ \text{J/MeV})$$

$$= 1.00 \times 10^{7}\ \text{W} = \boxed{10.0\ \text{MW}}$$

We could also compute this power directly. We assume that each electron had zero energy before being accelerated. Thus, by definition, each electron must have gone through a potential difference of 40.0 MV to acquire a final energy of 40.0 MeV. Hence, we have

$$\mathcal{P} = I\Delta V = (250 \times 10^{-3}\ \text{A})(40.0 \times 10^{6}\ \text{V}) = \boxed{10.0\ \text{MW}}$$

Example **The Cost of Making Dinner**

Estimate the cost of cooking a turkey for 4 h in an oven that operates continuously at 20.0 A and 240 V.

Solution The power used by the oven is

$$\mathcal{P} = I\Delta V = (20.0\ \text{A})(240\ \text{V}) = 4\,800\ \text{W} = 4.80\ \text{kW}$$

Because the energy consumed equals power × time, the amount of energy for which you must pay is

$$\text{Energy} = \mathcal{P}t = (4.80\ \text{kW})(4\ \text{h}) = 19.2\ \text{kWh}$$

If the energy is purchased at an estimated price of 8.00¢ per kilowatt hour, the cost is

$$\text{Cost} = (19.2\ \text{kWh})(\$0.080/\text{kWh}) = \boxed{\$1.54}$$

Example **Power in an Electric Heater**

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = \boxed{1.80\ \text{kW}}$$

If we doubled the applied potential difference, the current would double but the power would quadruple because $\mathcal{P} = (\Delta V)^2/R$.

Example **Drift speed a copper wire**

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31×10^{-6} m^2. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3}\ (1.00 \times 10^6\ \text{cm}^3/\text{m}^3)$$

$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$

$$= \frac{10.0\ \text{C/s}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.31 \times 10^{-6}\ \text{m}^2)}$$

$$= \boxed{2.22 \times 10^{-4}\ \text{m/s}}$$

Example **Electron Collisions in a Wire**

(a) Using the data and results from up Example and the classical model of electron conduction, estimate the average time between collisions for electrons in household copper wiring.

Solution from, we see that

$$\tau = \frac{m_e}{nq^2\rho}$$

Where $\rho = 1.7 \times 10^{-8}\ \Omega \cdot \text{m}$ for copper and the carrier density is $n=8.49\text{x}10^{28}$ electrons/m^3 for the wire described in. Substitution of these values into the expression above gives

$$\tau = \frac{(9.11 \times 10^{-31}\ \text{kg})}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.7 \times 10^{-8}\ \Omega \cdot \text{m})}$$

$$= \boxed{2.5 \times 10^{-14}\ \text{s}}$$

(b) Assuming that the average speed for free electrons in copper is $1.6\text{x}10^6$ m/s and using the result from part (a), calculate the mean free path for electrons in copper.

Solution

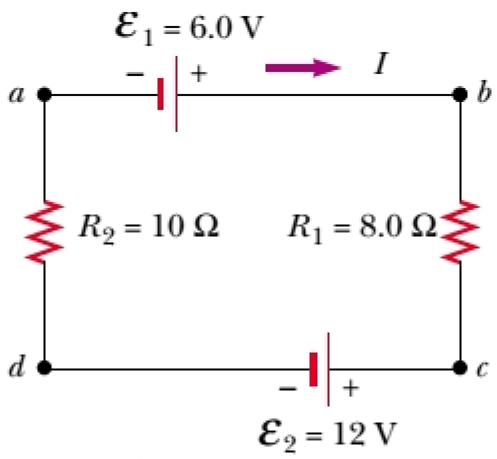
$$\ell = \bar{v}\tau = (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s})$$

$$= \boxed{4.0 \times 10^{-8}\ \text{m}}$$

<u>Example</u> **A Single-Loop Circuit**

A single-loop circuit contains two resistors and two batteries, as shown in figure. (Neglect the internal resistances of the batteries.) (a) Find the current in the circuit.

Solution We do not need Kirchhoff's rules to analyze this simple circuit, but let us use them anyway just to see how they are applied. There are no junctions in this single-loop circuit; thus, the current is the same in all elements. Let us assume that the current is clockwise, as shown in Figure. Traversing the circuit in the clockwise direction, starting at a, we see that $a \rightarrow b$ represents a potential change of $+\mathcal{E}_1$, $b \rightarrow c$ represents a potential change of $-IR_1$, $c \rightarrow d$ represents a potential change of $-\mathcal{E}_2$, and $d \rightarrow a$ represents a potential change of $-IR_2$. Applying Kirchhoff's loop rule gives



$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.13, we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction.

(b) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Solution

$$\mathcal{P}_1 = I^2R_1 = (0.33\text{ A})^2(8.0\ \Omega) = \boxed{0.87\text{ W}}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\text{ A})^2(10\ \Omega) = \boxed{1.1\text{ W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0$ W.

The 12-V battery delivers power $I\mathcal{E}_2 = 4.0$ W. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery. If we had included the internal resistances of the batteries in our analysis, some of the power would appear as internal energy in the batteries; as a result, we would have found that less power was being delivered to the 6-V battery.

Example **Applying Kirchhoff's Rules**

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure

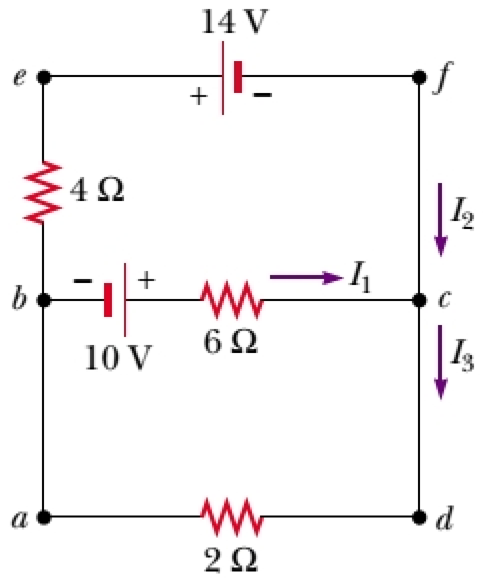
Solution Notice that we cannot reduce this circuit to a simpler form by means of the rules of adding resistances in series and in parallel. We must use Kirchhoff's rules to analyze this circuit. We arbitrarily choose the directions of the currents as labeled in Figure 28.14. Applying Kirchhoff's junction rule to junction c gives

$$(1) \qquad I_1 + I_2 = I_3$$

We now have one equation with three unknowns—I_1, I_2, and I_3. There are three loops in the circuit—*abcda, befcb,* and *aefda.* We therefore need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Applying Kirchhoff's loop rule to loops *abcda* and *befcb* and traversing these loops clockwise, we obtain the expressions

$$(2) \quad abcda \quad 10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} - (4\ \Omega)I_2 = 0$$



Note that in loop *befcb* we obtain a positive value when traversing the 6-Ω resistor because our direction of travel is opposite the assumed direction of I_1.

Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)\ (I_1 + I_2) = 0$$

$$(4) \qquad 10\text{ V} = (8\ \Omega)I_1 + (2\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \qquad -12\text{ V} = -(3\ \Omega)I_1 + (2\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22\text{ V} = (11\ \Omega)I_1$$

$$I_1 = \boxed{2\text{ A}}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2\ \Omega)I_2 = (3\ \Omega)I_1 - 12\text{ V} = (3\ \Omega)\ (2\text{ A}) - 12\text{ V} = -6\text{ V}$$

$$I_2 = \boxed{-3\text{ A}}$$

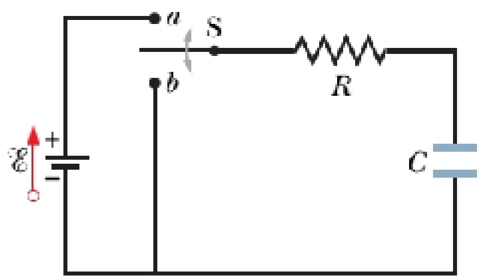
Finally,

$$I_3 = I_1 + I_2 = \boxed{-1\text{ A}}$$

The fact that I_2 and I_3 are both negative indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct.

RC Circuits

The capacitor of capacitance C in Fig. is initially uncharged. To charge it, we close switch (S) on point a. This completes an RC series circuit consisting of the capacitor, an ideal battery of emf $\mathscr{E}$, and a resistance R.



Charging a Capacitor

We already know that as soon as the circuit is complete, charge begins to flow (current exists) between a capacitor plate and a battery terminal on each side of the capacitor. This current increases the charge q on the plates and the potential difference V_C (=q/C) across the capacitor. When that potential difference equals the potential difference across the battery (which here is equal to the emf $\mathscr{E}$, and the current is zero). From (q=CV), the equilibrium (final) charge on the then fully charged capacitor is equal to $C\mathscr{E}$.

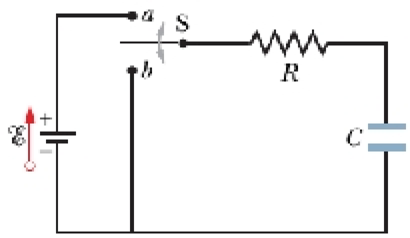


Fig. When switch S is closed on a, the capacitor is *charged* through the resistor. When the switch is afterward closed on b, the capacitor *discharges* through the resistor.

$$\boldsymbol{\varepsilon} - \frac{q}{C} - IR = 0$$

$$I_0 = \frac{\boldsymbol{\varepsilon}}{R} \qquad \text{(current at } t = 0\text{)}$$

$$\frac{dq}{dt} = \frac{\boldsymbol{\varepsilon}}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{C\boldsymbol{\varepsilon}}{RC} - \frac{q}{RC} = -\frac{q - C\boldsymbol{\varepsilon}}{RC}$$

$$\frac{dq}{q - C\boldsymbol{\varepsilon}} = -\frac{1}{RC}\, dt$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_0^q \frac{dq}{q - C\boldsymbol{\varepsilon}} = -\frac{1}{RC}\int_0^t dt$$

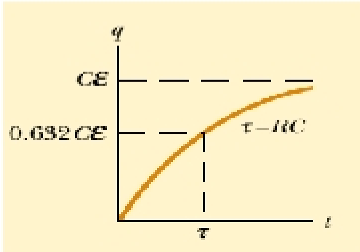
$$\ln\left(\frac{q - C\boldsymbol{\varepsilon}}{-C\boldsymbol{\varepsilon}}\right) = -\frac{t}{RC}$$

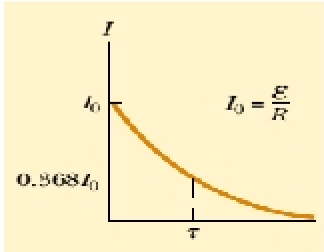
$$q = C\mathcal{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor)}$$

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC} \qquad \text{(charging a capacitor).}$$

$$V_C = \frac{q}{C} = \mathcal{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor).}$$

The time constant τ of the circuit. It represents the time it takes the current to decrease to 1/e of its initial value; that is, in a time τ. $I = e^{-1}I_0 = 0.368I_0$.
Likewise, in a time τ the charge increases from zero to $C\boldsymbol{\varepsilon}\,(1 - e^{-1}) = 0.632C\boldsymbol{\varepsilon}$.

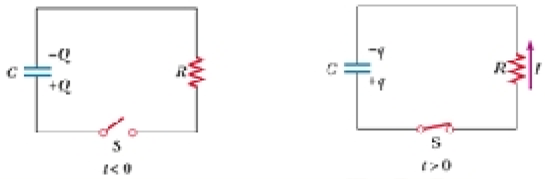




$$\tau = RC \qquad \text{(time constant).}$$

Discharging a Capacitor

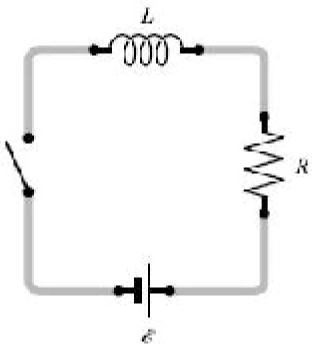
If the switch is closed at the capacitor begins to discharge through the resistor. At some time t during the discharge, the current in the circuit is *I* and the charge on the capacitor is q.



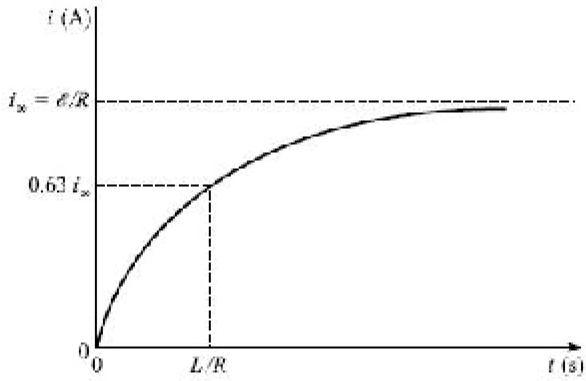
$$q = q_0 e^{-t/RC} \qquad \text{(discharging a capacitor),}$$

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \qquad \text{(discharging a capacitor)}$$

R-L **TIME CONSTANT:** Consider the circuit in Fig. (*a*). The symbol ⅏ represents a coil of self-inductance *L* henries. When the switch in the circuit is first closed, the current in the circuit rises as shown in Fig. (*b*). The current does not jump to its final value because the changing flux through the coil induces a back emf in the coil, which opposes the rising current. After L/R seconds, the current has risen to 0.632 of its final value i_∞. This time, $t = L/R$, is called the *time constant* of the *R-L* circuit. After a long time, the current is changing so slowly that the back emf in the inductor, $L(\Delta i/\Delta t)$, is negligible. Then $i = i_\infty = \mathcal{E}/R$.



(*a*)



(*b*)

SUMMARY

The **emf** of a battery is equal to the voltage across its terminals when the current is zero. That is, the emf is equivalent to the **open-circuit voltage** of the battery.

The **equivalent resistance** of a set of resistors connected in **series** is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

The **equivalent resistance** of a set of resistors connected in **parallel** is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

If it is possible to combine resistors into series or parallel equivalents, the preceding two equations make it easy to determine how the resistors influence the rest of the circuit.

Circuits involving more than one loop are conveniently analyzed with the use of **Kirchhoff's rules:**

1. The sum of the currents entering any junction in an electric circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out}$$

2. The sum of the potential differences across all elements around any circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0$$

The first rule is a statement of conservation of charge; the second is equivalent to a statement of conservation of energy.

When a resistor is traversed in the direction of the current, the change in potential ΔV across the resistor is $-IR$. When a resistor is traversed in the direction opposite the current, $\Delta V = +IR$. When a source of emf is traversed in the direction of the emf (negative terminal to positive terminal), the change in potential is $+\mathcal{E}$. When a source of emf is traversed opposite the emf (positive to negative), the change in potential is $-\mathcal{E}$. The use of these rules together with Equations 28.9 and 28.10 allows you to analyze electric circuits.

If a capacitor is charged with a battery through a resistor of resistance R, the charge on the capacitor and the current in the circuit vary in time according to the expressions

$$q(t) = Q(1 - e^{-t/RC})$$

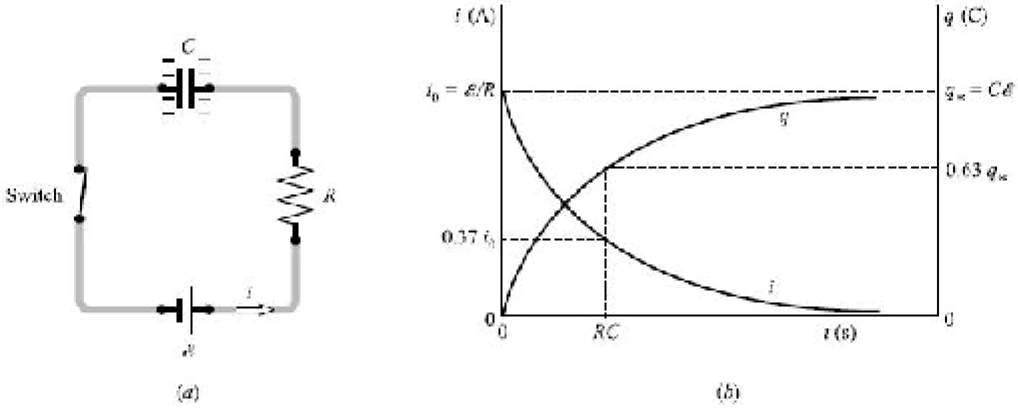
$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$

where $Q = C\mathcal{E}$ is the maximum charge on the capacitor. The product RC is called the **time constant** τ of the circuit. If a charged capacitor is discharged through a resistor of resistance R, the charge and current decrease exponentially in time according to the expressions

$$q(t) = Qe^{-t/RC}$$

$$I(t) = -\frac{Q}{RC}e^{-t/RC}$$

where Q is the initial charge on the capacitor and $Q/RC = I_0$ is the initial current in the circuit. Equations 28.14, 28.15, 28.17, and 28.18 permit you to analyze the current and potential differences in an RC circuit and the charge stored in the circuit's capacitor.



At the first instant after the switch is closed, $v_c = 0$ and $i = ℰ/R$. As time goes on, v_c increases and i decreases. The time, in seconds, taken for the current to drop to 1/2.718 or 0.368 of its initial value is RC,

(EX..) A coil has an inductance of 1.5 H and a resistance of 0.60 Ω. If the coil is suddenly connected across a 12-V battery, find the time required for the current to rise to 0.63 of its final value. What will be the final current through the coil?

The time required is the time constant of the circuit:

$$\text{Time constant} = \frac{L}{R} = \frac{1.5\text{ H}}{0.60\ \Omega} = 2.5\text{ s}$$

After a long time, the current will be steady and so no back emf will exist in the coil. Under those conditions,

$$I = \frac{ℰ}{R} = \frac{12\text{ V}}{0.60\ \Omega} = 20\text{ A}$$

(EX.) A 5.0-μF capacitor is charged to a potential difference of 20 kV between plates. After being disconnected from the power source, it is connected across a 7.0-MΩ resistor to discharge. What is the initial discharge current, and how long will it take for the capacitor voltage to decrease to 37 percent of the 20 kV?

The loop equation for the discharging capacitor is

$$v_c - iR = 0$$

where v_c is the p.d. across the capacitor. At the first instant, $v_c = 20$ kV, so

$$i = \frac{v_c}{R} = \frac{20\times10^3\text{ V}}{7.0\times10^6\ \Omega} = 2.9\text{ mA}$$

The potential across the capacitor, as well as the charge on it, will decrease to 0.37 of its original value in one time constant. The required time is

$$RC = (7.0\times10^6\ \Omega)(5.0\times10^{-6}\text{ F}) = 35\text{ s}$$

(EX.) A certain series circuit consists of a 12-V battery, a switch, a 1.0-MΩ resistor, and a 2.0-μF capacitor, initially uncharged. If the switch is now closed, find (*a*) the initial current in the circuit, (*b*) the time for the current to drop to 0.37 of its initial value, (*c*) the charge on the capacitor then, and (*d*) the final charge on the capacitor.

(*a*) The loop rule applied to the circuit at any instant gives

$$12\ \text{V} - iR - v_c = 0$$

where v_c is the p.d. across the capacitor. At the first instant, q is essentially zero and so $v_c = 0$. Then

$$12\ \text{V} - iR - 0 = 0 \qquad \text{or} \qquad i = \frac{12\ \text{V}}{1.0 \times 10^6\ \Omega} = 12\ \mu\text{A}$$

(*b*) The current drops to 0.37 of its initial value when

$$t = RC = (1.0 \times 10^6\ \Omega)(2.0 \times 10^{-6}\ \text{F}) = 2.0\ \text{s}$$

(*c*) At $t = 2.0$ s the charge on the capacitor has increased to 0.63 of its final value.

(*d*) The charge ceases to increase when $i = 0$ and $v_c = 12$ V. Therefore,

$$q_{\text{final}} = Cv_c = (2.0 \times 10^{-6}\ \text{F})(12\ \text{V}) = 24\ \mu\text{C}$$

Alternating Current

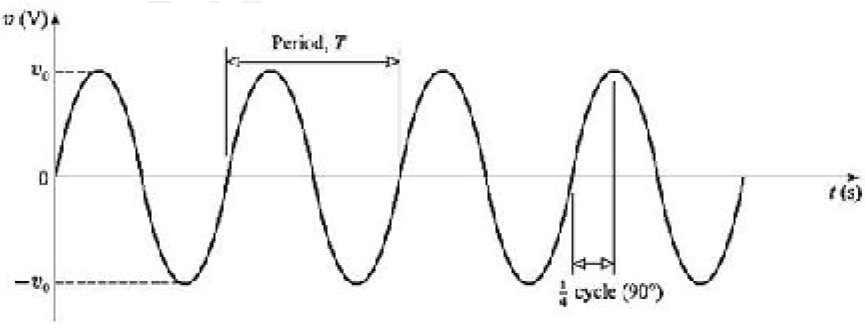
THE EMF GENERATED BY A ROTATING COIL in a magnetic field has a graph similar to the one shown in Fig. . It is called an *ac voltage* because there is a reversal of polarity (i.e., the voltage changes sign); ac voltages need not be sinusoidal. If the coil rotates with a frequency of f revolutions per second, then the emf has a frequency of f in hertz (cycles per second). The instantaneous voltage v that is generated has the form

$$v = v_0 \sin \omega t = v_0 \sin 2\pi f t$$

where v_0 is the amplitude (maximum value) of the voltage in volts, $\omega = 2\pi f$ is the angular velocity in rad/s, and f is the frequency in hertz. The frequency f of the voltage is related to its period T by

$$T = \frac{1}{f} \qquad \text{where } T \text{ is in seconds.}$$

An alternating current produced by a typical generator has a graph much like that for the voltage shown in Fig. . Its instantaneous value is i, and its amplitude is i_0. Often the current and voltage do not reach a maximum at the same time, even though they both have the same frequency.



METERS for use in ac circuits read the *effective*, or *root mean square* (rms), values of the current and voltage. These values are always positive and are related to the amplitudes of the instantaneous sinusoidal values through

$$V = V_{\text{rms}} = \frac{v_0}{\sqrt{2}} = 0.707 v_0$$

$$I = I_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$

It is customary to represent meter readings by capital letters (V, I), while instantaneous values are represented by small letters (v, i).

THE THERMAL ENERGY GENERATED OR POWER LOST by an rms current I in a resistor R is given by I^2R.

FORMS OF OHM'S LAW: Suppose that a sinusoidal current of frequency f with rms value I flows through a pure resistor R, or a pure inductor L, or a pure capacitor C. Then an ac voltmeter placed across the element in question will read an rms voltage V as follows:

$$\text{Pure resistor:} \quad V = IR$$

$$\text{Pure inductor:} \quad V = IX_L$$

where $X_L = 2\pi fL$ is called the *inductive reactance*. Its unit is ohms when L is in henries and f is in hertz.

$$\text{Pure capacitor:} \quad V = IX_C$$

where $X_C = 1/2\pi fC$ is called the *capacitive reactance*. Its unit is ohms when C is in farads.

PHASE: When an ac voltage is applied to a pure resistance, the voltage across the resistance and the current through it attain their maximum values at the same instant and their zero values at the same instant; the voltage and current are said to be *in-phase*.

When an ac voltage is applied to a pure inductance, the voltage across the inductance reaches its maximum value one-quarter cycle ahead of the current, i.e., when the current is zero. The back emf of the inductance causes the current through the inductance to lag behind the voltage by one-quarter cycle (or 90°), and the two are 90° *out-of-phase*.

When an ac voltage is applied to a pure capacitor, the voltage across it lags 90° behind the current flowing through it. Current must flow before the voltage across (and charge on) the capacitor can build up.

In more complicated situations involving combinations of R, L, and C, the voltage and current are usually (but not always) out-of-phase. The angle by which the voltage lags or leads the current is called the *phase angle*.

THE IMPEDANCE (Z) of a series circuit containing resistance, inductance, and capacitance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

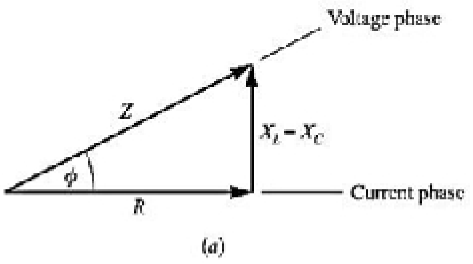
with Z in ohms. If a voltage V is applied to such a series circuit, then a form of Ohm's Law relates V to the current I through it:

$$V = IZ$$

The phase angle ϕ between V and I is given by

$$\tan\phi = \frac{X_L - X_C}{R} \qquad \text{or} \qquad \cos\phi = \frac{R}{Z}$$

PHASORS: A phasor is a quantity that behaves, in many regards, like a vector. Phasors are used to describe series R-L-C circuits because the above expression for the impedance can be associated with the Pythagorean theorem for a right triangle. As shown in Fig. (*a*), Z is the hypotenuse of the right triangle, while R and $(X_L - X_C)$ are its two legs. The angle labeled ϕ is the phase angle between the current and the voltage.



A similar relation applies to the voltages across the elements in the series circuit. As shown in Fig. (*b*), it is

$$V^2 = V_R^2 + (V_L - V_C)^2$$

Because of the phase differences a measurement of the voltage across a series circuit is not equal to the algebraic sum of the individual voltage readings across its elements. Instead, the above relation must be used.

RESONANCE occurs in a series *R-L-C* circuit when $X_L = X_C$. Under this condition $Z = R$ is minimum, so that I is maximum for a given value of V. Equating X_L to X_C, we find for the *resonant* (or *natural*) *frequency* of the circuit

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

POWER LOSS: Suppose that an ac voltage V is impressed across an impedance of any type. It gives rise to a current I through the impedance, and the phase angle between V and I is ϕ. The power loss in the impedance is given by

$$\text{Power loss} = VI\cos\phi$$

The quantity $\cos\phi$ is called the *power factor*. It is unity for a pure resistor; but it is zero for a pure inductor or capacitor (no power loss occurs in a pure inductor or capacitor).

(EX.1) A sinusoidal, 60.0-Hz, ac voltage is read to be 120 V by an ordinary voltmeter. (*a*) What is the maximum value the voltage takes on during a cycle? (*b*) What is the equation for the voltage?

(*a*) $$V = \frac{v_0}{\sqrt{2}} \quad \text{or} \quad v_0 = \sqrt{2}V = \sqrt{2}(120\text{ V}) = 170\text{ V}$$

(*b*) $$v = v_0 \sin 2\pi f t = (170\text{ V})\sin 120\pi t$$

where t is in s.

(EX.2) A 120-V ac voltage source is connected across a 2.0-μF capacitor. Find the current to the capacitor if the frequency of the source is (*a*) 60 Hz and (*b*) 60 kHz. (*c*) What is the power loss in the capacitor?

(*a*) $$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\text{ s}^{-1})(2.0\times10^{-6}\text{ F})} = 1.33\text{ k}\Omega$$

Then $$I = \frac{V}{X_C} = \frac{120\text{ V}}{1330\ \Omega} = 0.090\text{ A}$$

(*b*) Now $X_C = 1.33\ \Omega$, so $I = 90$ A. Notice that the impedance effect of a capacitor varies inversely with the frequency.

(*c*) $$\text{Power loss} = VI\cos\phi = VI\cos 90^\circ = 0$$

A TRANSFORMER is a device to raise or lower the voltage in an ac circuit. It consists of a primary and a secondary coil wound on the same iron core. An alternating current in one coil creates a continuously changing magnetic flux through the core. This change of flux induces an alternating emf in the other coil.

The efficiency of a transformer is usually very high. Thus, we may usually *neglect losses* and write

$$\text{Power in primary} = \text{power in secondary}$$
$$V_1 I_1 = V_2 I_2$$

The voltage ratio is the ratio of the numbers of turns on the two coils; the current ratio is the inverse ratio of the numbers of turns:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \text{and} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

(EX.3) A 120-V ac voltage source is connected across a pure 0.700-H inductor. Find the current through the inductor if the frequency of the source is (*a*) 60.0 Hz and (*b*) 60.0 kHz. (*c*) What is the power loss in the inductor?

(*a*) $$X_L = 2\pi f L = 2\pi(60.0\ \text{s}^{-1})(0.700\ \text{H}) = 264\ \Omega$$

Then $$I = \frac{V}{X_L} = \frac{120\ \text{V}}{264\ \Omega} = 0.455\ \text{A}$$

(*b*) Now $X_L = 264 \times 10^3\ \Omega$, so $I = 0.455 \times 10^{-3}$ A. Notice that the impedance effect of an inductor varies directly with the frequency.

(*c*) $$\text{Power loss} = VI\cos\phi = VI\cos 90° = 0$$

(EX.4) A coil having inductance 0.14 H and resistance of 12 Ω is connected across a 110-V, 25-Hz line. Compute (*a*) the current in the coil, (*b*) the phase angle between the current and the supply voltage, (*c*) the power factor, and (*d*) the power loss in the coil.

(*a*) $$X_L = 2\pi f L = 2\pi(25)(0.14) = 22.0\ \Omega$$

and $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (22-0)^2} = 25.1\ \Omega$$

so $$I = \frac{V}{Z} = \frac{110\ \text{V}}{25.1\ \Omega} = 4.4\ \text{A}$$

(*b*) $$\tan\phi = \frac{X_L - X_C}{R} = \frac{22-0}{12} = 1.83 \qquad \text{or} \qquad \phi = 61.3°$$

The voltage leads the current by 61°.

(*c*) $$\text{Power factor} = \cos\phi = \cos 61.3° = 0.48$$

(*d*) $$\text{Power loss} = VI\cos\phi = (110\ \text{V})(4.4\ \text{A})(0.48) = 0.23\ \text{kW}$$

Or, since power loss occurs only because of the resistance of the coil,

$$\text{Power loss} = I^2 R = (4.4\ \text{A})^2(12\ \Omega) = 0.23\ \text{kW}$$

(EX.5) A capacitor is in series with a resistance of 30 Ω and is connected to a 220-V ac line. The reactance of the capacitor is 40 Ω. Determine (*a*) the current in the circuit, (*b*) the phase angle between the current and the supply voltage, and (*c*) the power loss in the circuit.

(*a*)
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(30)^2 + (0 - 40)^2} = 50\ \Omega$$

so
$$I = \frac{V}{Z} = \frac{220\ \text{V}}{50\ \Omega} = 4.4\ \text{A}$$

(*b*)
$$\tan\phi = \frac{X_L - X_C}{R} = \frac{0 - 40}{30} = -1.33 \qquad \text{or} \qquad \phi = -53^\circ$$

The minus sign tells us that the voltage *lags* the current by 53°. The angle ϕ in Fig. (a) & (b) would lie below the horizontal axis.

(*c*) **Method 1**

$$\text{Power loss} = VI\cos\phi = (220)(4.4)\cos(-53^\circ) = (220)(4.4)\cos 53^\circ = 0.58\ \text{kW}$$

Method 2

Because the power loss occurs only in the resistor, and not in the pure capacitor,

$$\text{Power loss} = I^2R = (4.4\ \text{A})^2(30\ \Omega) = 0.58\ \text{kW}$$

(EX.6) A series circuit consisting of a 100-Ω noninductive resistor, a coil with a 0.10-H inductance and negligible resistance, and a 20-μF capacitor is connected across a 110-V, 60-Hz power source. Find (*a*) the current, (*b*) the power loss, (*c*) the phase angle between the current and the source voltage, and (*d*) the voltmeter readings across the three elements.

(*a*) For the entire circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with

$$R = 100\ \Omega$$
$$X_L = 2\pi fL = 2\pi(60\ \text{s}^{-1})(0.10\ \text{H}) = 37.7\ \Omega$$
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\ \text{s}^{-1})(20\times10^{-6}\ \text{F})} = 132.7\ \Omega$$

from which

$$Z = \sqrt{(100)^2 + (38 - 133)^2} = 138\ \Omega \qquad \text{and} \qquad I = \frac{V}{Z} = \frac{110\ \text{V}}{138\ \Omega} = 0.79\ \text{A}$$

(*b*) The power loss all occurs in the resistor, so

$$\text{Power loss} = I^2R = (0.79\ \text{A})^2(100\ \Omega) = 63\ \text{W}$$

(*c*)
$$\tan\phi = \frac{X_L - X_C}{R} = \frac{-95\ \Omega}{100\ \Omega} = -0.95 \qquad \text{or} \qquad \phi = -44^\circ$$

The voltage lags the current.

(*d*)
$$V_R = IR = (0.79\ \text{A})(100\ \Omega) = 79\ \text{V}$$
$$V_C = IX_C = (0.79\ \text{A})(132.7\ \Omega) = 0.11\ \text{kV}$$
$$V_L = IX_L = (0.79\ \text{A})(37.7\ \Omega) = 30\ \text{V}$$

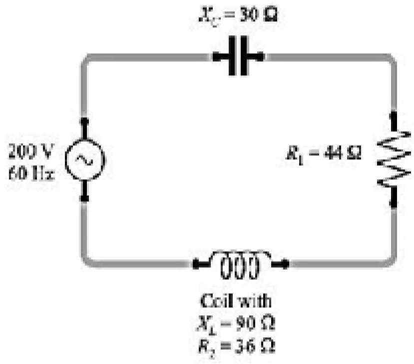
Notice that $V_C + V_L + V_R$ does not equal the source voltage. From Fig. (*b*), the correct relationship is

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(79)^2 + (-75)^2} = 109\ \text{V}$$

(EX.7) As shown in Fig., a series circuit connected across a 200-V, 60-Hz line consists of a capacitor of capacitive reactance 30 Ω, a noninductive resistor of 44 Ω, and a coil of inductive reactance 90 Ω and resistance 36 Ω. Determine (*a*) the current in the circuit, (*b*) the potential difference across each element, (*c*) the power factor of the circuit, and (*d*) the power absorbed by the circuit.

(*a*) $Z = \sqrt{(R_1 + R_2)^2 + (X_L - X_C)^2} = \sqrt{(44+36)^2 + (90-30)^2} = 0.10\ \text{k}\Omega$

so $I = \dfrac{V}{Z} = \dfrac{200\ \text{V}}{100\ \Omega} = 2.0\ \text{A}$



(*b*) p.d. across capacitor $= IX_C = (2.0\ \text{A})(30\ \Omega) = 60\ \text{V}$

p.d. across resistor $= IR_1 = (2.0\ \text{A})(44\ \Omega) = 88\ \text{V}$

Impedance of coil $= \sqrt{R_2^2 + X_L^2} = \sqrt{(36)^2 + (90)^2} = 97\ \Omega$

p.d. across coil $= (2.0\ \text{A})(97\ \Omega) = 0.19\ \text{kV}$

(*c*) Power factor $= \cos\phi = \dfrac{R}{Z} = \dfrac{80}{100} = 0.80$

(*d*) Power used $= VI\cos\phi = (200\ \text{V})(2\text{A})(0.80) = 0.32\ \text{kW}$

or Power used $= I^2R = (2\ \text{A})^2(80\ \Omega) = 0.32\ \text{kW}$

(EX.8) Calculate the resonant frequency of a circuit of negligible resistance containing an inductance of 40.0 mH and a capacitance of 600 pF.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(40.0\times10^{-3}\ \text{H})(600\times10^{-12}\ \text{F})}} = 32.5\ \text{kHz}$$

(EX.9) A step-up transformer is used on a 120-V line to furnish 1800 V. The primary has 100 turns. How many turns are on the secondary?

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{or} \quad \frac{120\ \text{V}}{1800\ \text{V}} = \frac{100\ \text{turns}}{N_2}$$

from which $N_2 = 1.50 \times 10^3$ turns.

(EX.10) A transformer used on a 120-V line delivers 2.0 A at 900 V. What current is drawn from the line? Assume 100 percent efficiency.

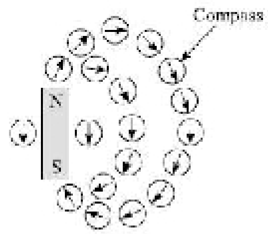
Power in primary = power in secondary

$$I_1(120\ \text{V}) = (2.0\ \text{A})(900\ \text{V})$$

$$I_1 = 15\ \text{A}$$

Forces in Magnetic Fields

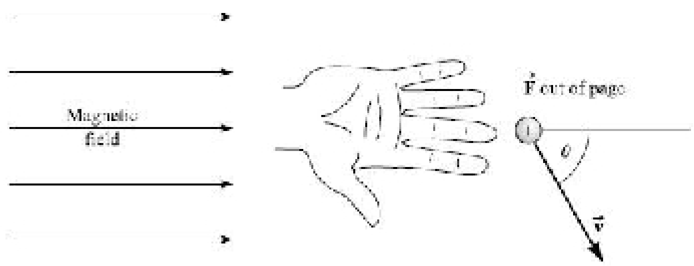
MAGNETIC FIELD LINES drawn in a region provide a means for showing the direction in which a compass needle placed in the region will point. A method for determining the field lines near a bar magnet is shown in Fig. . By tradition, we take the direction of the compass needle to be the direction of the field.



A MAGNET have two poles, although it must have one *north pole* and one *south pole*. Because a compass needle points away from a north pole and toward a south pole (S), magnetic field lines exit north poles and enter south poles.

MAGNETIC POLES of the same type (north or south) repel each other, while unlike poles attract each other.

THE DIRECTION OF THE FORCE acting on a charge $+q$ moving in a magnetic field can be found from a *right-hand rule* :



The force direction on a negative charge is opposite to that on a positive charge.

THE MAGNETIC FIELD AT A POINT is represented by a vector $\vec{B}$ that is variously called the *magnetic induction*, the *magnetic flux density*, or simply the *magnetic field*.

We define the magnitude of $\vec{B}$ and its units by way of the equation

$$F_M = qvB \sin\theta$$

where F_M is in newtons, q is in coulombs, v is in m/s, and B is the magnetic field in a unit called the *tesla* **(T)**. For reasons we will see later, a tesla is also expressed as a *weber per square meter*: $1\text{ T} = 1\text{ Wb/m}^2$ Still encountered is the cgs unit for B, the *gauss* (G), where

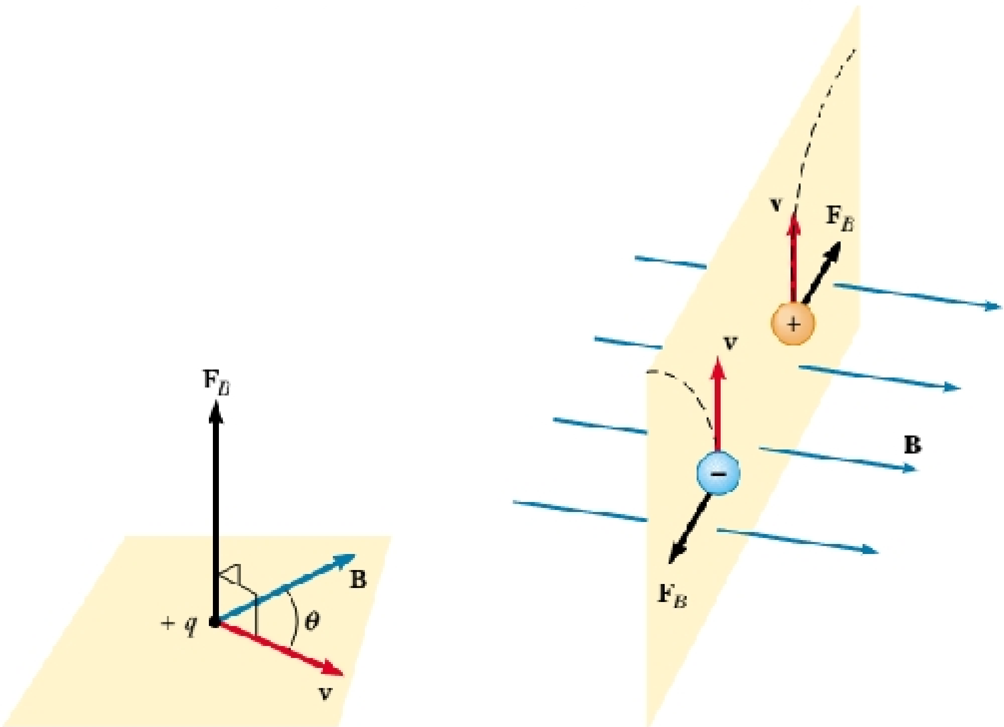
$$1\text{ G} = 10^{-4}\text{ T}$$

The Earth's magnetic field is a few tenths of a gauss. Also note that

$$1\text{ T} = 1\text{ Wb/m}^2 = 1\frac{\text{N}}{\text{C}\cdot(\text{m/s})} = 1\frac{\text{N}}{\text{A}\cdot\text{m}}$$

We can define a magnetic field B at some point in space in terms of the magnetic force F_B that the field exerts on a test object, for which we use a charged particle moving with a velocity v. For the time being, let us assume that no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

• The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle
• The magnitude and direction of F_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field B.
• When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
• When the particle's velocity vector makes any angle with the magnetic field, the magnetic force acts in a direction perpendicular to both vand B; that is, $F_B$is perpendicular to the plane formed by vand B



• The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction
• The magnitude of the magnetic force exerted on the moving particle is proportional to sin Θ, where $\Theta$is the angle the particle's velocity vector makes with the direction of B.
We can summarize these observations by writing the magnetic force in the Form

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

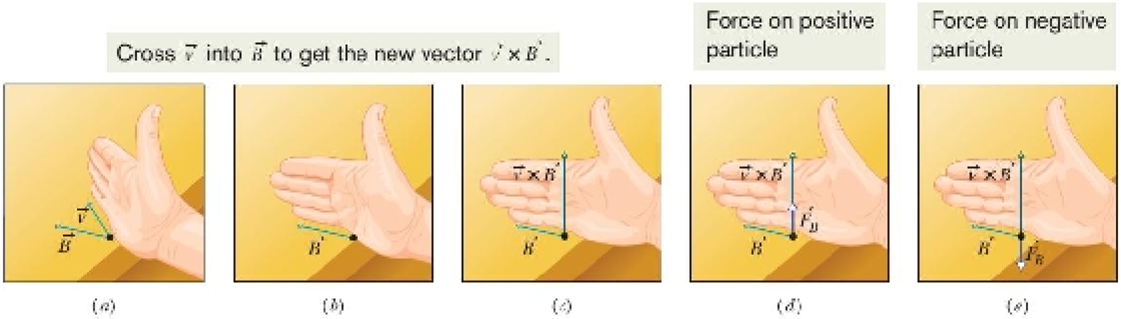


Fig. 28-2 (*a*) – (*c*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*d*) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*e*) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

Example

An electron in a television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the x axis . Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the x axis and lying in the xy plane. Calculate the magnetic force on and acceleration of the electron.

Solution Using Equation , we can find the magnitude of the magnetic force:

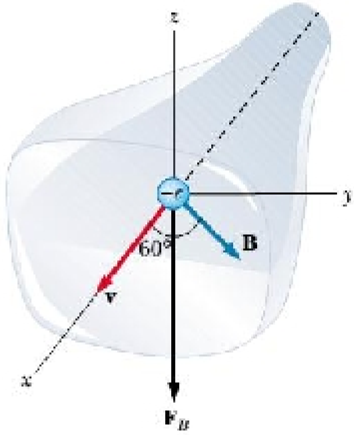
$$F_B = |q|vB\sin\theta$$
$$= (1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^6\ \text{m/s})(0.025\ \text{T})(\sin 60°)$$
$$= \boxed{2.8 \times 10^{-14}\ \text{N}}$$

Because **v** × **B** is in the positive z direction (from the right-hand rule) and the charge is negative, $\mathbf{F}_B$ is in the negative z direction.

The mass of the electron is 9.11×10^{-31} kg, and so its acceleration is

$$a = \frac{F_B}{m_e} = \frac{2.8 \times 10^{-14}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{3.1 \times 10^{16}\ \text{m/s}^2}$$

in the negative z direction.



FORCE ON A CURRENT IN A MAGNETIC FIELD:

The magnitude ΔF_M of the force on a small length ΔL of wire carrying current I is given by

$$\Delta F_M = I(\Delta L)B \sin\theta$$

where θ is the angle between the direction of the current I and the direction of the field. For a straight wire of length L in a uniform magnetic field, this becomes

$$F_M = ILB \sin\theta$$

★ Notice that the force is zero if the wire is in line with the field lines. The force is maximum if the field lines are perpendicular to the wire.

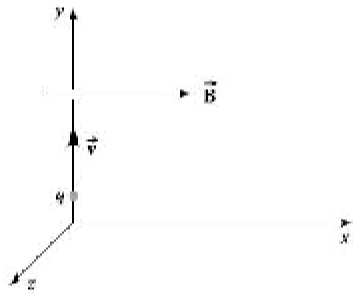
(EX.1) A uniform magnetic field, $B = 3.0$ G, exists in the $+x$-direction. A proton ($q = +e$) shoots through the field in the $+y$-direction with a speed of 5.0×10^6 m/s. (*a*) Find the magnitude and direction of the force on the proton. (*b*) Repeat with the proton replaced by an electron.

(*a*) The situation is shown in Fig. 30-4. We have, after changing 3.0 G to 3.0×10^{-4} T,

$$F_M = qvB \sin\theta = (1.6 \times 10^{-19}\ \text{C})(5.0 \times 10^6\ \text{m/s})(3.0 \times 10^{-4}\ \text{T}) \sin 90° = 2.4 \times 10^{-16}\ \text{N}$$

The force is perpendicular to the xy-plane, the plane defined by the field lines and $\vec{v}$. The right-hand rule tells us that the force is directed into the page, in the $-z$-direction.

(*b*) The magnitude of the force is the same as in (*a*), 2.4×10^{-16} N. But, because the electron is negative, the force direction is reversed. The force is in the $+z$-direction.



(EX.2) A cathode ray beam (an electron beam; $m_e = 9.1 \times 10^{-31}$ kg, $q = -e$) is bent in a circle of radius 2.0 cm by a uniform field with $B = 4.5 \times 10^{-3}$ T. What is the speed of the electrons?

★ To describe a circle like this, the particles must be moving perpendicular to $\vec{B}$. The force qvB is radially inward and supplies the centripetal force for the circular motion: $F_M = qvB = ma = mv^2/r$ and

$$r = \frac{mv}{qB}$$

$$v = \frac{rqB}{m} = \frac{(0.020\ \text{m})(1.6 \times 10^{-19}\ \text{C})(4.5 \times 10^{-3}\ \text{T})}{9.1 \times 10^{-31}\ \text{kg}} = 1.58 \times 10^7\ \text{m/s} = 1.6 \times 10^4\ \text{km/s}$$

(F.X.3) Alpha particles ($m_\alpha = 6.68 \times 10^{-27}$ kg, $q = +2e$) are accelerated from rest through a p.d. of 1.0 kV. They then enter a magnetic field $B = 0.20$ T perpendicular to their direction of motion. Calculate the radius of their path.

Their final KE is equal to the electric potential energy they lose during acceleration, Vq:

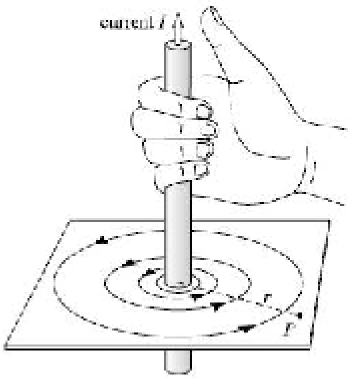
$$\frac{1}{2}mv^2 = Vq \qquad \text{or} \qquad v = \sqrt{\frac{2Vq}{m}}$$

By (F.X.2) , they follow a circular path in which

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2Vq}{m}} = \frac{1}{B}\sqrt{\frac{2Vm}{q}}$$

$$= \frac{1}{0.20\ \text{T}}\sqrt{\frac{2(1000\ \text{V})(6.68 \times 10^{-27}\ \text{kg})}{3.2 \times 10^{-19}\ \text{C}}} = 0.032\ \text{m}$$

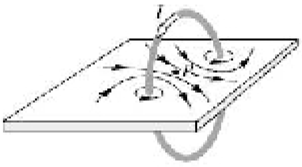
Figure shows the nature of the magnetic fields produced by several current configurations. Below each is given the value of B at the indicated point P. The constant $\mu_0 = 4\pi \times 10^{-7}$ T·m/A is called the *permeability of free space*. It is assumed that the surrounding material is vacuum or air.



(a) Long straight wire:

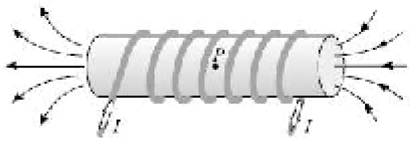
$$B = \frac{\mu_0 I}{2\pi r}$$

where r is distance to P from the axis of the wire



(b) Center of a circular coil with radius a and N loops:

$$B = \frac{\mu_0 N I}{2a}$$



(c) Interior point of long solenoid with n loops per meter:

$$B = \mu_0 n I$$

It is constant in the interior

(d) Interior point of toroid having N loops:

$$B = \frac{\mu_0 N I}{2\pi r}$$

where r is the radius of the circle on which P lies

$$F_B = |q|vB \sin \phi$$

keep the charge moving in a circle:

$$\sum F = ma_r$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

That is, the radius of the path is proportional to the linear momentum mv of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field. The angular speed of the particle

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

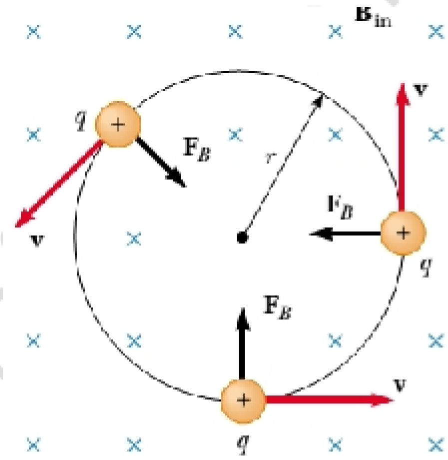


Figure When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to **B**. The magnetic force $\mathbf{F}_B$ acting on the charge is always directed toward the center of the circle.

Example

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

Solution From Equation 29.13, we have

$$v = \frac{qBr}{m_p} = \frac{(1.60 \times 10^{-19}\ \text{C})(0.35\ \text{T})(14 \times 10^{-2}\ \text{m})}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= \boxed{4.7 \times 10^{6}\ \text{m/s}}$$

Example

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm. shows such a curved beam of electrons.) If the magnetic field is perpendicular to the beam, (a) what is the magnitude of the field?

Solution First we must calculate the speed of the electrons. We can use the fact that the increase in their kinetic energy must equal the decrease in their potential energy $|e|\Delta V$ (because of conservation of energy). Then we can use Equation to find the magnitude of the magnetic field. Because $K_i = 0$ and $K_f = m_e v^2/2$, we have

$$\tfrac{1}{2} m_e v^2 = |e|\Delta V$$

$$v = \sqrt{\frac{2|e|\Delta V}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})(350\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= 1.11 \times 10^{7}\ \text{m/s}$$

$$B = \frac{m_e v}{|e| r} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.11 \times 10^{7}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.075\ \text{m})}$$

$$= \boxed{8.4 \times 10^{-4}\ \text{T}}$$

Example

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm. (Fig. shows such a curved beam of electrons.) If the magnetic field is perpendicular to the beam, (a) what is the magnitude of the field?

Solution First we must calculate the speed of the electrons. We can use the fact that the increase in their kinetic energy must equal the decrease in their potential energy $|e|\Delta V$ (because of conservation of energy). Then we can use Equation 29.13 to find the magnitude of the magnetic field. Because $K_i = 0$ and $K_f = m_e v^2/2$, we have

$$\frac{1}{2}m_e v^2 = |e|\Delta V$$

$$v = \sqrt{\frac{2|e|\Delta V}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})(350\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= 1.11 \times 10^{7}\ \text{m/s}$$

$$B = \frac{m_e v}{|e|r} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.11 \times 10^{7}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.075\ \text{m})}$$

$$= \boxed{8.4 \times 10^{-4}\ \text{T}}$$

(b) What is the angular speed of the electrons?
Solution Using Equation in the following, we find that

$$\omega = \frac{v}{r} = \frac{1.11 \times 10^7 \text{ m/s}}{0.075 \text{ m}} = \boxed{1.5 \times 10^8 \text{ rad/s}}$$

Exercise What is the period of revolution of the electrons?

(The period of the motion (the time that the particle takes to complete one revolution) is equal to the circumference of the circle divided by the linear speed of the particle:

The frequency f (the number of revolutions per unit time) is

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency).}$$

The angular frequency ω of the motion is then

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency).}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Helical Paths

If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field

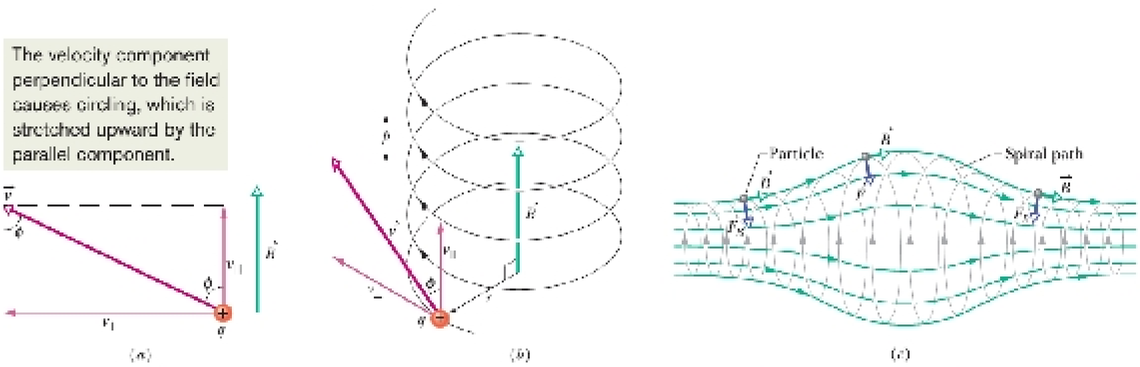


Fig. (*a*) A charged particle moves in a uniform magnetic field $\vec{B}$, the particle's velocity $\vec{v}$ making an angle ϕ with the field direction. (*b*) The particle follows a helical path of radius r and pitch p. (*c*) A charged particle spiraling in a nonuniform magnetic field. (The particle can become trapped, spiraling back and forth between the strong field regions at either end.) Note that the magnetic force vectors at the left and right sides have a component pointing toward the center of the figure.

vector. Figure a, for example, shows the velocity vector $\vec{v}$ of such a particle resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it:

$$v_{\parallel} = v \cos \phi \quad \text{and} \quad v_{\perp} = v \sin \phi.$$

The parallel component determines the *pitch p* of the helix—that is, the distance between adjacent turns (Fig. b). The perpendicular component determines the radius of the helix and is the quantity to be substituted for v in Eq.

Figure c shows a charged particle spiraling in a nonuniform magnetic field. The more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there. When the field at an end is strong enough, the particle "reflects" from that end. If the particle reflects from both ends, it is said to be trapped in a *magnetic bottle.*

When charged particles move in a nonuniform magnetic field, the motion is complex. For example, in a magnetic field that is strong at the ends and weak in the middle, such as that shown in the following Figure, the particles can oscillate back and forth between the end points. A charged particle starting at one end spirals along the field lines until it reaches the other end, where it reverses its path and spirals back. This configuration is known as a magnetic bottle because charged particles can be trapped (محصورة) within it. The magnetic bottle has been used to confine (لحصر) a plasma, a gas consisting of ions and electrons. Such a plasma-confinement scheme could fulfill a crucial role in the control of nuclear fusion, a process that could supply us with an almost endless source of energy. Unfortunately(لسؤ الحظ) , the magnetic bottle has its problems. If a large number of particles are trapped, collisions between them cause the particles to eventually (في النهاية)leak from the system.

The Van Allen radiation belts (احزمة) consist of charged particles (mostly electrons and protons) surrounding the Earth in doughnut-shaped regions. The particles, trapped by the Earth's nonuniform magnetic field, spiral around the field lines from pole to pole, covering the distance in just a few seconds. These particles originate mainly from the Sun, but some come from stars and other heavenly (سماوي)objects. For this reason, the particles are called cosmic rays (اشعة الكونية) . Most cosmic rays are deflected by the Earth's magnetic field and never reach the atmosphere (الغلاف الجوي).However, some of the particles become trapped; it is these particles that make up the Van Allen belts. When the particles are located over the poles, they sometimes collide with atoms in the atmosphere, causing the atoms to emit visible light. Such collisions are the origin of the beautiful Aurora Borealis(الشفق الشمالي) , or Northern Lights, in the northern

hemisphere (نصف الكرة الأرضية) and the Aurora Australis in the southern hemisphere.

Auroras are usually confined to the polar regions because it is here that the Van Allen belts are nearest the Earth's surface. Occasionally(من حين لآخر), though, solar activity causes larger numbers of charged particles to enter the belts and significantly distort the normal magnetic field lines associated (المرتبطة)with the Earth. In these situations an aurora can sometimes be seen at lower latitudes(خطوط العرض) .

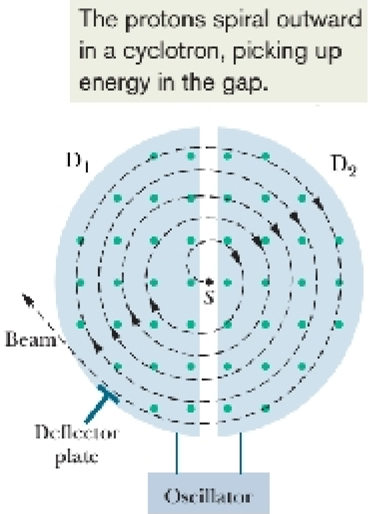
Cyclotrons and Synchrotrons

Beams of high-energy particles, such as high-energy electrons and protons, have been enormously (هائل) useful in probing(تقصي) atoms and nuclei to reveal the fundamental structure of matter. Such beams were instrumental (ذو دور فعال) in the discovery that atomic nuclei consist of protons and neutrons and in the discovery that protons and neutrons consist of quarks and gluons. The challenge of such beams is how to make and control them. Because electrons and protons are charged, they can be accelerated to the required high energy if they move through large potential differences. Because electrons have low mass, accelerating them in this way can be done in a reasonable distance(مسافة معقولة) . However, because protons (and other charged particles) have greater mass, the distance required for the acceleration is too long. A clever solution(محاليل ذكية) to this problem is first to let protons and other massive particles(كتل ضخمة) move through a modest potential difference (so that they gain a modest amount(مقدار معقول) of energy) and then use a magnetic field to cause them to circle back and move through a modest potential difference again. If this procedure is repeated thousands of times, the particles end up with a very large energy. Here we discuss two accelerators that employ a magnetic field to repeatedly bring particles back to an accelerating region, where they gain more and more energy until they finally emerge as a high-energy beam.

The Cyclotron

The Figure in the down is a top view of the region of a cyclotron in which the particles (protons, say) circulate. The two hollow D-shaped objects (each open on its straight edge) are made of sheet copper. These D_1 and D_2 as the shown in the figure, are part of an electrical oscillator that alternates(متناوبة) the electric potential difference across the gap between the D_1 and D_2. The electrical signs of the shape are alternated so that the electric field in the gap alternates in direction, first toward one (D_1) and then toward the other right (D_2), back and forth. The shape are immersed in a large magnetic field

directed out of the plane of the page. The magnitude B of this field is set via a control on the electromagnet producing the field.



The Fig. The elements of a cyclotron, showing the particle source S and the shape. A uniform magnetic field is directed up from the plane of the page. Circulating protons spiral outward within the hollow between D_1 andD_2, gaining energy every time they cross the gap between the $D_1$and D_2.

The Proton Synchrotron

At proton energies above **50 MeV**, the conventional cyclotron begins to **fail** because one of the assumptionsافتراض of its design—(that the frequency of revolution دورة of a charged particle circulating in a magnetic field is independent of the particle's speed)—is true only for **speeds** that are **much less than the speed of light**. At greater proton speeds (above about 10% of the speed of light), we must treat the problem relativisticallyنسبيا. According to relativity theory, as the speed of a circulating proton approaches that of light, the proton's frequency of revolution decreases steadilyتدريجيا. Thus, the proton gets outغير مماشي او يخرج of step with the cyclotron's oscillator—(whose frequency remains fixed at f_{osc})—and eventually the energy الطاقة في النهايةof the still circulating proton stops increasing.

The proton synchrotron is designed to meet these two difficulties. The magnetic field B and the oscillator frequency f_{osc}, instead of having fixed values as in the conventional cyclotron, are made to vary (تغير) with time during the accelerating cycle. When this is done properly, (1) the frequency of the circulating protons remains in step with the oscillator at all times (التردد لدوران البرتون يبقى مماشيا مع المذبذب في كل الأوقات), and (2) the protons follow a circular —(not a spiral)—path. Thus**, the magnet need extend only**

along that circular path, not over some $4x10^6 m^2$. **The circular path, however, still must be large if high energies are to be achieved**.

This resonance condition (f_{osc}) says that, if the energy of the circulating proton is to increase, energy must be fed(تغذى) to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field.

$$f = f_{osc} \qquad \text{(resonance condition)}$$

The key to the operation of the cyclotron is that the frequency (f) at which the proton circulates in the magnetic field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator

APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A CROSS FIELD

A charge moving with a velocity **v** in the presence of both an electric field **E** and a magnetic field **B** experiences both an electric force $q\mathbf{E}$ and a magnetic force $q\mathbf{v} \times \mathbf{B}$. The total force (called the Lorentz force) acting on the charge is

$$\sum \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

In many experiments involving moving charged particles, it is important that the particles all move with essentially the same velocity. This can be achieved by applying a combination of an electric field and a magnetic field oriented as shown in Figure down. A uniform electric field is directed vertically downward (in the plane of the page), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page). For q positive, the magnetic force is **upward**, and the electric force qE is **downward**. When the magnitudes of the two fields are chosen so that the particle moves in a **straight horizontal** line through the region of the fields. From the expression we find that

$$v = \frac{E}{B}$$

Only those particles having speed v pass undeflected through the mutually perpendicular electric and magnetic fields. The magnetic force exerted on particles moving at speeds greater than this is stronger than the electric force, and the particles are deflected upward. Those moving at speeds less than this are deflected downward.

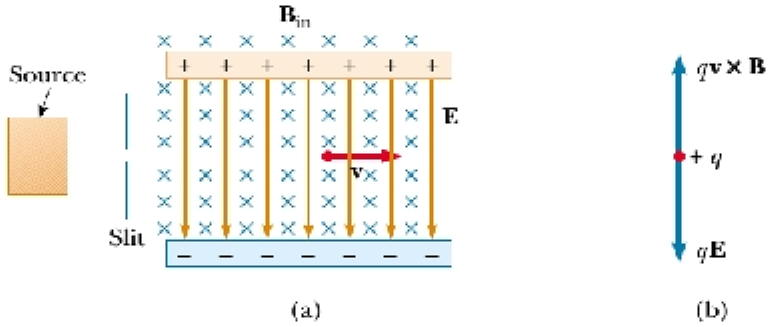


Figure (a) A velocity selector. When a positively charged particle is in the presence of a magnetic field directed into the page and an electric field directed downward, it experiences a downward electric force $q\mathbf{E}$ and an upward magnetic force $q\mathbf{v} \times \mathbf{B}$. (b) When these forces balance, the particle moves in a horizontal line through the fields.

Both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ can produce a force on a charged particle. When the two fields are perpendicular to each other, they are said to be *crossed fields*. Here we shall examine what happens to charged particles—namely, electrons—as they move through crossed fields. We use as our example the experiment that led to the discovery of the electron in 1897 by J. J. Thomson at Cambridge University. To measure the ratio e/m_e for electrons.

The Figure a shows the basic apparatus he used. Electrons are accelerated from the cathode and pass through two slits. They then drift into a region of perpendicular electric and magnetic fields. The magnitudes of the two fields are first adjusted to produce an undeflected beam. When the magnetic field is turned off, the electric field produces a measurable beam deflection that is recorded on the fluorescent screen. From the size of the deflection and the measured values of E and B, the charge-to-mass ratio can be determined. The results of this crucial experiment represent the discovery of the electron as a fundamental particle of nature.

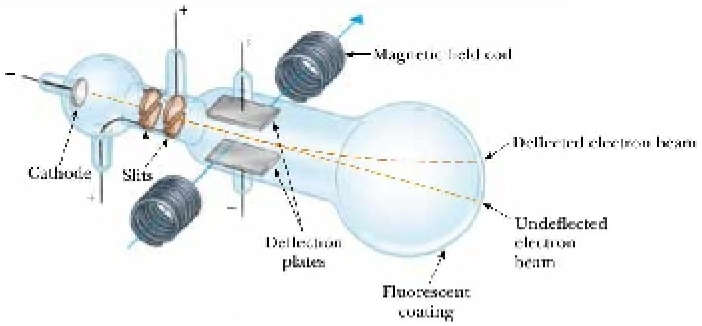


Figure Thomson's apparatus for measuring e/m_e. Electrons are accelerated from the cathode, pass through two slits, and are deflected by both an electric field and a magnetic field (directed perpendicular to the electric field). The beam of electrons then strikes a fluorescent screen.

Crossed Fields: The Hall Effect

In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can.
This Hall Effect allows us to find out whether(احد الاثنين) the charge carriers (حاملات الشحنات) in a conductor are positively or negatively charged. Beyond that, we can measure the number of such carriers per unit volume of the conductor. Figure (a) shows a copper strip(شريط) of width d, carrying a current i whose conventional direction(الاتجاه التقليدي) is from the top of the figure to the bottom. The charge carriers are electrons and, as we know, they drift (with drift speed v_d) in the opposite direction, from bottom to top. At the instant (لحظه) shown in Figure (a), an external magnetic field $\vec{B}$, pointing into the plane of the figure, has just been turned on. We see that a magnetic deflecting force $\vec{F}_B$ will act on each drifting electron, pushing it toward the right edge of the strip.
As time goes on, electrons move to the right, mostly piling up (تتكدس) on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge. The separation of positive charges on the left edge and negative charges on the right edge produces an electric field $\vec{E}$ within the strip, pointing from left to right in Figure (b). This field exerts an electric force on each electron, tending to push it to the left. Thus, this electric force $\vec{F}_E$ on the electrons, which opposes the magnetic force on them, begins to build up(تعزز) .
An equilibrium quickly develops in which the electric force on each electron has increased enough to match(تناظر) the magnetic force. When this happens, as Figure (b) shows, the force due to $\vec{B}$ and the force due to $\vec{E}$ are in balance. The drifting electrons then move along the strip toward the top of the page at velocity $\vec{v}_d$ with no further collection(تجمع) of electrons on the right edge of the strip and thus no further increase in the electric field $\vec{E}$.
A Hall potential difference V is associated with the electric field across strip width d.

$$V = Ed$$

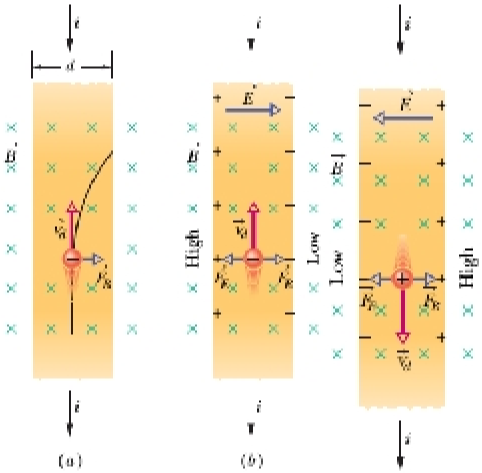


Figure. A strip of copper carrying a current i is immersed in a magnetic field $\vec{B}$. (a) The situation immediately after the magnetic field is turned on. The curved path that will then be taken by an electron is shown. (b) The situation at equilibrium, which quickly follows. Note that **negative** charges pile up (تتجمع) on the right side of the strip, leaving uncompensated(غير معوض) positive charges on the left. Thus, the left side is at a higher potential than the right side. (c) For the same current direction, if the charge carriers were **positively** charged, they would pile up on the right side, and the right side would be at the higher potential.

Sample Problem

Uniform circular motion of a charged particle in a magnetic field

Figure shows the essentials of a *mass spectrometer*, which can be used to measure the mass of an ion; an ion of mass m (to be measured) and charge q is produced in source S. The initially stationary ion is accelerated by the electric field due to a potential difference V. The ion leaves S and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000$ mT, $V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at $x = 1.6254$ m. What is the mass m of the individual ions, in atomic mass units ($1\ u = 1.6605 \times 10^{-27}$ kg)?

KEY IDEAS

(1) Because the (uniform) magnetic field causes the (charged) ion to follow a circular path, we can relate the ion's mass m to the path's radius r with Eq. ($r = mv/|q|B$). From Fig. we see that $r = x/2$ (the radius is half the diameter). From the problem statement, we know the magnitude B of the magnetic field. However, we lack the ion's speed v in the magnetic field after the ion has been accelerated due to the potential difference V. (2) To relate v and V, we use the fact that mechanical energy ($E_{mec} = K + U$) is conserved during the acceleration.

Finding speed: When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-V$. Thus, because the ion has positive charge q, its potential energy changes by $-qV$. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - qV = 0$$

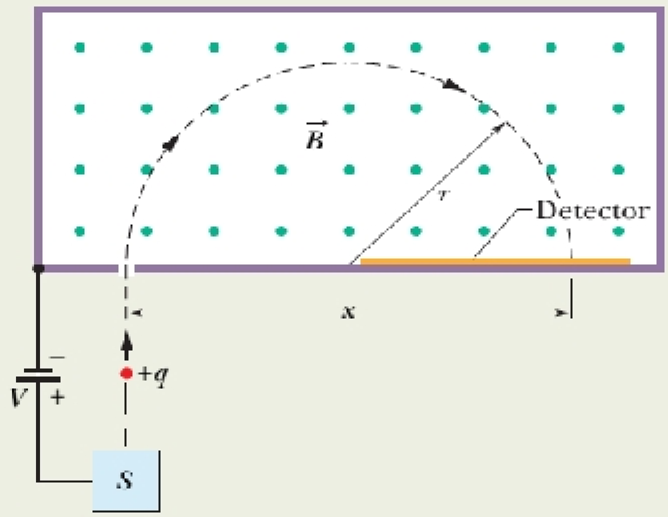


Fig. Essentials of a mass spectrometer. A positive ion, after being accelerated from its source S by a potential difference V, enters a chamber of uniform magnetic field $\vec{B}$. There it travels through a semicircle of radius r and strikes a detector at a distance x from where it entered the chamber.

detector at a distance x from where it entered the chamber.

or $$v = \sqrt{\frac{2qV}{m}}.$$

Finding mass: Substituting this value for v into Eq. gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus, $$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Solving this for m and substituting the given data yield

$$m = \frac{B^2 q x^2}{8V}$$
$$= \frac{(0.080000\ \text{T})^2(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})}$$
$$= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \qquad \text{(Answer)}$$

Example Accelerating a charged particle in a cyclotron

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a in part (D_1) radius R= 53 cm.

(a) What is the magnitude of the magetic field needed for deuterons to be accelerated in the cyclotron? The deuteron mass is $m=3.34\times10^{-27}$ kg (twice the proton mass).

KEY IDEA

For a given oscillator frequency fosc, the magnetic field magnitude B required to accelerate any particle in a cyclotron depends on the ratio m/|q| of mass to charge for the particle, according to Eq. ($|q|B = 2\pi m f_{osc}$).

Calculation: For deuterons and the oscillator frequency f_{osc} = 12 MHz, we find

$$B = \frac{2\pi m f_{osc}}{|q|} = \frac{(2\pi)(3.34 \times 10^{-27}\ \text{kg})(12 \times 10^6\ \text{s}^{-1})}{1.60 \times 10^{-19}\ \text{C}}$$
$$= 1.57\ \text{T} \approx 1.6\ \text{T}. \qquad \text{(Answer)}$$

Note that, to accelerate protons, B would have to be reduced by a factor of 2, provided the oscillator frequency remained fixed at 12 MHz.

(b) What is the resulting kinetic energy of the deuterons?

KEY IDEAS

(1) The kinetic energy ($\frac{1}{2}Mv^2$) of a deuteron exiting the cyclotron is equal to the kinetic energy it had just before exiting, when it was traveling in a circular path with a radius approximately equal to the radius R of the cyclotron shape. (2) We can find the speed v of the deuteron in that circular path with (r=mv/|q|B)

Calculations: Solving that equation for v, substituting R for r, and then substituting known data, we find

$$v = \frac{R|q|B}{m} = \frac{(0.53\text{ m})(1.60 \times 10^{-19}\text{ C})(1.57\text{ T})}{3.34 \times 10^{-27}\text{ kg}}$$

$$= 3.99 \times 10^{7}\text{ m/s.}$$

This speed corresponds to a kinetic energy of

$$K = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(3.34 \times 10^{-27}\text{ kg})(3.99 \times 10^{7}\text{ m/s})^2$$

$$= 2.7 \times 10^{-12}\text{ J,} \qquad \text{(Answer)}$$

or about 17 MeV.

<u>Magnetic Force on a Current-Carrying Wire</u>

Consider a length L of the wire in Fig. down. All the conduction electrons in this section of wire will drift past plane xx in Fig.(a) in a time t=L/vd. Thus, in that time a charge given by

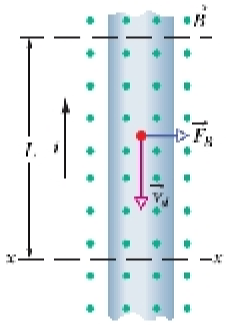
$$q = it = i\frac{L}{v_d}$$

If in state angle ϕ between the magnetic field and the wire is not perpendicular and opposite, is

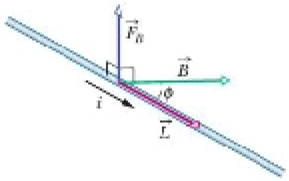
$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90^\circ$$

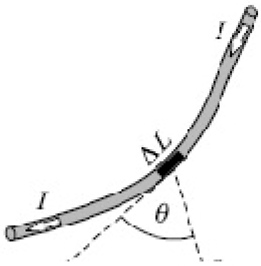
$$F_B = iLB$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad \text{(force on a current)}$$



a





b

If a wire is not straight or the field is not uniform, as Fig. up, we can imagine the wire broken up into small straight segments and apply the following Eq. to each segment. The force on the wire as a whole is then the vector sum of all the forces on the segments (لكل اجزاء) that make it up. In the differential limit, we can write

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}$$

<u>Example</u> **Magnetic force on a wire carrying current**

A straight, horizontal length of copper wire has a current i= 28 A through it. What are the magnitude and direction of the minimum magnetic $\vec{B}$ field needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig.). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (as Fig.) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$

Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg,$$

where mg is the magnitude of $\vec{F}_g$ and m is the mass of the wire.

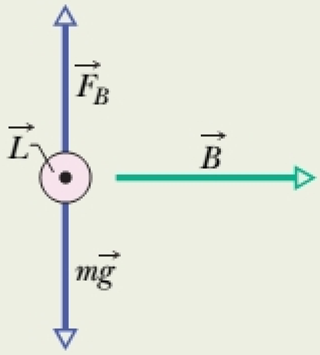


Fig. A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude B for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin\phi$ in . To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin\phi = 1$, so yields

$$B = \frac{mg}{iL\sin\phi} = \frac{(m/L)g}{i}.$$

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6\times10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$

$$= 1.6\times10^{-2}\ \text{T}. \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

(EX.) A flat circular coil with 40 loops of wire has a diameter of 32 cm. What current must flow in its wires to produce a field of $3.0\times10^{-4}\ \text{Wb/m}^2$ at its center?

$$B = \frac{\mu_0 N I}{2r} \quad \text{or} \quad 3.0\times10^{-4}\ \text{T} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(40)(I)}{2(0.16\ \text{m})} \qquad \text{which gives } I = 1.9\ \text{A}.$$

(EX.) An air-core solenoid with 2000 loops is 60 cm long and has a diameter of 2.0 cm. If a current of 5.0 A is sent through it, what will be the flux density within it?

$$B = \mu_0 n I = (4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2000}{0.60\ \text{m}}\right)(5.0\ \text{A}) = 0.021\ \text{T}$$

Calculating the Magnetic Field Due to a Current

THE BIOT – SAVART LAW

law of Biot and Savart the law of Biot and Savart (rhymes with “Leo and bazaar”). The law, which is experimentally deduced (استنتج) , is an inverse-square law. We shall use this law to calculate the net magnetic field produced at a point by various distributions of current.

We first mentally divide the wire into differential elements ds and then define for each element a length vector $\overrightarrow{ds}$ that has length ds and whose direction is the direction of the current in ds. We can then define a differential current-length element to be $i\overrightarrow{ds}$.

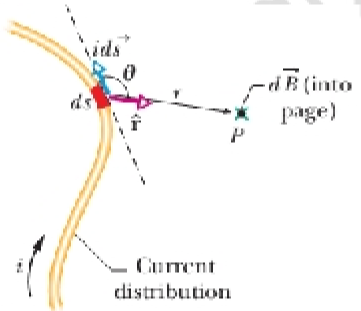


Fig. A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green × (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

a current-length element $i\,d\vec{s}$ producing a magnetic field is a vector, being the product of a scalar and a vector.

The magnitude of the field $d\vec{B}$ produced at point P at distance r by a current length element $i\,d\vec{s}$ turns out to be

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{r}}{r^2} \qquad \text{(Biot–Savart law).}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

where θ is the angle between the directions of $d\vec{s}$ and $\hat{r}$, a unit vector that points from ds toward P. Symbol μ_0 is a constant, called the *permeability constant,* whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\,\mathrm{T\cdot m/A}.$$

Magnetic Field Due to a Current in a Long Straight Wire

Shortly we shall use the law of Biot and Savart to prove that the magnitude of the magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire)}$$

To find the magnitude of the magnetic field at the center of a full circle of current by:-

$$B = \frac{\mu_0 i}{2R} \qquad \text{(at center of full circle)}$$

Force Between Two Parallel Currents

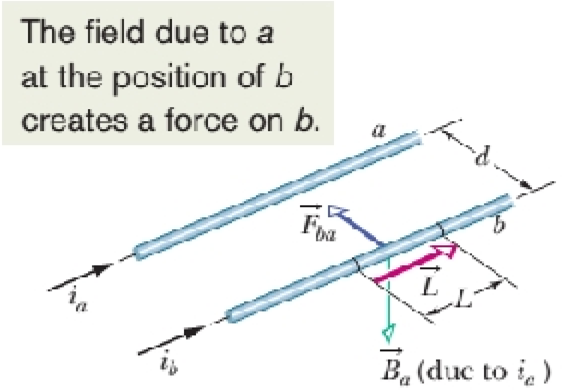


Fig. Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire b produced by the current in wire a. $\vec{F}_{ba}$ is the resulting force acting on wire b because it carries current in $\vec{B}_a$.

We seek first the force on wire b in Fig. due to the current in wire a. That current produces a magnetic field $\vec{B}_a$, and it is this magnetic field that actually causes the force we seek. To find the force, then, we need the magnitude and direction of the field $\vec{B}_a$ *at the site of wire b*. The magnitude of $\vec{B}_a$ at every point of wire b is

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

The curled–straight right-hand rule tells us that the direction of $\vec{B}_a$ at wire b is down, as Fig.

Now that we have the field, we can find the force it produces on wire b. the force $\vec{F}_{ba}$ on a length L of wire b due to the external magnetic field $\vec{B}_a$ is

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

where $\vec{L}$ is the length vector of the wire. In Fig. 29-9, vectors $\vec{L}$ and $\vec{B}_a$ are perpendicular to each other, and so with Eq. 29-11, we can write

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

The direction of $\vec{F}_{ba}$ is the direction of the cross product $\vec{L} \times \vec{B}_a$. Applying the right-hand rule for cross products to $\vec{L}$ and $\vec{B}_a$ in Fig. we see that $\vec{F}_{ba}$ is directly toward wire a, as shown.

Parallel currents attract each other, and antiparallel currents repel each other.

Ampere's Law

André-Marie Ampère (1775 – 1836), Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

We can find the net magnetic field due to any distribution of currents by first writing the differential magnetic field $d\vec{B}$ due to a current-length element and then summing the contributions of $d\vec{B}$ from all the elements. If the distribution has some symmetry, we may be able to apply **Ampere's law** to find the magnetic field with considerably less effort (جهد). This law, which can be derived from the Biot – Savart law.

Ampere's law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law)}$$

To see the meaning of the scalar product $\vec{B} \cdot d\vec{s}$ and its integral, let us first apply Ampere's law to the general situation of as the Figure. The figure shows cross sections of three long straight wires that carry currents i_1, i_2 and i_3 either directly into or directly out of the page. An arbitrary (اعتباطي) Amperian loop lying in the plane of the page encircles two of the currents but not the third. The counterclockwise (عكس عقارب الساعة) direction marked on the loop indicates the arbitrarily chosen direction of integration for upper Equation.

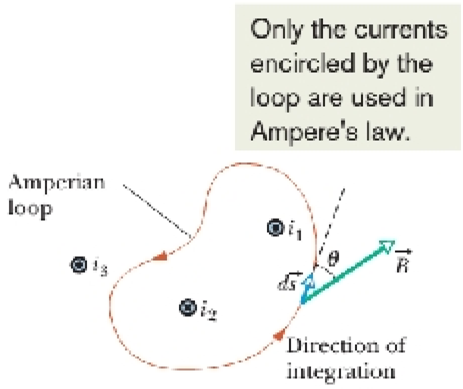


Fig. Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

With the indicated counterclockwise direction of integration, the net current encircled as the figure down, and the current i_3 is not encircled by the loop. Therefore, by the loop is

$$i_{enc} = i_1 - i_2$$

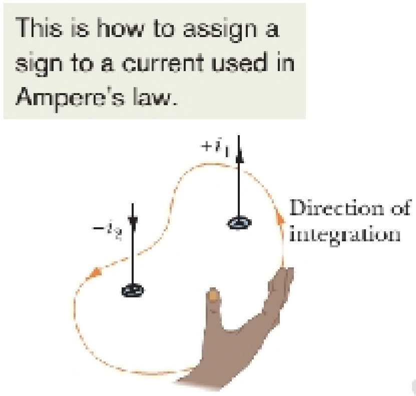


Figure A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop.

A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign. The net current encircled by the loop is

$$i_{enc} = i_1 - i_2$$

We can then rewrite

$$\oint B \cos\theta \, ds = \mu_0(i_1 - i_2)$$

Sample Problem

Magnetic field off to the side of two long straight currents

In the figure below, we show two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: i_1=15 A, i_2=32A, and d=5.3 cm.

KEY IDEAS

(1) The net magnetic field $\vec{B}$ at point P is the vector sum of the magnetic fields due to the currents in the two wires. (2) We can find the magnetic field due to any current by applying the Biot – Savart law to the current. For points near the current in a long straight wire.

Finding the vectors: In Fig. a, point P is distance R from both currents i_1 and i_2. Thus, figure a, tells us that at point P those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}$$

In the right triangle of Fig. 29-8a, note that the base angles (between sides Rand d) are both 45°.This allows us to write cos 45°=R/d and replace R with d cos 45°. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45^\circ} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45^\circ}$$

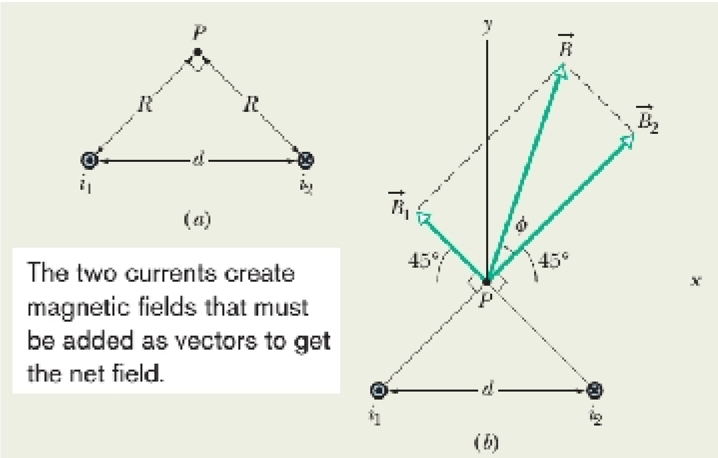


Figure (a) Two wires carry currents i1and i2in opposite directions (out of and into the page). Note the right angle at P. (b) The separate fields $\vec{B}_1$ and $\vec{B}_2$ are combined vectorially to yield the net field $\vec{B}$.

We want to combine $\vec{B}_1$ and $\vec{B}_2$ to find their vector sum, which is the net field $\vec{B}$ at P. To find the directions of $\vec{B}_1$ and $\vec{B}_2$ we apply the right-hand rule to each current in Fig. a. For wire 1, with current out of the page, we mentally grasp the wire with the right hand, with the thumb pointing out of the page. Then the curled fingers indicate that the field lines run counterclockwise. In particular, in the region of point P, they are directed upward to the left. Recall that the magnetic field at a point near a long, straight current-carrying wire must be directed perpendicular to a radial line between the point and the current. Thus, $\vec{B}_1$ must be directed upward to the left as drawn in Fig. b. (Note carefully the perpendicular symbol between vector $\vec{B}_1$ and the line connecting point P and wire 1.)

Repeating this analysis for the current in wire 2, we find that $\vec{B}_2$ is directed upward to the right as drawn in Fig. 29-8b. (Note the perpendicular symbol between vector $\vec{B}_2$ and the line connecting point P and wire 2.)

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point P, either by using a vector-capable calculator or by

resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. b, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean Theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)} \sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\sqrt{(15\ \text{A})^2 + (32\ \text{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \text{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\ \text{T} \approx 190\ \mu\text{T}. \qquad \text{(Answer)}$$

The angle Φ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. b, follows from

$$\phi = \tan^{-1} \frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1} \frac{i_1}{i_2} = \tan^{-1} \frac{15\ \text{A}}{32\ \text{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in fig. b Is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$

Magnetic Field Outside and Inside a Long Straight Wire with Current

a) **Outside a long straight wire**

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{(outside straight wire)}$$

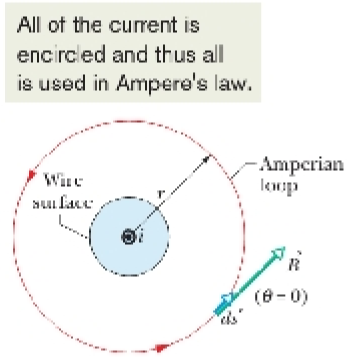


Fig. Using Ampere's law to find the magnetic field that a current (i) produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

b) Inside a long straight wire

where now $r < R$.

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \qquad \text{(inside straight wire)}$$

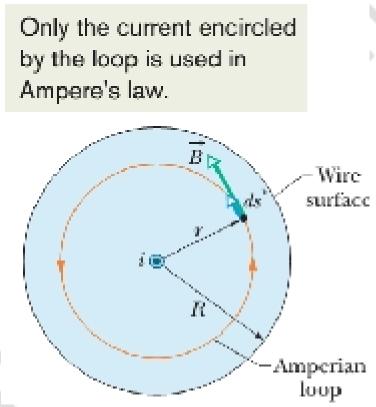


Fig. 29-14 Using Ampere's law to find the magnetic field that a current (i) produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Notes

Thus, inside the wire, the magnitude B of the magnetic field is proportional to r, is zero at the center, and is maximum at r = R(the surface).

Magnetic flux

To put Faraday's law to work, we need a way to calculate the amount of magnetic field that passes through a loop.

There we defined an **electric flux** $\Phi_E = \int \vec{E}\cdot d\vec{A}$. Here we define a magnetic flux: Suppose a loop enclosing an area A is placed in a magnetic field $\vec{B}$. Then the magnetic flux through the loop is

$$\Phi_B = \int \vec{B}\cdot d\vec{A} \qquad \text{(magnetic flux through area } A\text{)}$$

$d\vec{A}$ is a vector of magnitude dA that is perpendicular to a differential area dA.

Suppose that the loop lies in a plane and that the magnetic field is perpendicular to the plane of the loop. Then we can write the dot product in as B dAcos 0°=B dA. If the magnetic field is also uniform, then B can be brought out in front of the integral sign. The remaining then gives just the area A of the loop.

$$\Phi_B = BA \qquad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform})$$

The SI unit for magnetic flux is the tesla – square meter, which is called the weber

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T}\cdot\text{m}^2$$

<u>Faraday's law</u>

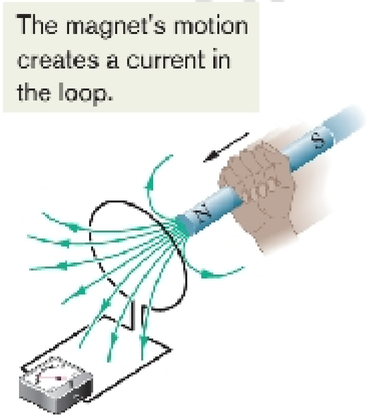


Fig. An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

The minus sign indicates that the induced emf opposite the change which produce it, as Lenz's Law.

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (closely packed), so that the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \qquad \text{(coil of } N \text{ turns).}$$

Here are the general means by which we can change the magnetic flux through a coil:

1) Change the magnitude B of the magnetic field within the coil.
2) Change either the total area of the coil or the portion of that area that lies within the magnetic field
3) Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil

Lenz's Law

He is determining the direction of an induced current in a loop.

An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

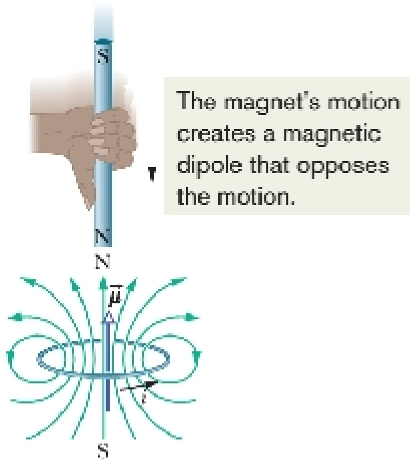


Fig. Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment

SELF-INDUCTANCE: A coil can induce an emf in itself. If the current in a coil changes, the flux through the coil due to the current also changes. As a result, the changing current in a coil induces an emf in that same coil.

Because an induced emf $\mathscr{E}$ is proportional to $\Delta\Phi_M/\Delta t$ and because $\Delta\Phi_M$ is proportional to Δi, where i is the current that causes the flux,

$$\mathscr{E} = -(\text{constant})\frac{\Delta i}{\Delta t} \quad \Longrightarrow \quad \mathscr{E} = L\frac{\Delta i}{\Delta t}$$

Here i is the current through the same coil in which $\mathscr{E}$ is induced.

The proportionality constant depends upon the geometry of the coil. We represent it by L and call it the *self-inductance* of the coil.

★ For $\mathscr{E}$ in units of V, i in units of A, and t in units of s, L is in *henries* (H).

MUTUAL INDUCTANCE: When the flux from one coil threads through another coil, an emf can be induced in either one by the other. The coil that contains the power source is called the *primary coil*. The other coil, in which an emf is induced by the changing current in the primary, is called the *secondary coil*. The induced secondary emf $\mathscr{E}_s$ is proportional to the time rate of change of the primary current, $\Delta i_p/\Delta t$:

$$\mathscr{E}_s = M\frac{\Delta i_p}{\Delta t}$$

where M is a constant called the *mutual inductance* of the two-coil system.

★ **ENERGY STORED IN AN INDUCTOR:** Stored energy $= \frac{1}{2}LI^2$

For L in units of H and I in units of A, the energy is in J.

Sample Problem

The figure shown a conducting loop consisting of a half-circle of radius r=0.20 m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B=4.0t^2+2.0t+3.0$, with Bin teslas and t in seconds. An ideal battery with emf $\mathscr{E}_{bat}=2.0$ V is connected to the loop. The resistance of the loop is 2.0Ω.

(a) What are the magnitude and direction of the emf $\mathscr{E}_{ind}$ induced around the loop by field $\vec{B}$ at t=10 s?

KEY IDEAS

1. According to Faraday's law, the magnitude of $\mathscr{E}_{ind}$ is equal to the rate $d\Phi_B/dt$ at which the magnetic flux through the loop changes.
2. The flux through the loop depends on how much of the loop's area lies within the flux and how the area is oriented in the magnetic field $\vec{B}$.
3. Because $\vec{B}$ is uniform and is perpendicular to the plane of the loop, the flux is given by Eq. ($\Phi_B = BA$). (We don't need to integrate B over the area to get the flux.)
4. The induced field B_{ind} (due to the induced current) must always oppose the *change* in the magnetic flux.

Magnitude: Using Eq. and realizing that only the field magnitude B changes in time (not the area A), we rewrite Faraday's law, Eq. as

$$\mathscr{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

Because the flux penetrates the loop only within the half-circle, the area A in this equation is $\frac{1}{2}\pi r^2$. Substituting this and the given expression for B yields

$$\mathscr{E}_{\text{ind}} = A\frac{dB}{dt} = \frac{\pi r^2}{2}\frac{d}{dt}(4.0t^2 + 2.0t + 3.0)$$
$$= \frac{\pi r^2}{2}(8.0t + 2.0).$$

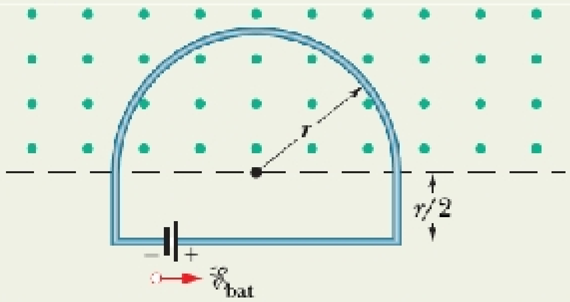


Fig. A battery is connected to a conducting loop that includes a half-circle of radius r lying in a uniform magnetic field. The field is directed out of the page; its magnitude is changing.

At $t = 10$ s, then,

$$\mathscr{E}_{\text{ind}} = \frac{\pi(0.20\text{ m})^2}{2}[8.0(10) + 2.0]$$
$$= 5.152\text{ V} \approx 5.2\text{ V}. \qquad \text{(Answer)}$$

Direction: To find the direction of $\mathscr{E}_{\text{ind}}$, we first note that in Fig. the flux through the loop is out of the page and increasing. Because the induced field B_{ind} (due to the induced current) must oppose that increase, it must be *into* the page. Using the curled–straight right-hand rule , we find that the induced current is clockwise around the loop, and thus so is the induced emf $\mathscr{E}_{\text{ind}}$.

(b) What is the current in the loop at $t = 10$ s?

KEY IDEA

The point here is that *two* emfs tend to move charges around the loop.

Calculation: The induced emf $\mathscr{E}_{ind}$ tends to drive a current clockwise around the loop; the battery's emf $\mathscr{E}_{bat}$ tends to drive a current counterclockwise. Because $\mathscr{E}_{ind}$ is greater than $\mathscr{E}_{bat}$, the net emf $\mathscr{E}_{net}$ is clockwise, and thus so is the current. To find the current at $t = 10$ s, we use Eq. 27-2 ($i = \mathscr{E}/R$):

$$i = \frac{\mathscr{E}_{net}}{R} = \frac{\mathscr{E}_{ind} - \mathscr{E}_{bat}}{R}$$

$$= \frac{5.152\text{ V} - 2.0\text{ V}}{2.0\ \Omega} = 1.58\text{ A} \approx 1.6\text{ A.} \qquad \text{(Answer)}$$

Induced Electric Fields

Let us place a copper ring of radius r in a uniform external magnetic field, the magnetic flux through the ring will then change at a steady rate and (by Faraday's law)an induced emf and thus an induced current will appear in the ring. From Lenz's law we can deduce (استنتج) that the direction of the induced current is counterclockwise, if there is a current in the copper ring, an electric field must be present along the ring because an electric field is needed to do the work of moving the conduction electrons. Moreover, the electric field must have been produced by the changing magnetic flux. Therefore, we rewrite Faraday's law.

A changing magnetic field produces an electric field.

Consider a particle of charge q_0 moving around the circular path, The work W done on it, is

$$W = \mathscr{E}q_0.$$

$$W = \int \vec{F} \cdot d\vec{s} = (q_0E)(2\pi r),$$

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}$$

Where $q_0E$is the magnitude of the force acting on the test charge and $2\pi r$ is the distance.

Substituting $\mathscr{E}q_0$ for W, we find that

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}$$

We can rewrite Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

(EX) A steady current of 2 A in a coil of 400 turns causes a flux of 10^{-4} Wb to link (pass through) the loops of the coil. Compute (*a*) the average back emf induced in the coil if the current is stopped in 0.08 s, (*b*) the inductance of the coil, and (*c*) the energy stored in the coil.

(*a*) $$|\mathscr{E}| = N\left|\frac{\Delta\Phi_M}{\Delta t}\right| = 400\frac{(10^{-4} - 0)\text{ Wb}}{0.08\text{ s}} = 0.5\text{ V}$$

(*b*) $$|\mathscr{E}| = L\left|\frac{\Delta i}{\Delta t}\right| \quad \text{or} \quad L = \left|\frac{\mathscr{E}\Delta t}{\Delta i}\right| = \frac{(0.5\text{ V})(0.08\text{ s})}{(2-0)\text{ A}} = 0.02\text{ H}$$

(*c*) $$\text{Energy} = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(0.02\text{ H})(2\text{ A})^2 = 0.04\text{ J}$$

(EX.) A flat circular coil with 40 loops of wire has a diameter of 32 cm. What current must flow in its wires to produce a field of 3.0×10^{-4} Wb/m^2 at its center?

$$B = \frac{\mu_0 NI}{2r} \quad \text{or} \quad 3.0 \times 10^{-4}\text{ T} = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(40)(I)}{2(0.16\text{ m})} \qquad \text{which gives } I = 1.9\text{ A.}$$

(EX.) An air-core solenoid with 2000 loops is 60 cm long and has a diameter of 2.0 cm. If a current of 5.0 A is sent through it, what will be the flux density within it?

$$B = \mu_0 nI = (4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{2000}{0.60\text{ m}}\right)(5.0\text{ A}) = 0.021\text{ T}$$

MAGNETIC EFFECTS OF MATTER:

Suppose that a very long solenoid or a toroid is located in vacuum. With a fixed current in the coil, the magnetic field at a certain point inside the solenoid or toroid is B_0, where the subscript $_0$ stands for vacuum. If now the solenoid or toroid core is filled with a material, the field at that point will be changed to a new value B. We define:

$$\textit{Relative permeability} \text{ of the material} = k_M = \frac{B}{B_0}$$

$$\textit{Permeability} \text{ of the material} = \mu = k_M\mu_0$$

Recall that μ_0 is the permeability of free space, $4\pi \times 10^{-7}$ T·m/A.

Diamagnetic materials have values for k_M slightly below unity (0.999 984 for solid lead, for example). They slightly decrease the value of B in the solenoid or toroid.

Paramagnetic materials have values for k_M slightly larger than unity (1.000 021 for solid aluminum, for example). They slightly increase the value of B in the solenoid or toroid.

Ferromagnetic materials, such as iron and its alloys, have k_M values of about 50 or larger. They greatly increase the value of B in the toroid or solenoid.

MOTION ALONG A STRAIGHT LINE

Position The *position* x of a particle on an x axis locates the particle with respect to the **origin,** or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The **positive direction** on an axis is the direction of increasing positive numbers; the opposite direction is the **negative direction** on the axis.

Displacement The *displacement* Δx of a particle is the change in its position:

$$\Delta x = x_2 - x_1.$$

Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the x axis and negative if the particle has moved in the negative direction.

Average Velocity When a particle has moved from position x_1 to position x_2 during a time interval $\Delta t = t_2 - t_1$, its *average velocity* during that interval is

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

The algebraic sign of v_{avg} indicates the direction of motion (v_{avg} is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.

On a graph of x versus t, the average velocity for a time interval Δt is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.

Average Speed The *average speed* s_{avg} of a particle during a time interval Δt depends on the total distance the particle moves in that time interval:

$$s_{avg} = \frac{\text{total distance}}{\Delta t}.$$

Instantaneous Velocity The *instantaneous velocity* (or simply **velocity**) v of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt},$$

The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of x versus t. Speed is the magnitude of instantaneous velocity.

Average Acceleration *Average acceleration* is the ratio of a change in velocity Δv to the time interval Δt in which the change occurs:

$$a_{avg} = \frac{\Delta v}{\Delta t}.$$

The algebraic sign indicates the direction of a_{avg}.

Instantaneous Acceleration *Instantaneous acceleration* (or simply **acceleration**) a is the first time derivative of velocity $v(t)$ and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

On a graph of v versus t, the acceleration a at any time t is the slope of the curve at the point that represents t.

Constant Acceleration The five equations describe the motion of a particle with constant acceleration:

$$v = v_0 + at,$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0),$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t,$$

$$x - x_0 = vt - \tfrac{1}{2}at^2.$$

These are not valid when the acceleration is not constant.

Free-Fall Acceleration An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant acceleration equations describe this motion, but we make two changes in notation: (1) we refer the motion to the vertical y axis with $+y$ vertically *up*; (2) we replace a with $-g$, where g is the magnitude of the free-fall acceleration. Near Earth's surface, $g = 9.8\ \text{m/s}^2$ ($= 32\ \text{ft/s}^2$).

Example

On a graph of x versus t, v_{avg} is the **slope** of the straight line that connects two particular points on the $x(t)$ curve: one is the point that corresponds to x_2 and t_2, and the other is the point that corresponds to x_1 and t_1. Like displacement, v_{avg} has both magnitude and direction (it is another vector quantity). Its magnitude is the magnitude of the line's slope. A positive v_{avg} (and slope) tells us that the line slants upward to the right; a negative v_{avg} (and slope) tells us that the line slants downward to the right. The average velocity v_{avg} always has the same sign as the displacement Δx because Δt in Eq is always positive.

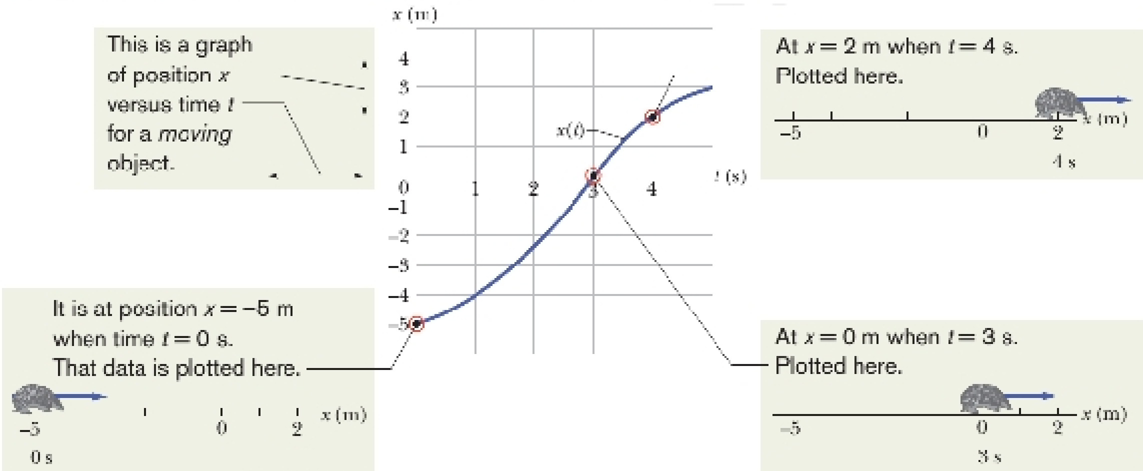


Fig. The graph of x(t) for a moving armadillo(تدريجي). The path associated with the graph is also shown, at three times

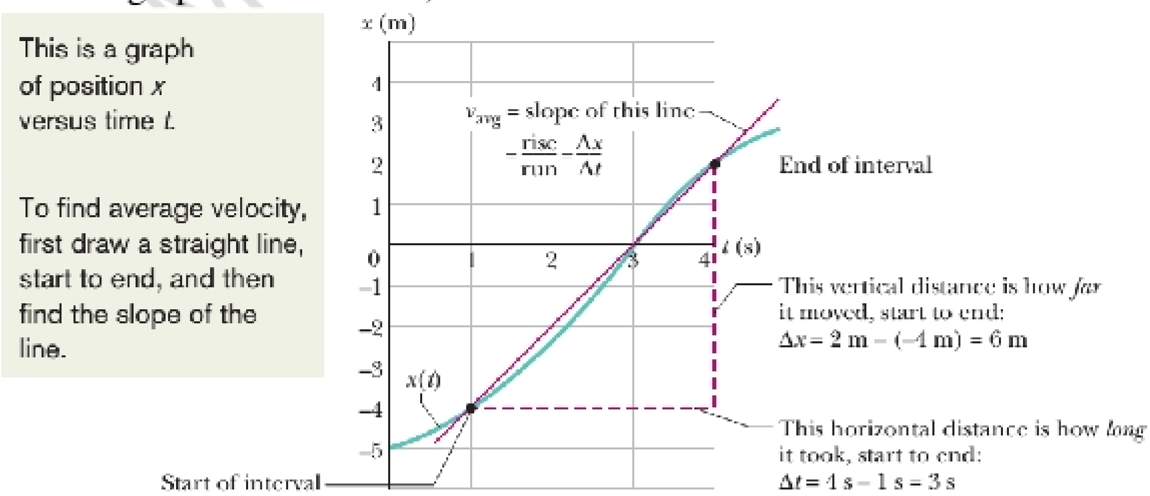


Fig. Calculation of the average velocity between t=1s and t=4 s as the slope of the line that connects the points on the x(t) curve representing those times.

$$v_{avg} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}.$$

Example

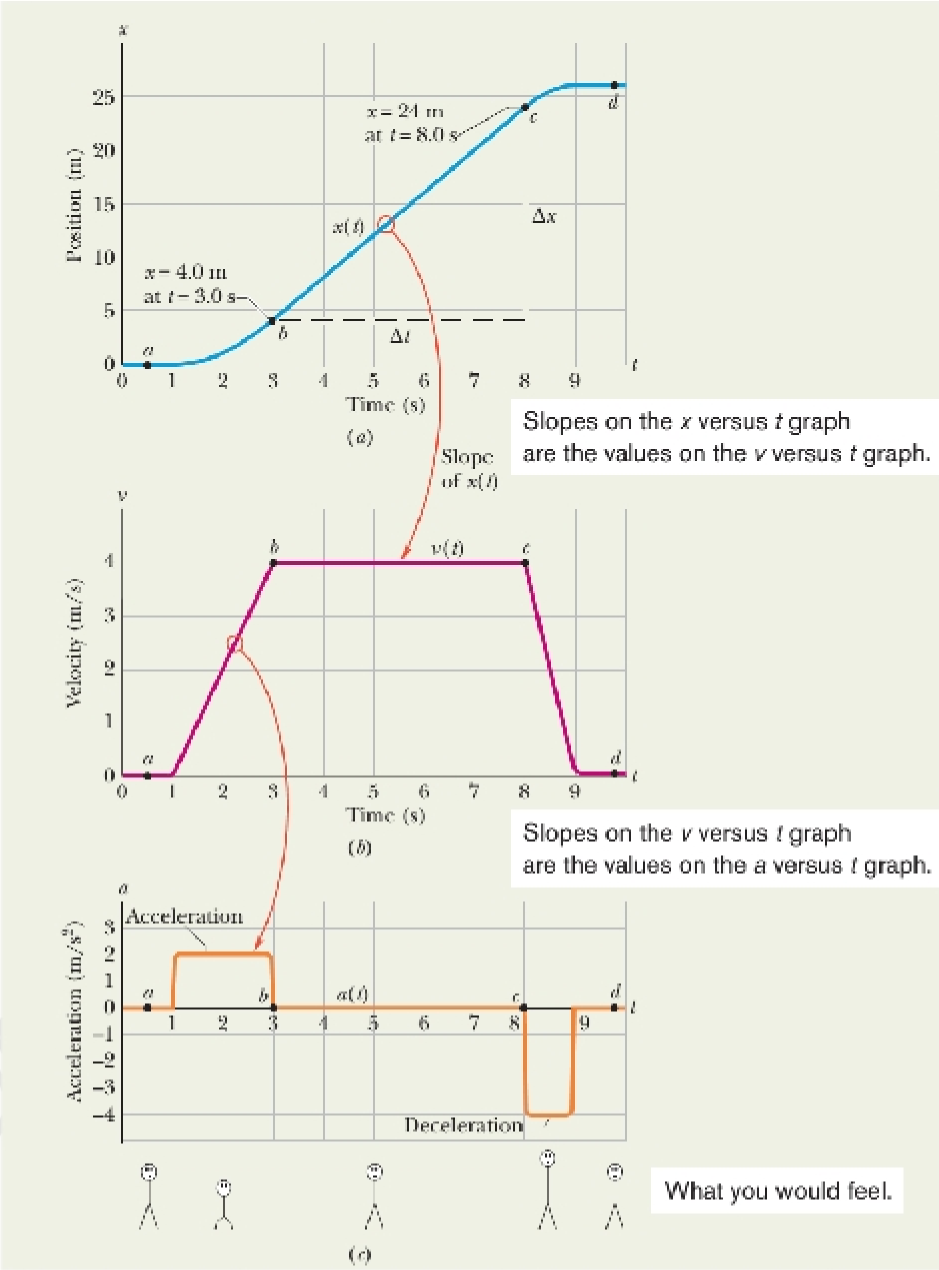


Fig. (a) The x(t) curve for an elevator cab that moves upward along an xaxis. (b)The v(t) curve for the cab. Note that it is the derivative of the x(t) curve (v=dx/dt). (c) The a(t) curve for the cab. It is the derivative of the v(t)

curve(a=dv/dt).The stick figures along the bottom suggest how a passenger's body might feel during the accelerations.

Examples

Change the speed 0.200 cm/s to units of kilometers per year.

$$0.200\frac{\text{cm}}{\text{s}} = \left(0.200\frac{\cancel{\text{cm}}}{\cancel{\text{s}}}\right)\left(10^{-5}\frac{\text{km}}{\cancel{\text{cm}}}\right)\left(3600\frac{\cancel{\text{s}}}{\cancel{\text{h}}}\right)\left(24\frac{\cancel{\text{h}}}{\cancel{\text{d}}}\right)\left(365\frac{\cancel{\text{d}}}{\text{y}}\right) = 63.1\frac{\text{km}}{\text{y}}$$

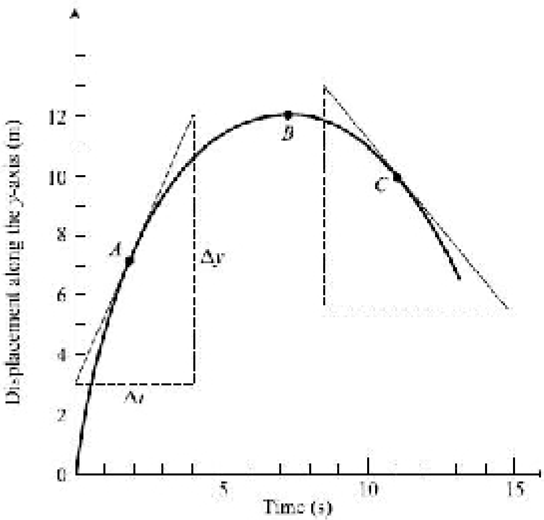
Examples

A runner makes one lap around a 200-m track in a time of 25 s. What were the runner's average speed

From the definition,

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time taken}} = \frac{200\ \text{m}}{25\ \text{s}} = 8.0\ \text{m/s}$$

(6) The vertical motion of an object is graphed in Fig. . Describe its motion qualitatively, and find its instantaneous velocity at points A, B, and C.



Recalling that the instantaneous velocity is given by the slope of the graph, we see that the object is moving fastest at $t = 0$. As it rises, it slows and finally stops at B. (The slope there is zero.) Then it begins to fall back downward at ever-increasing speed.

At point A, we have

$$v_A = \text{slope} = \frac{\Delta y}{\Delta t} = \frac{12.0\ \text{m} - 3.0\ \text{m}}{4.0\ \text{s} - 0\ \text{s}} = \frac{9.0\ \text{m}}{4.0\ \text{s}} = 2.3\ \text{m/s}$$

The velocity at A is positive, so it is in the $+y$-direction: $\vec{v}_A = 2.3$ m/s — UP. At points B and C,

$$v_B = \text{slope} = 0\,\text{m/s}$$

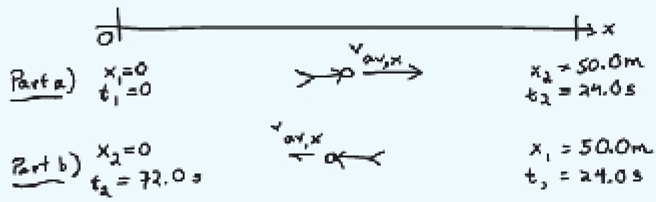
$$v_C = \text{slope} = \frac{\Delta y}{\Delta t} = \frac{5.5\ \text{m} - 13.0\ \text{m}}{15.0\ \text{s} - 8.5\ \text{s}} = \frac{-7.5\ \text{m}}{6.5\ \text{s}} = -1.2\ \text{m/s}$$

Because it is negative, the velocity at C is in the $-y$-direction: $\vec{v}_C = 1.2$ m/s — DOWN. Remember that velocity is a vector quantity and direction must be specified explicitly.

EXAMPLE 2.1 **Swim competition**

▲ **FIGURE 2.3**

During a freestyle competition, a swimmer performs the crawl stroke in a pool 50.0 m long, as shown in Figure 2.3. She swims a length at racing speed, taking 24.0 s to cover the length of the pool. She then takes twice that time to swim casually back to her starting point. Find (a) her average velocity for each length and (b) her average velocity for the entire swim.



▲ **FIGURE 2.4** Our sketch for this problem.

SOLUTION

SET UP As shown in Figure 2.4, we choose a coordinate system with the origin at the starting point (often a convenient choice) and x increasing to the right. We add the information given to the diagram we sketch for the problem.

SOLVE **Part (a):** For the first length, we have $x_1 = 0$, $x_2 = 50.0$ m, $t_1 = 0$, and $t_2 = 24.0$ s. Using the definition of average velocity given by Equation 2.3, we find that

$$v_{av,x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{50.0\ \text{m} - 0\ \text{m}}{24.0\ \text{s} - 0\ \text{s}} = 2.08\ \text{m/s}.$$

For the return trip, we have $x_1 = 50.0$ m, $x_2 = 0$, $t_1 = 24.0$ s, and $t_2 = 24.0\ \text{s} + 48.0\ \text{s} = 72.0$ s. Using the definition of average velocity again, we obtain

$$v_{av,x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0\ \text{m} - 50.0\ \text{m}}{72.0\ \text{s} - 24.0\ \text{s}} = -1.04\ \text{m/s}.$$

Part (b): The starting and finishing points are the same: $x_1 = x_2 = 0$. The average velocity for a round-trip is zero!

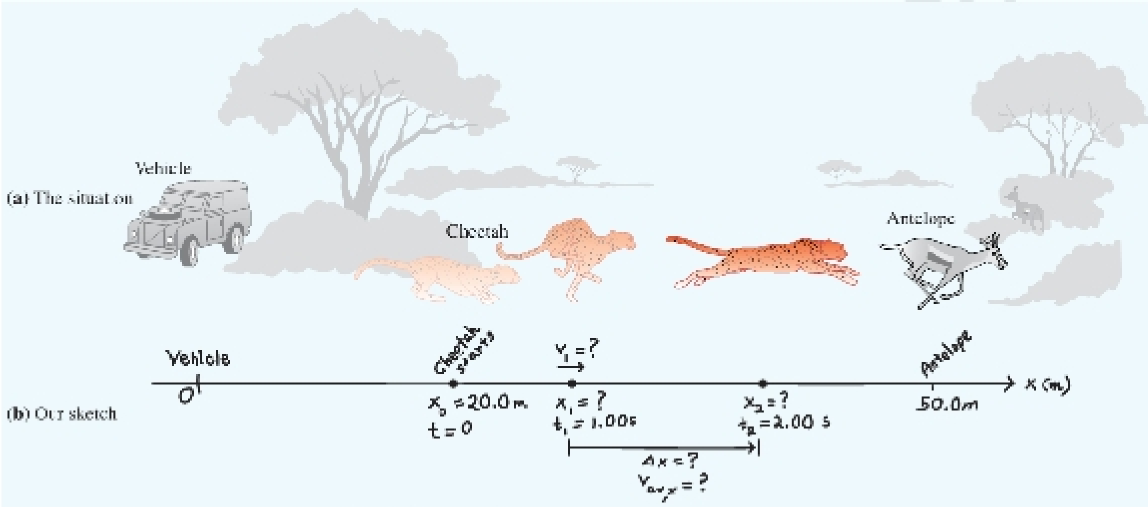
The average velocity during the two laps is zero because the total displacement is zero. But the average speed is the total distance traveled (100m), divided by the total time (72.0 s): 100.0 m/72.0 s = 1.39 m/s.

EXAMPLE **Average and instantaneous velocities**

A cheetah is crouched in ambush 20.0 m to the east of an observer's vehicle, as shown in Figure At time $t = 0$, the cheetah charges an antelope in a clearing 50.0 m east of the observer. The cheetah runs along a straight line; the observer estimates that, during the first 2.00 s of the attack, the cheetah's coordinate x varies with time t according to the equation

$$x = 20.0\ \text{m} + (5.00\ \text{m/s}^2)t^2.$$

(a) Find the displacement of the cheetah during the interval between $t_1 = 1.00$ s and $t_2 = 2.00$ s. **(b)** Find the average velocity during this time interval. **(c)** Estimate the instantaneous velocity at time $t_1 = 1.00$ s by taking $\Delta t = 0.10$ s.



SOLUTION

SET UP Figure 2.10b shows the diagram we sketch. First we define a coordinate system, orienting it so that the cheetah runs in the $+x$ direction. We add the points we are interested in, the values we know, and the values we will need to find for parts (a) and (b).

SOLVE **Part (a):** To find the displacement Δx, we first find the cheetah's positions (the values of x) at time $t_1 = 1.00$ s and at time $t_2 = 2.00$ s by substituting the values of t into the given equation. At time $t_1 = 1.00$ s, the cheetah's position x_1 is

$$x_1 = 20.0\ \text{m} + (5.00\ \text{m/s}^2)(1.00\ \text{s})^2 = 25.0\ \text{m}.$$

At time $t_2 = 2.00$ s, the cheetah's position x_2 is

$$x_2 = 20.0\ \text{m} + (5.00\ \text{m/s}^2)(2.00\ \text{s})^2 = 40.0\ \text{m}.$$

The displacement during this interval is

$$\Delta x = x_2 - x_1 = 40.0\ \text{m} - 25.0\ \text{m} = 15.0\ \text{m}.$$

Part (b): Having the displacement from 1.00 s to 2.00 s, we can now find the average velocity for that interval:

$$v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{40.0\ \text{m} - 25.0\ \text{m}}{2.00\ \text{s} - 1.00\ \text{s}} = \frac{15.0\ \text{m}}{1.00\ \text{s}} = 15.0\ \text{m/s}.$$

Part (c): The instantaneous velocity at 1.00 s is approximately (but not exactly) equal to the average velocity in the interval from $t_1 = 1.00$ s to $t_2 = 1.10$ s (i.e., $\Delta t = 0.10$ s). At $t_2 = 1.10$ s,

$$x_2 = 20.0\ \text{m} + (5.00\ \text{m/s})(1.10\ \text{s})^2 = 26.05\ \text{m},$$

so that

$$v_{\text{av},x} = \frac{\Delta x}{\Delta t} = \frac{26.05\ \text{m} - 25.0\ \text{m}}{1.10\ \text{s} - 1.00\ \text{s}} = 10.5\ \text{m/s}.$$

EXAMPLE 2.3 **Acceleration in a space walk**

An astronaut has left the space shuttle on a tether to test a new personal maneuvering device. She moves along a straight line directly away from the shuttle. Her onboard partner measures her velocity before and after certain maneuvers, and obtains the following results:

(a) $v_{1x} = 0.8$ m/s, $v_{2x} = 1.2$ m/s; (speeding up)
(b) $v_{1x} = 1.6$ m/s, $v_{2x} = 1.2$ m/s; (slowing down)
(c) $v_{1x} = -0.4$ m/s, $v_{2x} = -1.0$ m/s; (speeding up)
(d) $v_{1x} = -1.6$ m/s, $v_{2x} = -0.8$ m/s. (slowing down)

If $t_1 = 2$ s and $t_2 = 4$ s in each case, find the average acceleration for each set of data.

SOLUTION

SET UP We use the diagram in Figure to organize our data.

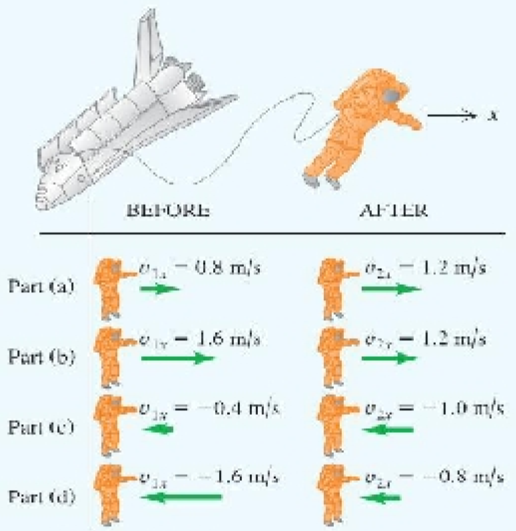
SOLVE To find the astronaut's average acceleration in each case, we will use the definition of average acceleration $a_{av,x} = \Delta v_x/\Delta t$. The time interval is $\Delta t = 2.0$ s in all cases; the change in velocity in each case is $\Delta v_x = v_{2x} - v_{1x}$.

Part (a): $a_{av,x} = \dfrac{1.2\ \text{m/s} - 0.8\ \text{m/s}}{4\ \text{s} - 2\ \text{s}} = +0.2\ \text{m/s}^2;$

Part (b): $a_{av,x} = \dfrac{1.2\ \text{m/s} - 1.6\ \text{m/s}}{4\ \text{s} - 2\ \text{s}} = -0.2\ \text{m/s}^2;$

Part (c): $a_{av,x} = \dfrac{-1.0\ \text{m/s} - (-0.4\ \text{m/s})}{4\ \text{s} - 2\ \text{s}} = -0.3\ \text{m/s}^2;$

Part (d): $a_{av,x} = \dfrac{-0.8\ \text{m/s} - (-1.6\ \text{m/s})}{4\ \text{s} - 2\ \text{s}} = +0.4\ \text{m/s}^2.$



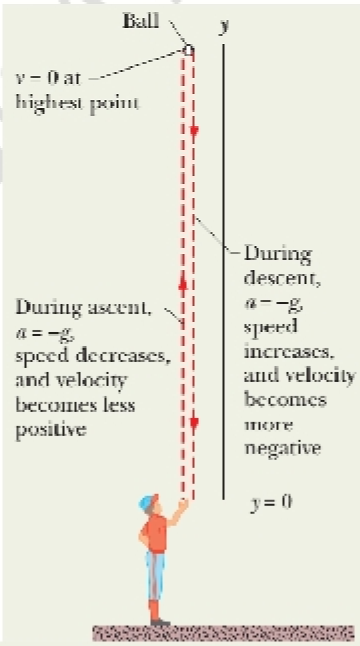
Example

In Fig., a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s. (a) How long does the ball take to reach its maximum height?

KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration a=-g. Because this is constant.
(2)The velocity vat the maximum height must be 0.

Calculation: Knowing v, a, and the initial velocity v_0=12 m/s, and seeking t, we solve Eq., which contains



those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12\ \text{m/s}}{-9.8\ \text{m/s}^2} = 1.2\ \text{s}. \qquad \text{(Answer)}$$

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in y notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for y. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 7.3\text{ m}. \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

Calculations: We know v_0, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we choose Eq. 2-15. Rewriting it for y and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2} g t^2,$$

or

$$5.0\text{ m} = (12\text{ m/s})t - (\tfrac{1}{2})(9.8\text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

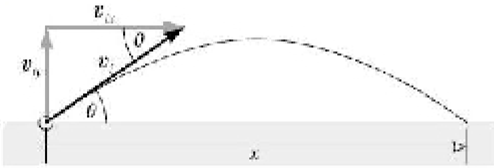
$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for t yields

$$t = 0.53\text{ s} \quad \text{and} \quad t = 1.9\text{ s}. \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

Ex (*a*) Find the range x of a gun which fires a shell with muzzle velocity v at an angle of elevation θ. (*b*) Find the angle of elevation θ of a gun which fires a shell with a muzzle velocity of 120 m/s and hits a target on the same level but 1300 m distant.



(*a*) Let t be the time it takes the shell to hit the target. Then, $x = v_{ix}t$ or $t = x/v_{ix}$. Consider the vertical motion alone, and take *up* as positive. When the shell strikes the target,

$$\text{Vertical displacement} = 0 = v_{iy}t + \tfrac{1}{2}(-g)t^2$$

Solving this equation gives $t = 2v_{iy}/g$. But $t = x/v_{ix}$, so

$$\frac{x}{v_{ix}} = \frac{2v_{iy}}{g} \qquad \text{or} \qquad x = \frac{2v_{ix}v_{iy}}{g} = \frac{2(v_i \cos\theta)(v_i \sin\theta)}{g}$$

The formula $2 \sin\theta \cos\theta = \sin 2\theta$ can be used to simplify this. After substitution, we get

$$x = \frac{v_i^2 \sin 2\theta}{g}$$

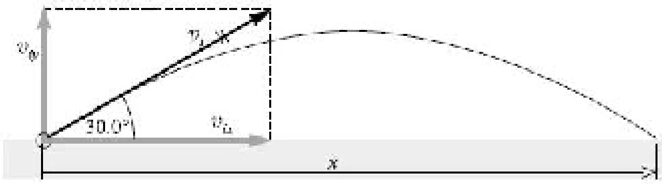
The maximum range corresponds to $\theta = 45°$, since $\sin 2\theta$ has a maximum value of 1 when $2\theta = 90°$ or $\theta = 45°$.

(*b*) From the range equation found in (*a*), we have

$$\sin 2\theta = \frac{gx}{v_i^2} = \frac{(9.81\ \text{m/s}^2)(1300\ \text{m})}{(120\ \text{m/s})^2} = 0.885$$

Therefore, $2\theta = \arcsin 0.885 = 62°$ and so $\theta = 31°$.

Ex A baseball is thrown with an initial velocity of 100 m/s at an angle of 30.0° above the horizontal, as shown in Fig. . How far from the throwing point will the baseball attain its original level?



We divide the problem into horizontal and vertical parts, for which

$$v_{ix} = v_i \cos 30.0° = 86.6\ \text{m/s} \qquad \text{and} \qquad v_{iy} = v_i \sin 30.0° = 50.0\ \text{m/s}$$

where *up* is being taken as positive.

In the vertical problem, $y = 0$ since the ball returns to its original height. Then

$$y = v_{iy}t + \tfrac{1}{2}a_y t^2 \qquad \text{or} \qquad 0 = (50.0\ \text{m/s}) + \tfrac{1}{2}(-9.81\ \text{m/s}^2)t$$

and $t = 10.2$ s.

In the horizontal problem, $v_{ix} = v_{fx} = v_x = 86.6$ m/s. Therefore,

$$x = v_x t = (86.6\ \text{m/s})(10.2\ \text{s}) = 884\ \text{m}$$

Sample Problem

Cannonball to pirate ship

Figure 4-15 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

KEY IDEAS

(1) A fired cannonball is a projectile. We want an equation that relates the launch angle θ_0 to the ball's horizontal displacement as it moves from cannon to ship. (2) Because the cannon and the ship are at the same height, the horizontal displacement is the range.

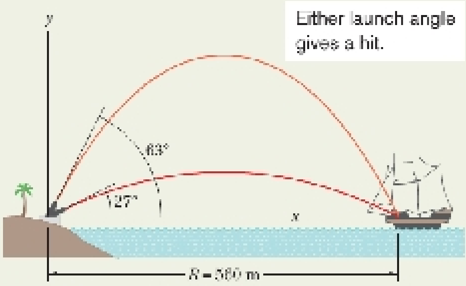


Fig. 4-15 A pirate ship under fire.

Calculations: We can relate the launch angle θ_0 to the range R with Eq. 4-26 which, after rearrangement, gives

$$\theta_0 = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} = \frac{1}{2}\sin^{-1}\frac{(9.8\ \text{m/s}^2)(560\ \text{m})}{(82\ \text{m/s})^2}$$
$$= \frac{1}{2}\sin^{-1} 0.816. \qquad (4\text{-}33)$$

One solution of $\sin^{-1}$ (54.7°) is displayed by a calculator; we subtract it from 180° to get the other solution (125.3°). Thus, Eq. 4-33 gives us

$$\theta_0 = 27° \quad \text{and} \quad \theta_0 = 63°. \qquad \text{(Answer)}$$

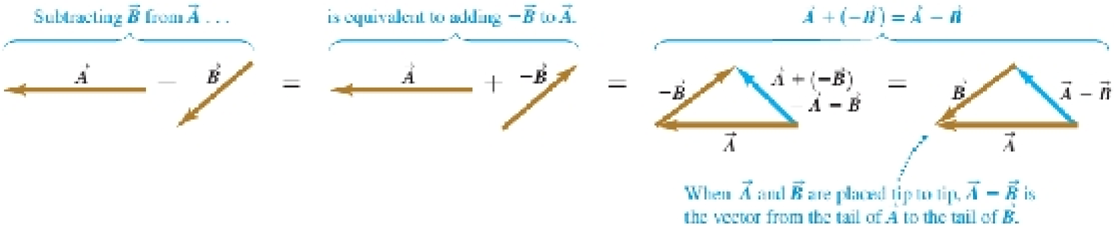
(b) What is the maximum range of the cannonballs?

Calculations: We have seen that maximum range corresponds to an elevation angle θ_0 of 45°. Thus,

$$R = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(82\ \text{m/s})^2}{9.8\ \text{m/s}^2}\sin(2 \times 45°)$$
$$= 686\ \text{m} \approx 690\ \text{m}. \qquad \text{(Answer)}$$

As the pirate ship sails away, the two elevation angles at which the ship can be hit draw together, eventually merging at $\theta_0 = 45°$ when the ship is 690 m away. Beyond that distance the ship is safe. However, the cannonballs could go farther if the cannon were higher.

Note

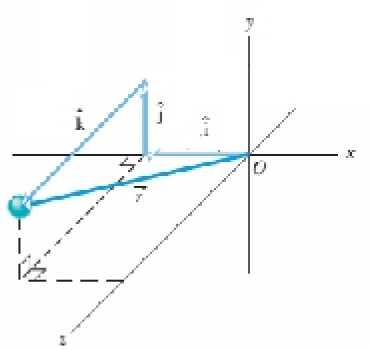


MOTION IN TWOAND THREE DIMENSIONS

Position and Displacement

The locating of particle is with a position $\vec{r}$ vector, which is a vector that extends from a reference point (usually the origin) to the particle. The vector $\vec{r}$ can be written:

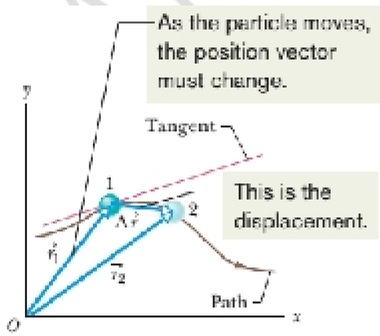
$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}},$$



If the position vector changes say, from to during a certain time interval then the particle's displacement $\Delta\vec{r}$ during that time interval is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}}$$



Average Velocity and Instantaneous Velocity

If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt, then its average velocity $\Delta\vec{v}_{avg}$ is:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}}$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}.$$

$$\vec{v}_{avg} = \frac{\Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\,\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\,\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\,\hat{\mathrm{k}}$$

When spoken about instantaneous velocity $\vec{v}$ of a particle. This is the value that approaches in the limit as we shrink the time interval Δt to 0 about that instant. We may write $\vec{v}$ as the derivative:

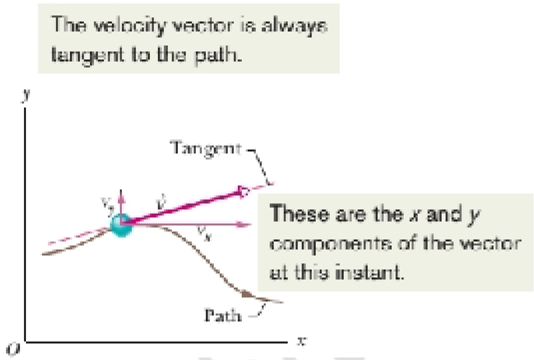
$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}$$

Where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}$$

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}$$



<u>Average Acceleration and Instantaneous Acceleration</u>

When a particle's velocity changes from $\vec{v}_1$to $\vec{v}_2$ in a time interval Δt, its average acceleration $\vec{a}_{\text{avg}}$ during Δt is:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}.$$

If we shrink Δt to zero about some instant, then in the limit $\vec{a}_{\text{avg}}$ approaches the instantaneous acceleration(or acceleration) $\vec{a}$ at that instant; that is,

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}})$$

$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}$$

We can rewrite this as

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}$$

Where the scalar components are:

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}.$$

Example

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$

and

$$y = 0.22t^2 - 9.1t + 30.$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

Calculations: We can write

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}.$$

(We write $\vec{r}(t)$ rather than $\vec{r}$ because the components are functions of t, and thus $\vec{r}$ is also.)

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and $$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so $$\vec{r} = (66 \text{ m})\hat{\imath} - (57 \text{ m})\hat{\jmath}, \qquad \text{(Answer)}$$

which is drawn in Fig. To get the magnitude and angle of $\vec{r}$, we use Eq.

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$
$$= 87 \text{ m}, \qquad \text{(Answer)}$$

and $$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41°. \qquad \text{(Answer)}$$

b) at velocity t=15sec

KEY IDEA

We can find $\vec{v}$ by taking derivatives of the components of the rabbit's position vector.

Calculations: Applying the v_x part of Eq. to Eq. , we find the x component of $\vec{v}$ to be

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$
$$= -0.62t + 7.2.$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1.$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation then yields

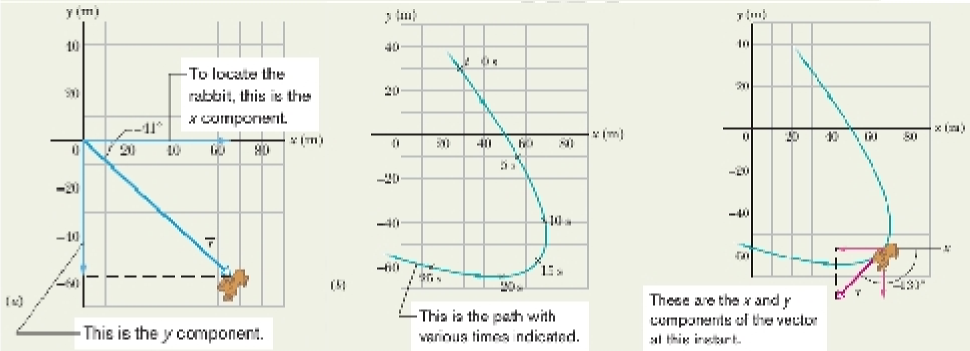
$$\vec{v} = (-2.1\ \text{m/s})\hat{\text{i}} + (-2.5\ \text{m/s})\hat{\text{j}}, \qquad \text{(Answer)}$$

which is shown in Fig. , tangent to the rabbit's path and in the direction the rabbit is running at $t = 15$ s.

To get the magnitude and angle of $\vec{v}$, either we use a vector-capable calculator or we follow Eq. to write

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1\ \text{m/s})^2 + (-2.5\ \text{m/s})^2}$$
$$= 3.3\ \text{m/s} \qquad \text{(Answer)}$$

and $$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5\ \text{m/s}}{-2.1\ \text{m/s}}\right)$$
$$= \tan^{-1} 1.19 = -130°. \qquad \text{(Answer)}$$



c) Find the acceleration at time t=15sec.

KEY IDEA

We can find $\vec{a}$ by taking derivatives of the rabbit's velocity components.

Calculations: Applying the a_x part we find the x component of $\vec{a}$ to be

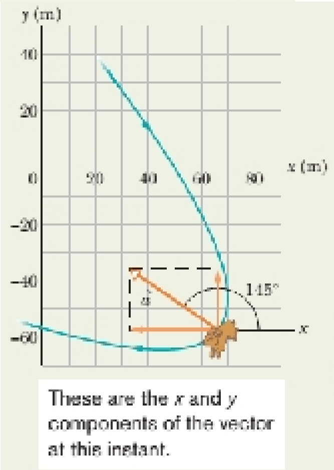
$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62\ \text{m/s}^2.$$

Similarly, applying the a_y part yields the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

We see that the acceleration does not vary with time (it is a constant)because the time variable t does not appear in the expression for either acceleration component, then yields

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{\text{i}} + (0.44 \text{ m/s}^2)\hat{\text{j}}, \qquad \text{(Answer)}$$



Uniform Circular Motion

The figure shows the relationship between the velocity and acceleration vectors at various stages during uniform circular motion. Both vectors have constant magnitude, but their directions change continuously. The velocity is always directed tangent to the circle in the direction of motion. The acceleration is always directed radially inward. Because of this, the acceleration associated with uniform circular motion is called a centripetal (meaning "center seeking") acceleration. The magnitude of this acceleration $\vec{a}$ is:

$$a = \frac{v^2}{r} \qquad \text{(centripetal acceleration),}$$

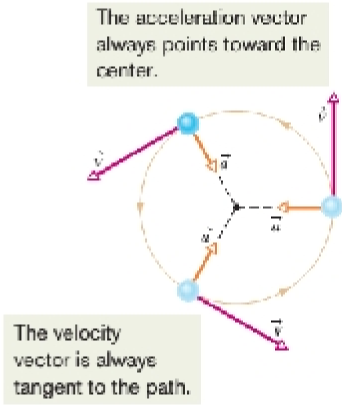


Fig. Velocity and acceleration vectors for uniform circular motion.

Where r is the radius of the circle and υ is the speed of the particle.

In addition, during this acceleration at constant speed, the particle travels the circumference of the circle (a distance of 2πr) in time

$$T = \frac{2\pi r}{v} \qquad \text{(period).}$$

Example

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(1.496 \times 10^{11}\ \text{m})}{(1\ \text{yr})^2}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right)^2$$

$$= \boxed{5.93 \times 10^{-3}\ \text{m/s}^2}$$

Example

The passengers in a carnival ride travel in a circle with radius 5.0 m. They make one complete circle in a time. What is their acceleration?

SOLUTION

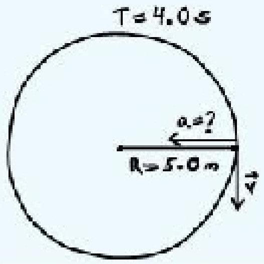
SET UP Figure 3.28 shows our diagram.

SOLVE We again use Equation 3.16: $a = v^2/R$. To find the speed v, we use the fact that a passenger travels a distance equal to the circumference of the circle $(2\pi R)$ in the time T for one revolution:

$$v = \frac{2\pi R}{T} = \frac{2\pi(5.0\ \text{m})}{4.0\ \text{s}} = 7.9\ \text{m/s}.$$

The centripetal acceleration is

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{(7.9\ \text{m/s})^2}{5.0\ \text{m}} = 12\ \text{m/s}^2.$$



Example

Sample Problem

Top gun pilots in turns

"Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g-LOC for "g-induced loss of consciousness."

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{i} + 500\hat{j})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{i} - 500\hat{j})$ m/s?

KEY IDEAS

We assume the turn is made with uniform circular motion. Then the pilot's acceleration is centripetal and has magnitude a given by Eq. 4-34 ($a = v^2/R$), where R is the circle's radius. Also, the time required to complete a full circle is the period given by Eq. 4-35 ($T = 2\pi R/v$).

Calculations: Because we do not know radius R, let's solve Eq. 4-35 for R and substitute into Eq. 4-34. We find

$$a = \frac{2\pi v}{T}.$$

Speed v here is the (constant) magnitude of the velocity during the turning. Let's substitute the components of the initial velocity into Eq. 3-6:

$$v = \sqrt{(400\ \text{m/s})^2 + (500\ \text{m/s})^2} = 640.31\ \text{m/s}.$$

To find the period T of the motion, first note that the final velocity is the reverse of the initial velocity. This means the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given 24.0 s. Thus a full circle would have taken $T = 48.0$ s. Substituting these values into our equation for a, we find

$$a = \frac{2\pi(640.31\ \text{m/s})}{48.0\ \text{s}} = 83.81\ \text{m/s}^2 \approx 8.6g. \qquad \text{(Answer)}$$

Summary

Position Vector The location of a particle relative to the origin of a coordinate system is given by a *position vector* $\vec{r}$, which in unit-vector notation is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

Here $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$ are the vector components of position vector $\vec{r}$, and x, y, and z are its scalar components (as well as the coordinates of the particle). A position vector is described either by a magnitude and one or two angles for orientation, or by its vector or scalar components.

Displacement If a particle moves so that its position vector changes from $\vec{r}_1$ to $\vec{r}_2$, the particle's *displacement* $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

The displacement can also be written as

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}.$$

Average Velocity and Instantaneous Velocity If a particle undergoes a displacement $\Delta\vec{r}$ in time interval Δt, its *average velocity* $\vec{v}_{avg}$ for that time interval is

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}.$$

As Δt in Eq. UP is shrunk to 0, $\vec{v}_{avg}$ reaches a limit called either the *velocity* or the *instantaneous velocity* $\vec{v}$:

$$\vec{v} = \frac{d\vec{r}}{dt},$$

which can be rewritten in unit-vector notation as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k},$$

where $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$. The instantaneous velocity $\vec{v}$ of a particle is always directed along the tangent to the particle's path at the particle's position.

Average Acceleration and Instantaneous Acceleration If a particle's velocity changes from $\vec{v}_1$ to $\vec{v}_2$ in time interval Δt, its *average acceleration* during Δt is

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}.$$

As Δt in Eq. 4-15 is shrunk to 0, $\vec{a}_{avg}$ reaches a limiting value called either the *acceleration* or the *instantaneous acceleration* $\vec{a}$:

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

In unit-vector notation,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k},$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.

Projectile Motion *Projectile motion* is the motion of a particle that is launched with an initial velocity $\vec{v}_0$. During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$. (Upward is taken to be a positive direction.) If $\vec{v}_0$ is expressed as a magnitude (the speed v_0) and an angle θ_0 (measured from the horizontal), the particle's equations of motion along the horizontal x axis and vertical y axis are

$$x - x_0 = (v_0 \cos\theta_0)t,$$
$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2,$$
$$v_y = v_0 \sin\theta_0 - gt,$$
$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The **trajectory** (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2},$$

if x_0 and y_0 of Eqs. 4-21 to 4-24 are zero. The particle's **horizontal range** R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g}\sin 2\theta_0.$$

Uniform Circular Motion If a particle travels along a circle or circular arc of radius r at constant speed v, it is said to be in *uniform circular motion* and has an acceleration $\vec{a}$ of constant magnitude

$$a = \frac{v^2}{r}.$$

The direction of $\vec{a}$ is toward the center of the circle or circular arc, and $\vec{a}$ is said to be *centripetal*. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v}.$$

T is called the *period of revolution*, or simply the *period*, of the motion.

Relative Motion When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B. The two measured velocities are related by

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA},$$

where $\vec{v}_{BA}$ is the velocity of B with respect to A. Both observers measure the same acceleration for the particle:

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

FORCE AND MOTION

Newtonian Mechanics

Newton's First Law

First Law When all external influences on a particle are removed, the particle moves with constant velocity. [This velocity may be zero in which case the particle remains at rest.]

In other words, if the body is at rest, it stays at rest. If it is moving, it continues to move with the same velocity (same magnitude and same direction). (That mean, the acceleration is zero, in two caces)

That is, if the net force acting on the object is zero, then the object either remains at rest or continues to move with constant velocity. When the velocity of an object is constant (including the case in which the object remains at rest), the object is said to be in equilibrium.

$$\mathbf{a} = 0 \text{ and } \Sigma\mathbf{F} = 0$$

Mass and Newton's Second Law

Second Law When a force $\boldsymbol{F}$ acts on a particle of mass m, the particle moves with instantaneous acceleration $\boldsymbol{a}$ given by the formula

$$\boldsymbol{F} = m\boldsymbol{a},$$

where the unit of force is implied by the units of mass and acceleration.

The net force on a body is equal to the product of the body's mass and its acceleration.

Newton's Third Law

Third Law When two particles exert forces upon each other, these forces are (i) equal in magnitude, (ii) opposite in direction, and (iii) parallel to the straight line joining the two particles.

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}.$$

To every action there is always opposed an equal reaction; or, the mutual actions of two objects upon each other are always equal, and directed to contrary parts.

Mass

The mass of a body is the characteristic of that body that relates the body's acceleration to the net force causing the acceleration. Masses are scalar quantities.

Force

The concept of force gives us a quantitative description of the interaction between two objects or between an object and its environment. When you push on a car that is stuck in the snow, you exert a force on it, a locomotive exerts a force on the train it is pulling or pushing, a steel cable exerts a force on the beam it is lifting at a construction site, and so on. Each of these examples shows that force is a push or a pull acting on an object. In this chapter, we'll encounter several kinds of forces.

Types of force

Contact force ($\vec{F}$) (When a force involves direct contact between two objects), **Normal force** ($\vec{n}$) (When an object rests on a surface, there is always a component of force perpendicular to the surface), **friction force** ($\vec{f}$) (This force be a component of force parallel to the surface, and often (though not always) acts to resist sliding of the object on the surface. When a rope (الحبل) or cord is attached to an object and pulled, the corresponding force applied to the object is referred to as a **tension** ($\vec{T}$). When there force of the gravitational attraction of the earth is act on object is call **Weight** ($\vec{w}$) and, also called is (The **Gravitational Force** F_g**).**

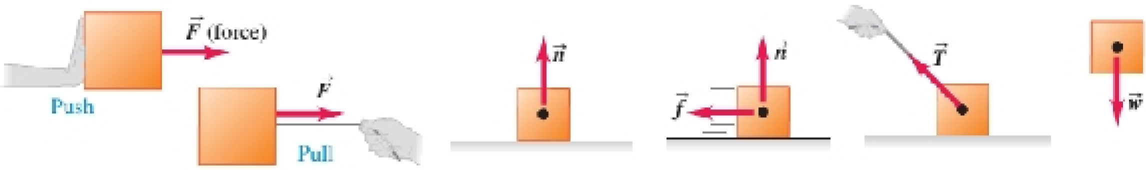


Fig. That forms appear types of the force

Tension

The tension force $\vec{T}$ is pulling force applied at body ends as figure (a). If the cord wraps (ملفوف) halfway around a pulley (البكرة), as in Figure(c), the net force on the pulley from the cord has the magnitude 2T.

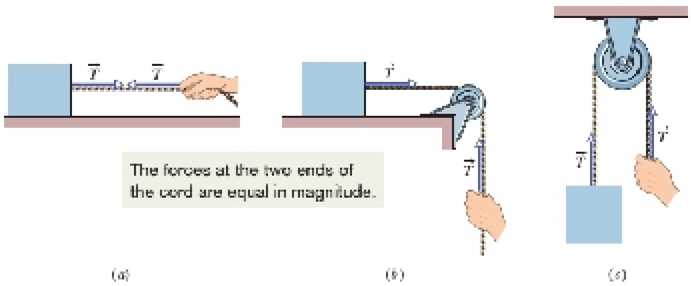


Fig. (a) The cord, pulled taut, is under tension. If its mass is negligible, the cord pulls on the body and the hand with force $\vec{T}$, even if the cord runs around a massless, frictionless pulley as in (b) and (c).

Weight

The weight W of a body is the magnitude of the net force required to prevent the body from falling freely,

$$W = F_g$$

$$W = mg \qquad \text{(weight)}$$

The Normal Force

is the push on you from the floor is a normal force, and The name comes from the mathematical term normal, meaning perpendicular.
When the body is motion due act force (in other order, Newton's second law),

$$F_N - F_g = ma_y$$

$$F_N - mg = ma_y.$$

$$F_N = mg + ma_y = m(g + a_y)$$

If the table and block are not accelerating relative to the ground, a=0 is yield

$$F_N = mg$$

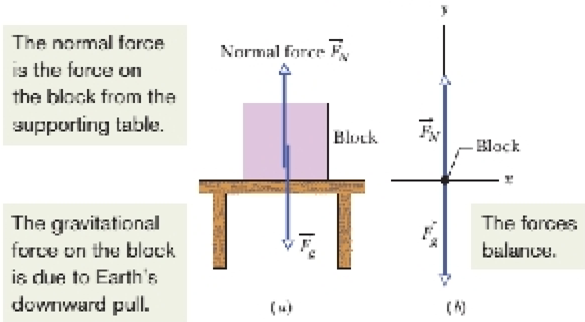


Fig. (*a*) A block resting on a table experiences a normal force $\vec{F}_N$ perpendicular to the tabletop. (*b*) The free-body diagram for the block.

Frictional force

The friction force is resisted of motion the body due action force on it, by a bonding between the body and the surface. The direction of f_k is opposite of the body motion, and there, frictional coefficient μ_k is

$$f_k = \mu_k F_N$$

Where, μ_k is the coefficient of kinetic friction. f_k frictional force, F_N of the normal force

Resultant of Forces

Experiment shows that when two forces $\vec{F_1}$ and $\vec{F_2}$ act at the same time on the same point of an object, the effect is the same as the effect of a single force equal to the vector sum of the original forces. This vector sum is often

called the resultant of the forces or the net force, denoted by $\vec{R}$. That is,
$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

More generally, the effect of any number of forces applied at a point on an object is the same as the effect of a single force equal to the vector sum of the original forces. This important principle goes by the name superposition of forces.

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2 + \vec{\boldsymbol{F}}_3 + \cdots = \sum \vec{\boldsymbol{F}}, \quad \text{(resultant, or vector sum, of forces)}$$

$$R_x = \sum F_x, \qquad R_y = \sum F_y, \qquad \text{(components of vector sum of forces)}$$

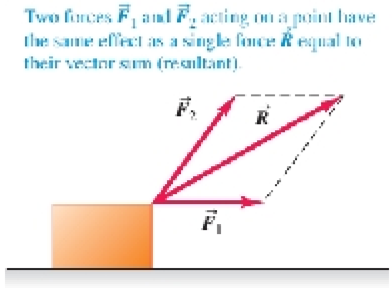
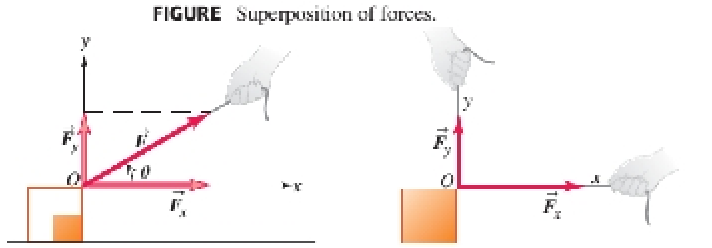


FIGURE Superposition of forces.



(a) Component vectors: $\vec{F}_x$ and $\vec{F}_y$
Components: $F_x = F \cos\theta$ and $F_y = F \sin$

(b) Component vectors $\vec{F}_x$ and $\vec{F}_y$ together have the same effect as original force $\vec{F}$.

FIGURE A force can be represented by its components.

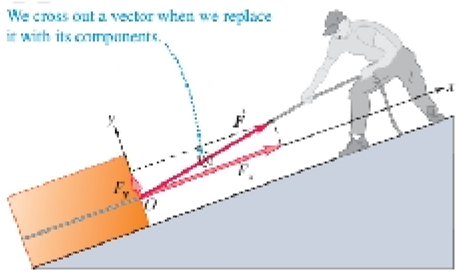


FIGURE The pull exerted by the mason can be replaced by components parallel and perpendicular to the direction of motion.

Units in Newton's Second Law

System	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
CGS	dyne	gram (g)	cm/s^2

$1\ \text{dyne} = 1\ \text{g}\cdot\text{cm/s}^2.$ $\quad 1\ \text{N} \equiv 1\ \text{kg}\cdot\text{m/s}^2$

EXAMPLE **Quick stop of a heavy car**

A big luxury car weighing 1.96×10^4 N (about 4400 lb), traveling in the $+x$ direction, makes a fast stop; the x component of the net force acting on it is -1.50×10^4 N. What is its acceleration?

SOLUTION

SET UP Figure shows our diagram. We draw the weight as a force with magnitude w. Because the car does not accelerate in the y direction, the road exerts an upward normal force of equal magnitude on the car; for completeness, we include that force in our diagram.

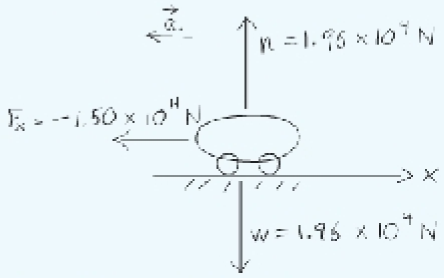


FIGURE Our diagram for this problem.

SOLVE To find the acceleration, we'll use Newton's second law, $\sum F_x = ma_x$. First, however, we need the car's mass. Since we know the car's weight, we can find its mass m from Equation , $w = mg$. We obtain

$$m = \frac{w}{g} = \frac{1.96 \times 10^4\ \text{N}}{9.80\ \text{m/s}^2} = \frac{1.96 \times 10^4\ \text{kg}\cdot\text{m/s}^2}{9.80\ \text{m/s}^2} = 2000\ \text{kg}.$$

Then, $\sum F_x = ma_x$ gives

$$a_x = \frac{F_x}{m} = \frac{-1.50 \times 10^4\ \text{N}}{2000\ \text{kg}} = \frac{-1.50 \times 10^4\ \text{kg}\cdot\text{m/s}^2}{2000\ \text{kg}}$$
$$= -7.5\ \text{m/s}^2.$$

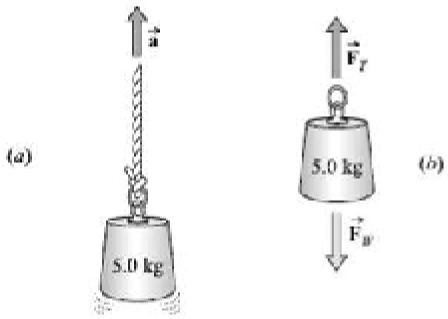
Example

(1) A 5.0 kg object is to be given an upward acceleration of 0.30 m/s^2 by a rope pulling straight upward on it. What must be the tension in the rope?

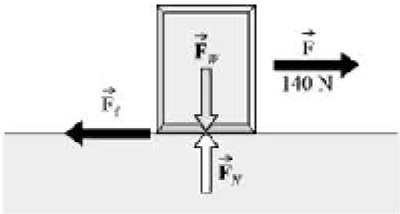
The free-body diagram for the object is shown in Fig. . The tension in the rope is F_T, and the weight of the object is $F_W = mg = (5.0\ \text{kg})(9.81\ \text{m/s}^2) = 49.1$ N. Using $\Sigma F_y = ma_y$ with *up* taken as positive, we have

$$F_T - mg = ma_y \qquad \text{or} \qquad F_T - 49.1\ \text{N} = (5.0\ \text{kg})(0.30\ \text{m/s}^2)$$

from which $F_T = 50.6\ \text{N} = 51\ \text{N}$. As a check, we notice that F_T is larger than F_W as it must be if the object is to accelerate upward.



(2) A horizontal force of 140 N is needed to pull a 60.0 kg box across the horizontal floor at constant speed. What is the coefficient of friction between floor and box? Determine it to three significant figures even though that's quite unrealistic.



The free-body diagram for the box is shown in Fig. . Because the box does not move up or down, $a_y = 0$. Therefore,

$$\Sigma F_y = ma_y \qquad \text{gives} \qquad F_N - mg = (m)(0\ \text{m/s}^2)$$

from which we find that $F_N = mg = (60.0\ \text{kg})(9.81\ \text{m/s}^2) = 588.6\ \text{N}$. Further, because the box is moving horizontally at constant speed, $a_x = 0$ and so

$$\Sigma F_x = ma_x \qquad \text{gives} \qquad 140\ \text{N} - F_f = 0$$

from which the friction force is $F_f = 140$ N. We then have

$$\mu_k = \frac{F_f}{F_N} = \frac{140\ \text{N}}{588.6\ \text{N}} = 0.238$$

(3) A 400-g block with an initial speed of 80 cm/s slides along a horizontal tabletop against a friction force of 0.70 N. (*a*) How far will it slide before stopping? (*b*) What is the coefficient of friction between the block and the tabletop?

(*a*) We take the direction of motion as positive. The only unbalanced force acting on the block is the friction force, −0.70 N. Therefore,

$$\Sigma F = ma \qquad \text{becomes} \qquad -0.70\ \text{N} = (0.400\ \text{kg})(a)$$

from which $a = -1.75\ \text{m/s}^2$. (Notice that m is always in kilograms.) To find the distance the block slides, we have $v_{ix} = 0.80$ m/s, $v_{fx} = 0$, and $a = -1.75\ \text{m/s}^2$. Then $v_{fx}^2 - v_{ix}^2 = 2ax$ gives

$$x = \frac{v_{fx}^2 - v_{ix}^2}{2a} = \frac{(0 - 0.64)\ \text{m}^2/\text{s}^2}{(2)(-1.75\ \text{m/s}^2)} = 0.18\ \text{m}$$

(*b*) Because the vertical forces on the block must cancel, the upward push of the table F_N must equal the weight mg of the block. Then

$$\mu_k = \frac{\text{friction force}}{F_N} = \frac{0.70\ \text{N}}{(0.40\ \text{kg})(9.81\ \text{m/s}^2)} = 0.18$$

(4) A 600-kg car is moving on a level road at 30 m/s. (*a*) How large a retarding force (assumed constant) is required to stop it in a distance of 70 m? (*b*) What is the minimum coefficient of friction between tires and roadway if this is to be possible? Assume the wheels are not locked, in which case we are dealing with static friction – there's no sliding.

(*a*) We must first find the car's acceleration from a motion equation. It is known that $v_{ix} = 30$ m/s, $v_{fx} = 0$, and $x = 70$ m. We use $v_{fx}^2 = v_{ix}^2 + 2ax$ to find

$$a = \frac{v_{fx}^2 - v_{ix}^2}{2x} = \frac{0 - 900\ \text{m}^2/\text{s}^2}{140\ \text{m}} = -6.43\ \text{m/s}^2$$

Now we can write

$$F = ma = (600\ \text{kg})(-6.43\ \text{m/s}^2) = -3860\ \text{N} = -3.9\ \text{kN}$$

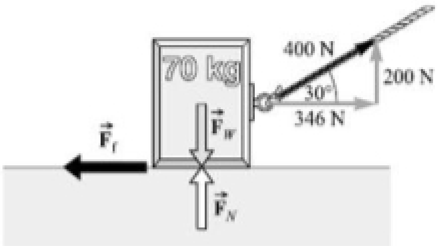
(*b*) The force found in (*a*) is supplied as the friction force between the tires and roadway. Therefore, the magnitude of the friction force on the tires is $F_f = 3860$ N. The coefficient of friction is given by $\mu_s = F_f/F_N$, where F_N is the normal force. In the present case, the roadway pushes up on the car with a force equal to the car's weight. Therefore,

$$F_N = F_W = mg = (600\ \text{kg})(9.81\ \text{m/s}^2) = 5886\ \text{N}$$

so that
$$\mu_s = \frac{F_f}{F_N} = \frac{3860}{5886} = 0.66$$

The coefficient of friction must be at least 0.66 if the car is to stop within 70 m.

(5) Suppose, as shown in Fig. , that a 70-kg box is pulled by a 400-N force at an angle of 30° to the horizontal. The coefficient of kinetic friction is 0.50. Find the acceleration of the box.



Because the box does not move up or down, we have $\Sigma F_y = ma_y = 0$. From Fig. , we see that this equation is

$$F_N + 200\ \text{N} - mg = 0$$

But $mg = (70\ \text{kg})(9.81\ \text{m/s}^2) = 687$ N, and it follows that $F_N = 486$ N.

We next find the friction force acting on the box:

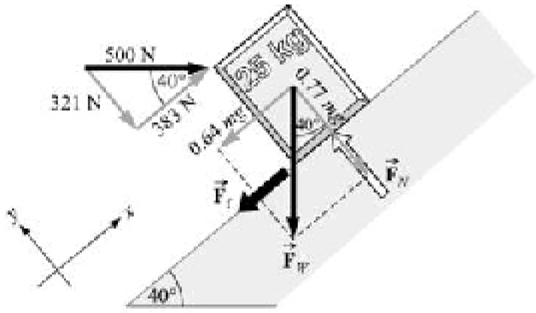
$$F_f = \mu_k F_N = (0.50)(486\ \text{N}) = 243\ \text{N}$$

Now let us write $\Sigma F_x = ma_x$ for the box. It is

$$(346 - 243)\ \text{N} = (70\ \text{kg})(a_x)$$

from which $a_x = 1.5\ \text{m/s}^2$.

(6) When a force of 500 N pushes on a 25-kg box as shown in Fig. , the acceleration of the box up the incline is $0.75\ \text{m/s}^2$. Find the coefficient of kinetic friction between box and incline.



The acting forces and their components are shown in Fig. . Notice how the x- and y-axes are taken. Since the box moves up the incline, the friction force (which always acts to retard the motion) is directed down the incline.

Let us first find F_f by writing $\Sigma F_x = ma_x$. From Fig. 3-12, using $\sin 40^\circ = 0.643$,

$$383\ \text{N} - F_f - (0.64)(25)(9.81)\ \text{N} = (25\ \text{kg})(0.75\ \text{m/s}^2)$$

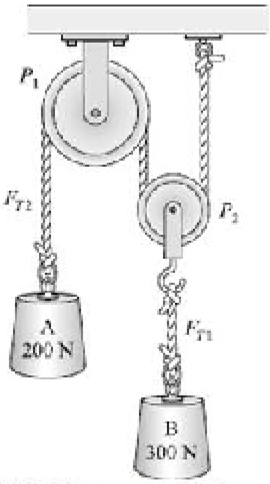
from which $F_f = 207$ N.

We also need F_N. Writing $\Sigma F_y = ma_y = 0$, and using $\cos 40^\circ = 0.766$, we get

$$F_N - 321\ \text{N} - (0.77)(25)(9.81)\ \text{N} = 0 \qquad \text{or} \qquad F_N = 510\ \text{N}$$

Then $$\mu_k = \frac{F_f}{F_N} = \frac{207}{510} = 0.41$$

(7) In Fig. the weights of the objects are 200 N and 300 N. The pulleys are essentially frictionless and massless. Pulley P_1 has a stationary axle, but pulley P_2 is free to move up and down. Find the tensions F_{T1} and F_{T2} and the acceleration of each body.



Mass B will rise and mass A will fall. You can see this by noticing that the forces acting on pulley P_2 are $2F_{T2}$ up and F_{T1} down. Since the pulley has no mass, it can have no acceleration, and so $F_{T1} = 2F_{T2}$ (the inertialess object transmits the tension). Twice as large a force is pulling upward on B as on A.

Let a be the downward acceleration of A. Then $a/2$ is the upward acceleration of B. (Why?) We now write $\Sigma F_y = ma_y$ for each mass in turn, taking the direction of motion as positive in each case. We have

$$F_{T1} - 300\ \text{N} = (m_B)(\tfrac{1}{2}a) \qquad \text{and} \qquad 200\ \text{N} - F_{T2} = m_A a$$

But $m = F_W/g$ and so $m_A = (200/9.81)$ kg and $m_B = (300/9.81)$ kg. Further $F_{T1} = 2F_{T2}$. Substitution of these values in the two equations allows us to compute F_{T2} and then F_{T1} and a. The results are

$$F_{T1} = 327\ \text{N} \qquad F_{T2} = 164\ \text{N} \qquad a = 1.78\ \text{m/s}^2$$

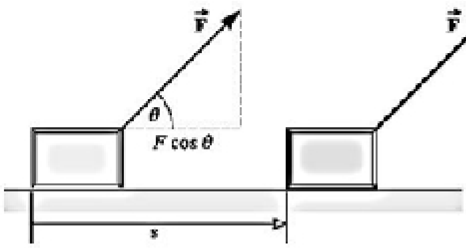
KINETIC ENERGY AND WORK

Work

THE WORK done by a force is defined as the product of that force times the parallel distance over which it acts. Consider the simple case of straight-line motion shown in Fig. where a force $\vec{F}$ acts on a body that simultaneously undergoes a vector displacement $\vec{s}$. The component of $\vec{F}$ in the direction of $\vec{s}$ is $F\cos\theta$. The work W done by the force $\vec{F}$ is defined to be the component of $\vec{F}$ in the direction of the displacement, multiplied by the displacement:

$$\textbf{Work} = (F\cos\theta)(s) = Fs\cos\theta$$

Notice that θ is the angle between the force and displacement vectors. Work is a scalar quantity.



If $\vec{F}$ and $\vec{s}$ are in the same direction, $\cos\theta = \cos 0° = 1$ and $W = Fs$. But, if $\vec{F}$ and $\vec{s}$ are in opposite directions, then $\cos\theta = \cos 180° = -1$ and $W = -Fs$; the work is negative. Forces such as friction often slow the motion of an object and are then opposite in direction to the displacement. Such forces usually do negative work. Inasmuch as the friction force opposes the motion of an object the work done in overcoming friction (along any path, curved or straight) equals the product of F_f and the path-length traveled. Thus, if an object is dragged against friction, back to the point where the journey started, work is done even if the net displacement is zero.

Work is the transfer of energy from one entity to another by way of the action of a force applied over a distance. The point of application of the force must move if work is to be done.

THE UNIT OF WORK in the SI is the *newton-meter*, called the *joule* (J). One joule is the work done by a force of 1 N when it displaces an object 1 m in the direction of the force. Other units sometimes used for work are the *erg*, where 1 erg $= 10^{-7}$ J, and the *foot-pound* (ft·lb), where 1 ft·lb = 1.355 J.

Kinetic Energy

The concept of **energy** appears throughout every area of physics, yet it's not easy to define just what energy is. This concept is the cornerstone (حجر الزاوية) of a fundamental law of nature called **conservation of energy**, which states that the total energy in any isolated system is constant.

System, A system usually consists of one or more objects that can interact, move, and undergo deformations.

Isolated system is when there a system has no interaction with its surroundings.

Conservation of energy is the total energy in any isolated system is constant Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is conserved). Energy cannot be created or destroyed. This statement is

believed to be an absolute conservation law; no exception has ever been observed. It is one of the core principles of physics.

Energy is measure of the change imparted (ممنوح او مشارك) to a system. It is given to an object when a force does work on the object. The amount of energy transferred to object equals the work done. Further, when an object does work, it loses an amount energy equals to the work it does. Energy and work have the same units, joules. Energy, like work, is scalar quantity. An object that is capable of doing work possesses (يمتلك) energy.

KINETIC ENERGY (KE) is the energy possessed by an object because it is in motion. If an object of mass m is moving with a speed v, it has translational KE given by

$$KE = \frac{1}{2}mv^2$$

When m is in kg and v is in m/s, the units of KE are joules.

$$\text{work}=Fx \;,\quad F=ma \Rightarrow \text{work}=m.a.x$$

$$V^2=V_o^2+2ax \Rightarrow ax=\frac{V^2-V_o^2}{2}$$

$$\text{work or Kinetic Energy}=m\,\frac{V^2-V_o^2}{2}$$

$$K.E=\frac{1}{2}mv^2$$

Work – Kinetic Energy Theorem

If the change in kinetic energy of the body (from an initial energy $K_i = \frac{mv^2}{2}$ to $K_f = \frac{mv^2}{2}$ a later) to the work $W = F_X d$. Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W,$$

which says that

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

We can also write

$$K_f = K_i + W,$$

which says that

$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

GRAVITATIONAL POTENTIAL ENERGY (PE_G) is the energy possessed by an object because of the gravitational interaction. In falling through a vertical distance h, a mass m can do work in the amount mgh. We define the PE_G of an object relative to an arbitrary zero level, often the Earth's surface. If the object is at a height h above the zero (or reference) level, its

$$PE_G = mgh$$

where g is the acceleration due to gravity. Notice that mg is the weight of the object. The units of PE_G are joules when m is in kg, g is in m/s^2, and h is in m.

POWER is the time rate of doing work:

$$\text{Average power} = \frac{\text{work done by a force}}{\text{time taken to do this work}} = \text{force} \times \text{speed}$$

where the speed is measured in the direction of the force applied to the object. More generally, power is the rate of transfer of energy. In the SI, the unit of power is the *watt* (W), and 1 W = 1 J/s.

Another unit of power often used is the *horsepower*: 1 hp = 746 W.

(1) Compute the work done against gravity by a pump that discharges 600 liters of fuel oil into a tank 20 m above the pump's intake. One cubic centimeter of fuel oil has a mass of 0.82 g. One liter is $1000\ cm^3$.

The mass lifted is

$$(600\ \text{liters})\left(1000\ \frac{cm^3}{\text{liter}}\right)\left(0.82\ \frac{g}{cm^3}\right) = 492\,000\ g = 492\ kg$$

The lifting work is then

$$\text{Work} = (mg)(h) = (492\ kg \times 9.81\ m/s^2)(20\ m) = 96\ kJ$$

2)

Work Done by a Constant Force

A particle moving in the xy plane undergoes a displacement $\mathbf{d} = (2.0\mathbf{i} + 3.0\mathbf{j})$ m as a constant force $\mathbf{F} = (5.0\mathbf{i} + 2.0\mathbf{j})$ N acts on the particle. (a) Calculate the magnitude of the displacement and that of the force.

Solution

$$d = \sqrt{x^2 + y^2} = \sqrt{(2.0)^2 + (3.0)^2} = \boxed{3.6\ m}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = \boxed{5.4\ N}$$

(b) Calculate the work done by **F**.

Solution Substituting the expressions for **F** and **d** into Equations 7.4 and 7.5, we obtain

$$W = \mathbf{F}\cdot\mathbf{d} = (5.0\mathbf{i} + 2.0\mathbf{j})\cdot(2.0\mathbf{i} + 3.0\mathbf{j})\ N\cdot m$$
$$= 5.0\mathbf{i}\cdot 2.0\mathbf{i} + 5.0\mathbf{i}\cdot 3.0\mathbf{j} + 2.0\mathbf{j}\cdot 2.0\mathbf{i} + 2.0\mathbf{j}\cdot 3.0\mathbf{j}$$
$$= 10 + 0 + 0 + 6 = 16\ N\cdot m = \boxed{16\ J}$$

3)

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\ m)\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\ N)\hat{i} + (-6.0\ N)\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

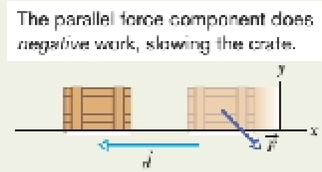


Fig. 7-5 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

KEY IDEA

Because we can treat the crate as a particle and because the wind force is constant ("steady") in both magnitude and direction during the displacement, we can use either Eq. 7-7 ($W = Fd\cos\phi$) or Eq. 7-8 ($W = \vec{F}\cdot\vec{d}$) to calculate the work. Since we know $\vec{F}$ and $\vec{d}$ in unit-vector notation, we choose Eq. 7-8.

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\ N)\hat{i} + (-6.0\ N)\hat{j}]\cdot[(-3.0\ m)\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i}\cdot\hat{i}$, $\hat{j}\cdot\hat{j}$, and $\hat{k}\cdot\hat{k}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\ N)(-3.0\ m)\hat{i}\cdot\hat{i} + (-6.0\ N)(-3.0\ m)\hat{j}\cdot\hat{i}$$
$$= (-6.0\ J)(1) + 0 = -6.0\ J. \qquad \text{(Answer)}$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

KEY IDEA

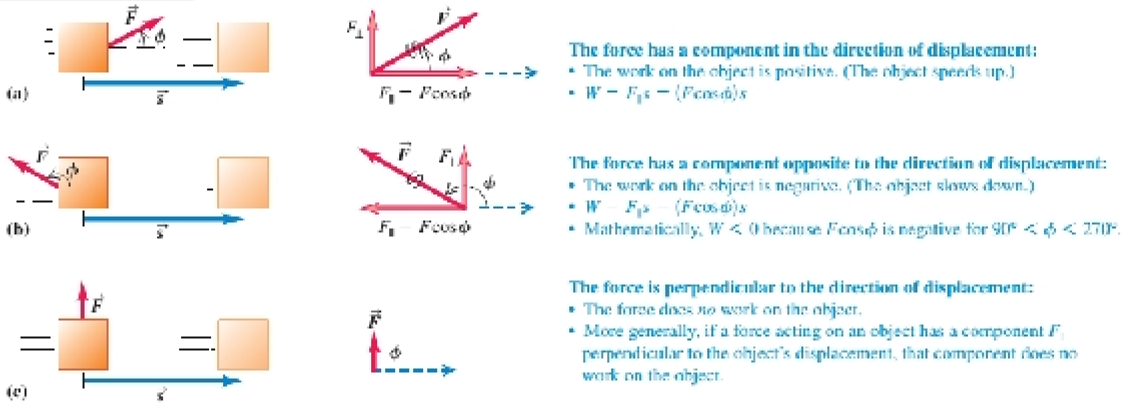
Because the force does negative work on the crate, it reduces the crate's kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10\ J + (-6.0\ J) = 4.0\ J. \qquad \text{(Answer)}$$

Less kinetic energy means that the crate has been slowed.

General cases



4)

Farmer Johnson hitches his tractor to a sled loaded with firewood and pulls it a distance of 20.0 m along level frozen ground (Figure a). The total weight of sled and load is 14,700 N. The tractor exerts a constant force $\vec{F}_T$ with magnitude 5000 N at an angle of $= 36.9°$ above the horizontal, as shown. A constant 3500 N friction force opposes the motion. Find the work done on the sled by each force individually and the total work done on the sled by all the forces.

SOLUTION

SET UP (Figure b) shows a free-body diagram and a coordinate system, identifying all the forces acting on the sled. As in the preceding example, we point the x axis in the direction of displacement.

SOLVE The work W_w done by the weight is zero because itsdirection is perpendicular to the displacement. (The angle between the two directions is 90°, and the cosine of the angle is zero.) For the same reason, the work done W_n by the normal force $\vec{n}$ (which, incidentally, is not equal in magnitude to the weight) is also zero. So $W_w = W_n = 0$.

That leaves F_T and f. From Equation 7.1, the work W_T done by the tractor is (with $\cos\phi = \cos 36.9° = 0.800$)

$$W_T = (F_T \cos\phi)s$$
$$= (5000\ \text{N})(0.800)(20.0\ \text{m}) = 80{,}000\ \text{N}\cdot\text{m} = 80.0\ \text{kJ}.$$

The friction force f is opposite to the displacement, so, for this force, $\phi = 180°$ and $\cos\phi = -1$. The work W_f done by the friction force is

$$W_f = fs\cos 180° = -(3500\ \text{N})(20.0\ \text{m})$$
$$= -70{,}000\ \text{N}\cdot\text{m} = -70.0\ \text{kJ}.$$

The total work W_T done by all of the forces on the sled is the algebraic sum (not the vector sum) of the work done by the individual forces:

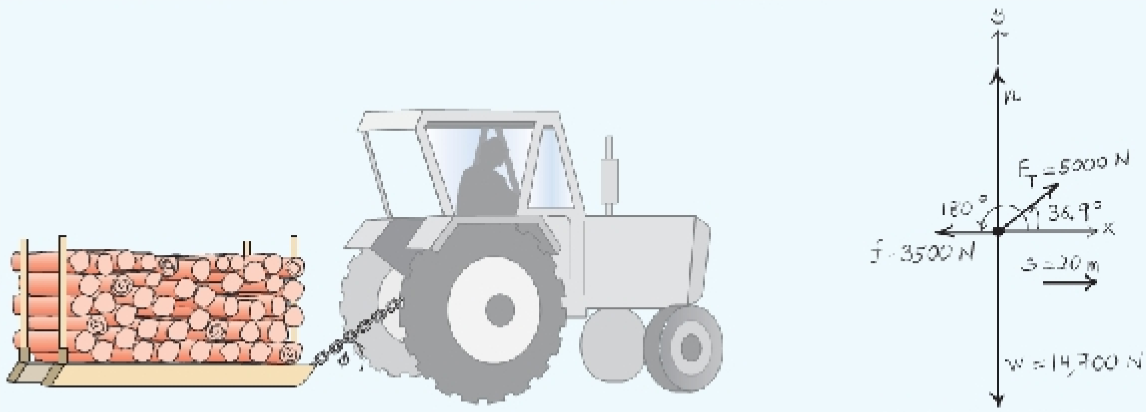
$$W_{\text{total}} = W_T + W_w + W_n + W_f$$
$$= 80.0\text{ kJ} + 0 + 0 + (-70.0\text{ kJ}) = 10.0\text{ kJ}.$$

Alternative Solution: In the alternative approach, we first find the vector sum (resultant) of the forces and then use it to compute the total work. The vector sum is best found by using components. From (Figure b),

$$\sum F_x = (5000\text{ N})\cos 36.9° - 3500\text{ N} = 500\text{ N},$$
$$\sum F_y = (5000\text{ N})\sin 36.9° + n + (-14{,}700\text{ N}).$$

We don't really need the second equation; we know that the y component of force is perpendicular to the displacement, so it does no work. Besides, there is no y component of acceleration, so $\sum F_y$ has to be zero anyway. The work done by the total x component is therefore the total work:

$$W_{\text{total}} = (500\text{ N})(20.0\text{ m}) = 10{,}000\text{ J} = 10.0\text{ kJ}.$$



(a) (b) Free body diagram of sled

Figure (a,b) of a tractor pulls a sled.

5)

Let's revisit the sled from Example '4 . The free-body diagram is shown again in Figure We found that the total work done on the sled by all the forces is 10,000 J = 10.0 kJ, so the kinetic energy of the sled must increase by 10.0 kJ. The mass of the sled is $m = (14{,}700\text{ N})/(9.80\text{ m/s}^2) = 1500\text{ kg}$. Suppose the sled's initial speed v_i is 2.00 m/s. What is its final speed?

SOLUTION

SET UP Steps 1 and 2 of the problem-solving strategy were done in Example 7.3, where we found that $W_{total} = 10.0$ kJ. The initial kinetic energy K_i is

$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1500\text{ kg})(2.00\text{ m/s})^2 = 3000\text{ kg}\cdot\text{m}^2/\text{s}^2 = 3000\text{ J}.$$

The final kinetic energy K_f is

$$K_f = \tfrac{1}{2}(1500\text{ kg})v_f^2,$$

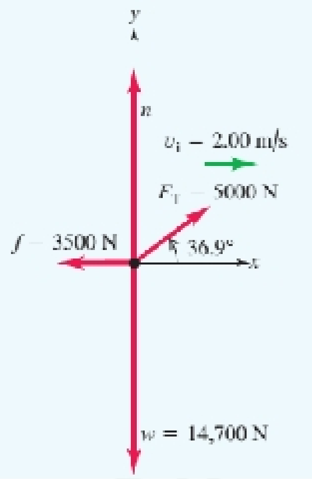
where v_f is the unknown final speed that we want to find.

SOLVE Equation 7.4 gives

$$K_f = K_i + W_{total},$$
$$\tfrac{1}{2}(1500\text{ kg})v_f^2 = 3000\text{ J} + 10{,}000\text{ J} = 13{,}000\text{ J}.$$

Solving for v_f, we find

$$v_f = 4.16\text{ m/s}.$$



Work Done by a Spring Force

The Spring Force

Figure a shows a spring in its relaxed state—that is, neither compressed nor extended. One end is fixed, and a particle-like object —a block, say —is attached to the other, free end. If we stretch the spring by pulling the block to the right as in Fig. b, the spring pulls on the block toward the left. (Because a spring force acts to restore the relaxed state, it is sometimes said to be a restoring force.) If we compress the spring by pushing the block to the left as in Fig. c, the spring now pushes on the block toward the right.

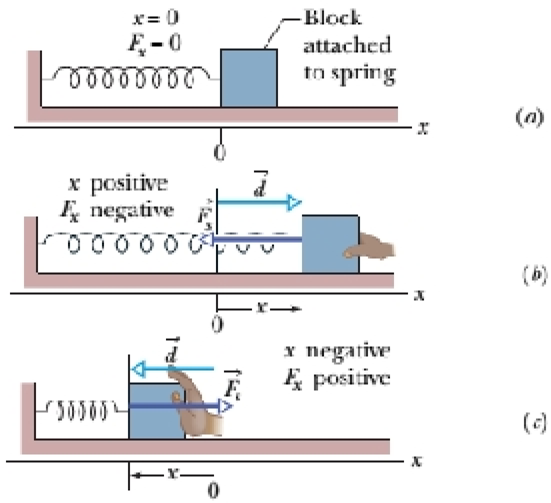


Fig. (*a*) A spring in its relaxed state. The origin of an *x* axis has been placed at the end of the spring that is attached to a block. (*b*) The block is displaced by $\vec{d}$, and the spring is stretched by a positive amount *x*. Note the restoring force $\vec{F}_s$ exerted by the spring. (*c*) The spring is compressed by a negative amount *x*. Again, note the restoring force.

To a good approximation for many springs, the force $\vec{F}_s$ from a spring is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in the relaxed state. The spring force is given by

$$\vec{F}_s = -k\vec{d} \qquad \text{(Hooke's law)}$$

Where k is called the spring constant (or force constant) and is a measure of the stiffness of the spring.

In Fig. an x axis has been placed parallel to the length of the spring, with the origin (x=0) at the position of the free end when the spring is in its relaxed state. For this common arrangement, we can write

$$F_x = -kx \qquad \text{(Hooke's law)}$$

The net work W_s done by the spring, from x_i to x_f, is the sum of all these works:

$$W_s = \sum -F_{xj}\,\Delta x$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force)}$$

If x_i=0 the work down became

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).}$$

6)

Work done by spring to change kinetic energy

In Fig. a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. ($W_s = -\frac{1}{2}kx^2$), with d replacing x.
2. The work W_s is also related to the kinetic energy of the canister by Eq. ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

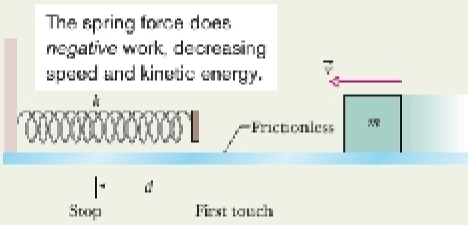


Fig. A canister of mass m moves at velocity $\vec{v}$ toward a spring that has spring constant k.

Substituting according to the third key idea gives us this expression

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\ \text{m/s})\sqrt{\frac{0.40\ \text{kg}}{750\ \text{N/m}}}$$
$$= 1.2 \times 10^{-2}\ \text{m} = 1.2\ \text{cm}. \qquad \text{(Answer)}$$

7)

Figure shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

KEY IDEA

We want an instantaneous power, not an average power over a time period. Also, we know the box's velocity (rather than the work done on it).

Calculation: We use Eq. for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v\cos\phi_1 = (2.0\ \text{N})(3.0\ \text{m/s})\cos 180°$$
$$= -6.0\ \text{W}. \qquad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v\cos\phi_2 = (4.0\ \text{N})(3.0\ \text{m/s})\cos 60°$$
$$= 6.0\ \text{W}. \qquad \text{(Answer)}$$

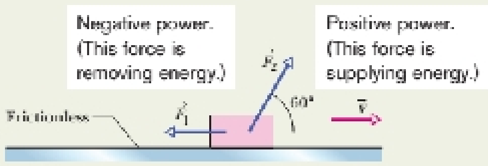


Fig. Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a box that slides rightward across a frictionless floor. The velocity of the box is $\vec{v}$.

This positive result tells us that force $\vec{F}_2$ is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers:

$$P_{net} = P_1 + P_2$$
$$= -6.0\ \text{W} + 6.0\ \text{W} = 0, \qquad \text{(Answer)}$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s. With neither the forces $\vec{F}_1$ and $\vec{F}_2$ nor the velocity $\vec{v}$ changing, we see from Eq. that P_1 and P_2 are constant and thus so is P_{net}.

Impulse and Momentum

THE LINEAR MOMENTUM ($\vec{p}$) of a body is the product of its mass (m) and velocity (v):

$$\text{Linear momentum} = (\text{mass of body})\,(\text{velocity of body})$$
$$\vec{p} = m\vec{v}$$

Momentum is a vector quantity whose direction is that of the velocity. The units of momentum are kg·m/s in the SI.

AN IMPULSE is the product of a force ($\vec{F}$) and the time interval (Δt) over which the force acts:

$$\text{Impulse} = (\text{force})\,(\text{length of time the force acts})$$

Impulse is a vector quantity whose direction is that of the force. Its units are N·s in the SI.

AN IMPULSE CAUSES A CHANGE IN MOMENTUM: The change of momentum produced by an impulse is equal to the impulse in both magnitude and direction. Thus, if a constant force $\vec{F}$ acting for a time Δt on a body of mass m changes its velocity from an initial value $\vec{v}_i$ to a final value $\vec{v}_f$, then

$$\text{Impulse} = \text{change in momentum}$$
$$\vec{F}\,\Delta t = m(\vec{v}_f - \vec{v}_i)$$

Newton's Second Law, as he gave it, is $\vec{F} = \Delta\vec{p}/\Delta t$ from which it follows that $\vec{F}\,\Delta t = \Delta\vec{p}$. Moreover, $\vec{F}\,\Delta t = \Delta(m\vec{v})$ and if m is constant $\vec{F}\,\Delta t = m(\vec{v}_f - \vec{v}_i)$.

IN COLLISIONS AND EXPLOSIONS, the vector sum of the momenta just before the event equals the vector sum of the momenta just after the event. The vector sum of the momenta of the objects involved does not change during the collision or explosion.

Thus, when two bodies of masses m_1 and m_2 collide,

$$\text{Total momentum before impact} = \text{total momentum after impact}$$
$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $\vec{u}_1$ and $\vec{u}_2$ are the velocities before impact, and $\vec{v}_1$ and $\vec{v}_2$ are the velocities after. In one dimension, in component form,

$$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$$

and similarly for the y- and z-components.

A PERFECTLY ELASTIC COLLISION is one in which the sum of the translational KEs of the objects is not changed during the collision. In the case of two bodies,

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

(1) A 2.0-kg brick is moving at a speed of 6.0 m/s. How large a force F is needed to stop the brick in a time of 7.0×10^{-4} s?

Let us solve this by use of the impulse equation:

$$\text{Impulse on brick} = \text{change in momentum of brick}$$
$$F\,\Delta t = mv_f - mv_i$$
$$F(7.0 \times 10^{-4}\ \text{s}) = 0 - (2.0\ \text{kg})(6.0\ \text{m/s})$$

from which $F = -1.7 \times 10^4$ N. The minus sign indicates that the force opposes the motion.

(2) A 7500-kg truck traveling at 5.0 m/s east collides with a 1500-kg car moving at 20 m/s in a direction 30° south of west. After collision, the two vehicles remain tangled together. With what speed and in what direction does the wreckage begin to move?

The original momenta are shown in Fig. (*a*), while the final momentum $M\vec{v}$ is shown in Fig. (*b*). Momentum must be conserved in both the north and east directions. Therefore,

$$(\text{momentum before})_E = (\text{momentum after})_E$$
$$(7500\ \text{kg})(5.0\ \text{m/s}) - (1500\ \text{kg})[(20\ \text{m/s})\cos 30°] = Mv_E$$

where $M = 7500\ \text{kg} + 1500\ \text{kg} = 9000$ kg, and v_E is the scalar eastward component of the velocity of the wreckage.

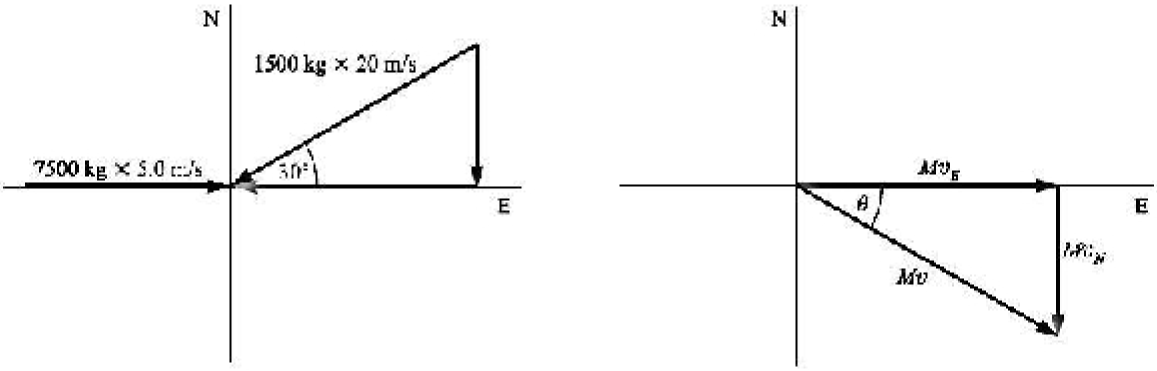
$$(\text{momentum before})_N = (\text{momentum after})_N$$
$$(7500\ \text{kg})(0) - (1500\ \text{kg})[(20\ \text{m/s})\sin 30°] = Mv_N$$

The first equation gives $v_E = 1.28$ m/s, and the second gives $v_N = -1.67$ m/s. The resultant is

$$v = \sqrt{(1.67\ \text{m/s})^2 + (1.28\ \text{m/s})^2} = 2.1\ \text{m/s}$$

The angle θ in Fig. 8-3(*b*) is

$$\theta = \arctan\left(\frac{1.67}{1.28}\right) = 53°$$



(*a*) (*b*)

Angular Momentum

A particle passing through point A has linear momentum $\vec{\ell} = m\vec{v}$,with the vector $\vec{P}$ lying in an xy plane. The particle has angular momentum

$\vec{\ell} = \vec{r} \times \vec{P}$ with respect to the origin O. By the right-hand rule, the angular momentum vector points in the positive direction of z. .(a) The magnitude of

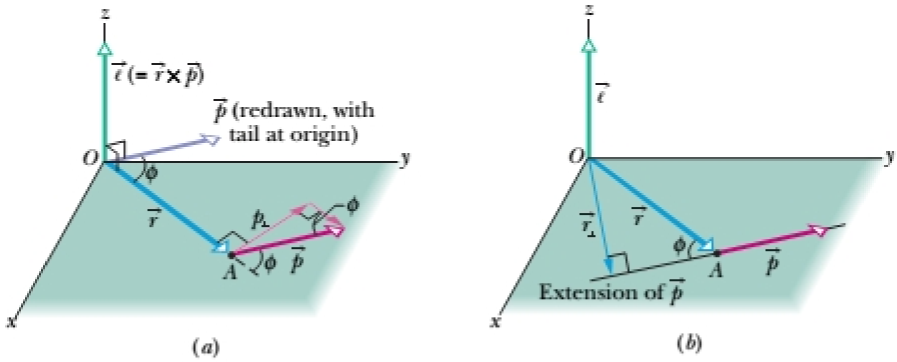


Figure . (a) The magnitude of $\vec{\ell}$ is given by $\vec{\ell} = rP_\perp = rmv_\perp$. (b) The magnitude of $\vec{\ell}$ is also given by $\vec{\ell} = r_\perp P = r_\perp mv$.

Where $\vec{r}$ is the position vector of the particle with respect to O. As the particle moves relative to O in the direction of its momentum $\vec{P} = m\vec{v}$, position vector $\vec{r}$ rotates around O. The SI unit of angular momentum is the kilogram meter-squared per second (kg.m^2/s), equivalent to the joule-second (J.s)

$$\vec{\ell} = \vec{r} \times \vec{P} = m(\vec{r} \times \vec{v}) \quad \text{angular momentum}$$

To find the magnitude of $\vec{\ell}$,we use the general result of Eq. to write

$$\ell = rmv \sin\emptyset$$

Where $\emptyset$ is the smaller angle between $\vec{r}$ and $\vec{P}$ when these two vectors are tail to tail.

Definition of angular momentum when a rigid body with moment of inertia I (with respect to a specified symmetry axis) rotates with angular velocity ω about that axis, the angular momentum of the body with respect to the axis is the product of the moment of inertia I about the axis and the angular velocity ω. We denote angular momentum by L:

$$L = Iw$$

The sign of angular momentum depends on the sign of ω thus, according to our usual convention (تحويلات), it is positive for counterclockwise (عكس عقرب الساعة) rotation and negative for clockwise (اتجاه عقرب الساعة) rotation.

Note

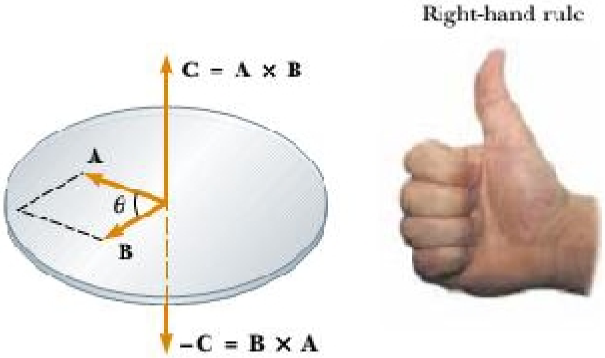


Figure 11.2 The vector product **A × B** is a third vector **C** having a magnitude $AB \sin\theta$ equal to the area of the parallelogram shown. The direction of **C** is perpendicular to the plane formed by **A** and **B**, and this direction is determined by the right-hand rule.

Estimate the magnitude of the angular momentum of a bowling ball spinning at 10 rev/s, as shown in Figure

Solution We start by making some estimates of the relevant physical parameters and model the ball as a uniform solid sphere. A typical bowling ball might have a mass of 6.0 kg and a radius of 12 cm. The moment of inertia of a solid sphere about an axis through its center is, from Table ,

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(6.0\ \text{kg})(0.12\ \text{m})^2 = 0.035\ \text{kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L_z = I\omega = (0.035\ \text{kg}\cdot\text{m}^2)(10\ \text{rev/s})(2\pi\ \text{rad/rev}) = 2.2\ \text{kg}\cdot\text{m}^2/\text{s}$$

Because of the roughness of our estimates, we probably want to keep only one significant figure, and so $L_z \approx 2\ \text{kg}\cdot\text{m}^2/\text{s}$.

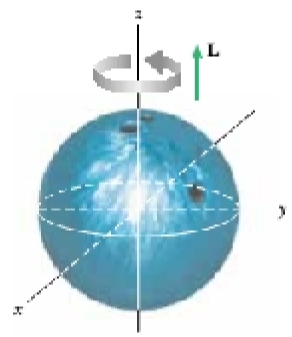


Figure (Example 11.5) A bowling ball that rotates about the z axis in the direction shown has an angular momentum **L** in the positive z direction. If the direction of rotation is reversed, **L** points in the negative z direction.

THE CENTER OF MASS of an object (of mass m) is the single point that moves in the same way as a point mass (of mass m) would move when subjected to the same external forces that act on the object. That is, if the resultant force acting on an object (or system of objects) of mass m is $\vec{F}$, the acceleration of the center of mass of the object (or system) is given by $\vec{a}_{cm} = \vec{F}/m$.

If the object is considered to be composed of tiny masses m_1, m_2, m_3, and so on, at coordinates (x_1, y_1, z_1), (x_2, y_2, z_2), and so on, then the coordinates of the center of mass are given by

$$x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m_i} \qquad y_{cm} = \frac{\Sigma y_i m_i}{\Sigma m_i} \qquad z_{cm} = \frac{\Sigma z_i m_i}{\Sigma m_i}$$

where the sums extend over all masses composing the object. In a uniform gravitational field, the center of mass and the center of gravity coincide.

(3) Three masses are placed on the x-axis: 200 g at $x = 0$, 500 g at $x = 30$ cm, and 400 g at $x = 70$ cm. Find their center of mass.

$$x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m_i} = \frac{(0)(0.20\text{ kg}) + (0.30\text{ m})(0.50\text{ kg}) + (0.70\text{ m})(0.40\text{ kg})}{(0.20 + 0.50 + 0.40)\text{ kg}} = 0.39\text{ m}$$

The y- and z-coordinates of the mass center are zero.

(4) A system consists of the following masses in the xy-plane: 4.0 kg at coordinates ($x = 0$, $y = 5.0$ m), 7.0 kg at (3.0 m, 8.0 m), and 5.0 kg at (−3.0 m, −6.0 m). Find the position of its center of mass.

$$x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m_i} = \frac{(0)(4.0\text{ kg}) + (3.0\text{ m})(7.0\text{ kg}) + (-3.0\text{ m})(5.0\text{ kg})}{(4.0 + 7.0 + 5.0)\text{ kg}} = 0.38\text{ m}$$

$$y_{cm} = \frac{\Sigma y_i m_i}{\Sigma m_i} = \frac{(5.0\text{ m})(4.0\text{ kg}) + (8.0\text{ m})(7.0\text{ kg}) + (-6.0\text{ m})(5.0\text{ kg})}{16\text{ kg}} = 2.9\text{ m}$$

and $z_{cm} = 0$.

Angular Motion in a Plane

ANGULAR DISPLACEMENT (θ) is usually expressed in radians, in degrees, or in revolutions:

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad} \qquad \text{or} \qquad 1 \text{ rad} = 57.3^\circ$$

One radian is the angle subtended at the center of a circle by an arc equal in length to the radius of the circle. Thus an angle θ in radians is given in terms of the arc length l it subtends on a circle of radius r by

$$\theta = \frac{l}{r}$$

The radian measure of an angle is a dimensionless number. Radians, like degrees, are not a physical unit – the radian is not expressable in meters, kilograms, or seconds. Nonetheless, we will use the abbreviation rad to remind us that we are working with radians.

THE ANGULAR SPEED (ω) of an object whose axis of rotation is fixed is the rate at which its angular coordinate, the angular displacement θ, changes with time. If θ changes from θ_i to θ_f in a time t, then the *average angular speed* is

$$\omega_{av} = \frac{\theta_f - \theta_i}{t}$$

The units of ω_{av} are exclusively rad/s. Since each complete turn or cycle of a revolving system carries it through 2π rad

$$\omega = 2\pi f$$

where f is the *frequency* in revolutions per second, rotations per second, or cycles per second. Accordingly, ω is also called the *angular frequency*. We can associate a direction with ω and thereby create a vector quantity $\vec{\omega}$. Thus if the fingers of the right hand curve around in the direction of rotation, the thumb points along the axis of rotation in the direction of $\vec{\omega}$, the *angular velocity* vector.

THE ANGULAR ACCELERATION (α) of an object whose axis of rotation is fixed is the rate at which its angular speed changes with time. If the angular speed changes uniformly from ω_i to ω_f in a time t, then the *angular acceleration* is constant and

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

The units of α are typically rad/s^2, rev/min^2, and such. It is possible to associate a direction with $\Delta\omega$, and therefore with α, thereby specifying the angular acceleration vector $\vec{\alpha}$, but we will have no need to do so here.

EQUATIONS FOR UNIFORMLY ACCELERATED ANGULAR MOTION are exactly analogous to those for uniformly accelerated linear motion. In the usual notation we have:

Linear	Angular
$v_{av} = \frac{1}{2}(v_i + v_f)$	$\omega_{av} = \frac{1}{2}(\omega_i + \omega_f)$
$s = v_{av}t$	$\theta = \omega_{av}t$
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$v_f^2 = v_i^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$
$s = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

RELATIONS BETWEEN ANGULAR AND TANGENTIAL QUANTITIES: When a wheel of radius r rotates about an axis whose direction is fixed, a point on the rim of the wheel is described in terms of the circumferential distance l it has moved, its tangential speed v, and its tangential acceleration a_T. These quantities are related to the angular quantities θ, ω, and α, which describe the rotation of the wheel, through the relations

$$l = r\theta \qquad v = r\omega \qquad a_T = r\alpha$$

provided radian measure is used for θ, ω, and α. By simple reasoning, l can be shown to be the length of belt wound on the wheel or the distance the wheel would roll (without slipping) if free to do so. In such cases, v and a_T refer to the tangential speed and acceleration of a point on the belt or of the center of the wheel.

CENTRIPETAL ACCELERATION (a_C): A point mass m moving with constant speed v around a circle of radius r is undergoing acceleration. Although the magnitude of its linear velocity is not changing, the direction of the velocity is continually changing. This change in velocity gives rise to an acceleration a_C of the mass, directed toward the center of the circle. We call this acceleration the *centripetal acceleration*; its magnitude is given by

$$a_C = \frac{(\text{tangential speed})^2}{\text{radius of circular path}} = \frac{v^2}{r}$$

where v is the speed of the mass around the perimeter of the circle.

Because $v = r\omega$, we also have $a_C = r\omega^2$, where ω must be in rad/s.

THE CENTRIPETAL FORCE ($\vec{F}_C$) is the force that must act on a mass m moving in a circular path of radius r to give it the centripetal acceleration v^2/r. From $F = ma$, we have

$$F_C = \frac{mv^2}{r} = mr\omega^2$$ where $\vec{F}_C$ is directed toward the center of the circular path.

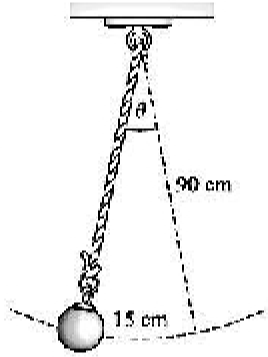
(5) Express each of the following in terms of the other angular measures: (*a*) 28°, (*b*) $\frac{1}{4}$ rev/s, (*c*) 2.18 rad/s^2.

(*a*)
$$28° = (28\text{ deg})\left(\frac{1\text{ rev}}{360\text{ deg}}\right) = 0.078\text{ rev}$$
$$= (28\text{ deg})\left(\frac{2\pi\text{ rad}}{360\text{ deg}}\right) = 0.49\text{ rad}$$

(*b*)
$$\frac{1}{4}\frac{\text{rev}}{\text{s}} = \left(0.25\frac{\text{rev}}{\text{s}}\right)\left(\frac{360\text{ deg}}{1\text{ rev}}\right) = 90\frac{\text{deg}}{\text{s}}$$
$$= \left(0.25\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{\pi}{2}\frac{\text{rad}}{\text{s}}$$

(*c*)
$$2.18\frac{\text{rad}}{\text{s}^2} = \left(2.18\frac{\text{rad}}{\text{s}^2}\right)\left(\frac{360\text{ deg}}{2\pi\text{ rad}}\right) = 125\frac{\text{deg}}{\text{s}^2}$$
$$= \left(2.18\frac{\text{rad}}{\text{s}^2}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 0.347\frac{\text{rev}}{\text{s}^2}$$

The bob of a pendulum 90 cm long swings through a 15-cm arc, as shown in Fig.. Find the angle θ, in radians and in degrees, through which it swings.



Recall that $l = r\theta$ applies only to angles in radian measure. Therefore, in radians

$$\theta = \frac{l}{r} = \frac{0.15\text{ m}}{0.90\text{ m}} = 0.167\text{ rad} = 0.17\text{ rad}$$

Then in degrees
$$\theta = (0.167\text{ rad})\left(\frac{360\text{ deg}}{2\pi\text{ rad}}\right) = 9.6°$$

(6) A spaceship orbits the Moon at a height of 20 000 m. Assuming it to be subject only to the gravitational pull of the Moon, find its speed and the time it takes for one orbit. For the Moon, $m_m = 7.34 \times 10^{22}$ kg and $r = 1.738 \times 10^6$ m.

The gravitational force of the Moon on the ship supplies the required centripetal force:

$$G\frac{m_s m_m}{R^2} = \frac{m_s v^2}{R}$$

where R is the radius of the orbit. Solving, we find that

$$v = \sqrt{\frac{Gm_m}{R}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.34 \times 10^{22}\ \text{kg})}{(1.738 + 0.0200) \times 10^6\ \text{m}}} = 1.67\ \text{km/s}$$

from which we find that

$$\text{Time for one orbit} = \frac{2\pi R}{v} = 6.62 \times 10^3\ \text{s} = 110\ \text{min}$$

(7) As shown in Fig. 9-3, a ball B is fastened to one end of a 24-cm string, and the other end is held fixed at point O. The ball whirls in the horizontal circle shown. Find the speed of the ball in its circular path if the string makes an angle of 30° to the vertical.

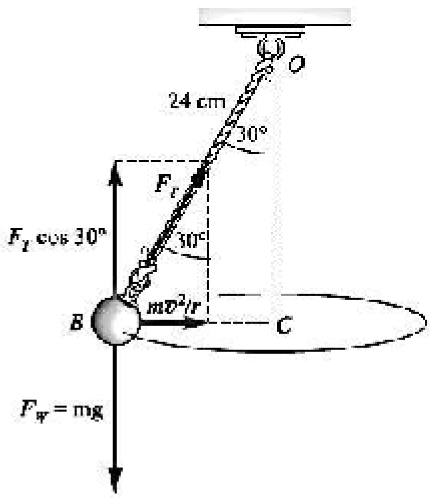
The only forces acting on the ball are the ball's weight mg and the tension F_T in the cord. The tension must do two things: (1) balance the weight of the ball by means of its vertical component, $F_T \cos 30°$; (2) supply the required centripetal force by means of its horizontal component, $F_T \sin 30°$. Therefore we can write

$$F_T \cos 30° = mg \qquad \text{and} \qquad F_T \sin 30° = \frac{mv^2}{r}$$

Solving for F_T in the first equation and substituting it in the second gives

$$\frac{mg \sin 30°}{\cos 30°} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{rg(0.577)}$$

However, $r = \overline{BC} = (0.24\ \text{m}) \sin 30° = 0.12$ m and $g = 9.81\ \text{m/s}^2$, from which $v = 0.82$ m/s.



Equilibrium Under the Action of Concurrent Forces

CONCURRENT FORCES are forces whose lines of action all pass through a common point. The forces acting on a point object are concurrent because they all pass through the same point, the point object.

AN OBJECT IS IN EQUILIBRIUM under the action of concurrent forces provided it is not accelerating.

THE FIRST CONDITION FOR EQUILIBRIUM is the requirement that $\Sigma \vec{\mathbf{F}} = 0$ or, in component form, that

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$$

That is, the resultant of all external forces acting on the object must be zero.

(1) In Fig. (*a*), the tension in the horizontal cord is 30 N as shown. Find the weight of the object.

The tension in cord 1 is equal to the weight of the object hanging from it. Therefore $F_{T1} = F_W$, and we wish to find F_{T1} or F_W.

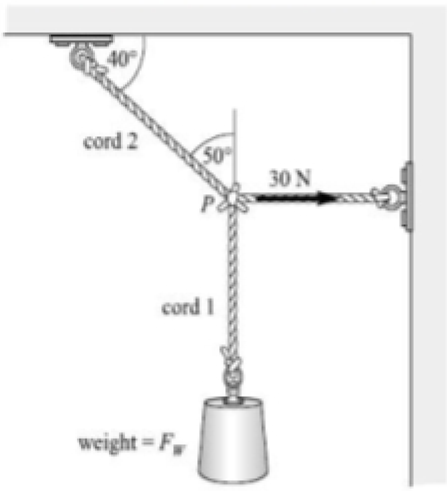
Notice that the unknown force F_{T1} and the known force of 30 N both pull on the knot at point *P*. It therefore makes sense to isolate the knot at *P* as our object. The free-body diagram showing the forces on the knot is drawn as in Fig. (*b*). The force components are also shown there.

We next write the first condition for equilibrium for the knot. From the free-body diagram,

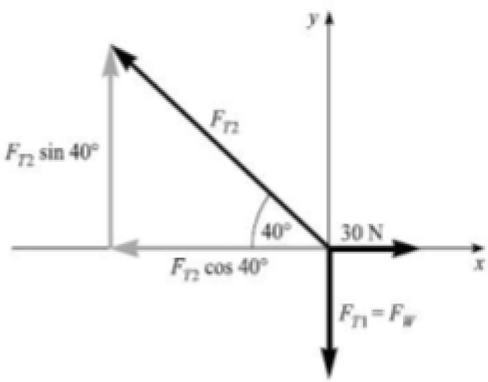
$$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad \text{becomes} \quad 30\text{ N} - F_{T2}\cos 40^\circ = 0$$

$$+\!\uparrow \Sigma F_y = 0 \quad \text{becomes} \quad F_{T2}\sin 40^\circ - F_W = 0$$

Solving the first equation for F_{T2} gives $F_{T2} = 39.2$ N. Substituting this value in the second equation gives $F_W = 25$ N as the weight of the object.



(*a*)



(*b*)

(2) A rope extends between two poles. A 90-N boy hangs from it as shown in Fig. (*a*). Find the tensions in the two parts of the rope.

We label the two tensions F_{T1} and F_{T2}, and isolate the rope at the boy's hands as the object. The free-body diagram for the object is shown in Fig. (*b*).

After resolving the forces into their components as shown, we can write the first condition for equilibrium:

$$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad \text{becomes} \quad F_{T2}\cos 5.0^\circ - F_{T1}\cos 10^\circ = 0$$

$$+\uparrow \Sigma F_y = 0 \quad \text{becomes} \quad F_{T2}\sin 5.0^\circ + F_{T1}\sin 10^\circ - 90\,\text{N} = 0$$

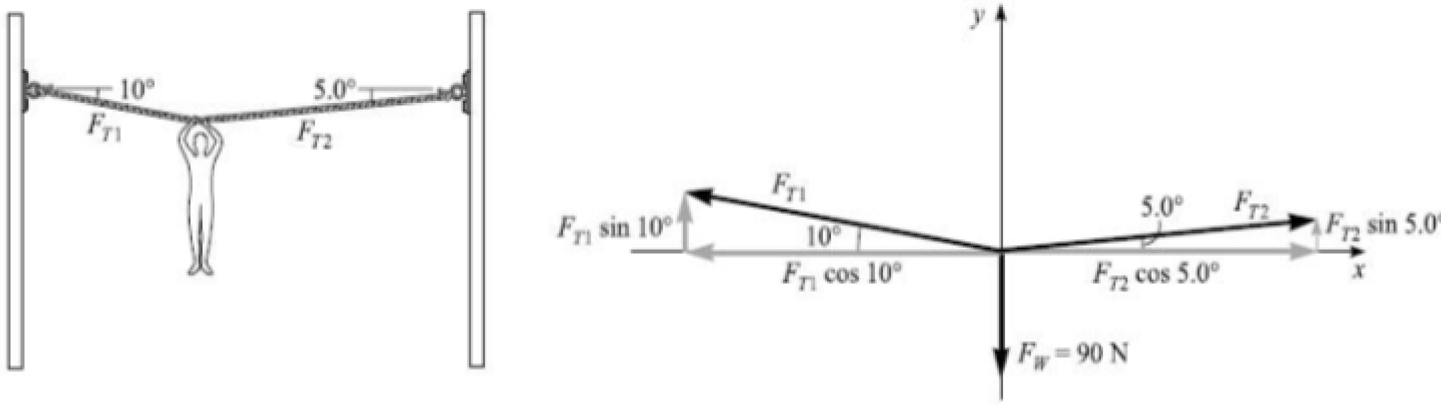
When we evaluate the sines and cosines, these equations become

$$0.996F_{T2} - 0.985F_{T1} = 0 \qquad \text{and} \qquad 0.087F_{T2} + 0.174F_{T1} - 90 = 0$$

Solving the first for F_{T2} gives $F_{T2} = 0.990F_{T1}$. Substituting this in the second equation gives

$$0.086F_{T1} + 0.174F_{T1} - 90 = 0$$

from which $F_{T1} = 0.35$ kN. Then, because $F_{T2} = 0.990F_{T1}$, we have $F_{T2} = 0.34$ kN.



(*a*) (*b*)

Kinetic Energy of Rotation

The body as made up of a lot of particles, with masses m_a, m_b, m_c,… at distances r_a, r_b, r_c,…, from the axis of rotation. The particles don't necessarily all lie in the same plane, so we specify that each r is the perpendicular distance from the particle to the axis.

When a rigid body rotates about a fixed axis, the speed υ of a typical particle is given by $\upsilon = r\omega$, where ω is the body's angular velocity in radians per second. Particle A has a **perpendicular distance $\mathbf{r_A}$** from the axis of rotation and has a speed given by $\upsilon_A = r_A\,\omega$ Different particles have different values of r and υ but ω is the same for all. (Otherwise the body wouldn't be rigid.) The kinetic energy K_A of particle A can be expressed as

$$K_A = \tfrac{1}{2}m_A\upsilon_A^2 = \tfrac{1}{2}m_A\left(r_A^2\omega^2\right).$$

The total kinetic energy *K* of the body is the sum of the kinetic energies of all its particles:

$$\begin{aligned}K &= \tfrac{1}{2}m_A\upsilon_A^2 + \tfrac{1}{2}m_B\upsilon_B^2 + \tfrac{1}{2}m_C\upsilon_C^2 + \cdots \\ &= \tfrac{1}{2}m_A\left(r_A^2\omega^2\right) + \tfrac{1}{2}m_B\left(r_B^2\omega^2\right) + \tfrac{1}{2}m_C\left(r_C^2\omega^2\right) + \cdots \\ &= \tfrac{1}{2}\left(m_Ar_A^2 + m_Br_B^2 + m_Cr_C^2 + \cdots\right)\omega^2.\end{aligned}$$

The quantity in parentheses (بين القوسين) in the last line, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation and adding all the products together, is called **the moment of inertia** of the body, denoted by *I*:

Definition of moment of inertia

A body's moment of inertia, *I*, describes how its mass is distributed in relation to an axis of rotation:

$$I = m_Ar_A^2 + m_Br_B^2 + m_Cr_C^2 + \cdots.$$

Unit: $kg \cdot m^2$

$$I = \sum m_i r_i^2 \qquad \text{(rotational inertia)}$$

$$K = \frac{1}{2}I\omega^2 \qquad \text{(radian measure)}$$

This is analogous to the expression $K = \frac{1}{2}mv^2$ for the kinetic energy of a particle; thus moment of inertia is analogous to mass, and angular velocity ω is analogous to speed υ. This kinetic energy is not a new form of energy; it's the same physical quantity that we expressed as $\frac{1}{2}mv^2$ for a single particle. But this Equation is much easier to use when we have to find the kinetic energy of a rotating body.

Combined Translation and Rotation of Energy Relationships

It's beyond the scope of this book to prove that the motion of a rigid body can always be divided into translation of the center of mass and rotation about the center of mass. But we can show that this is true for the *kinetic energy* of a rigid body that has both translational and rotational motions. In this case, the body's kinetic energy is the sum of a part $\frac{1}{2}Mv_{cm}^2$ associated with motion of the center of mass and a part $\frac{1}{2}I_{cm}\omega^2$ associated with rotation about an axis through the center of mass:

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

(rigid body with both translation and rotation)

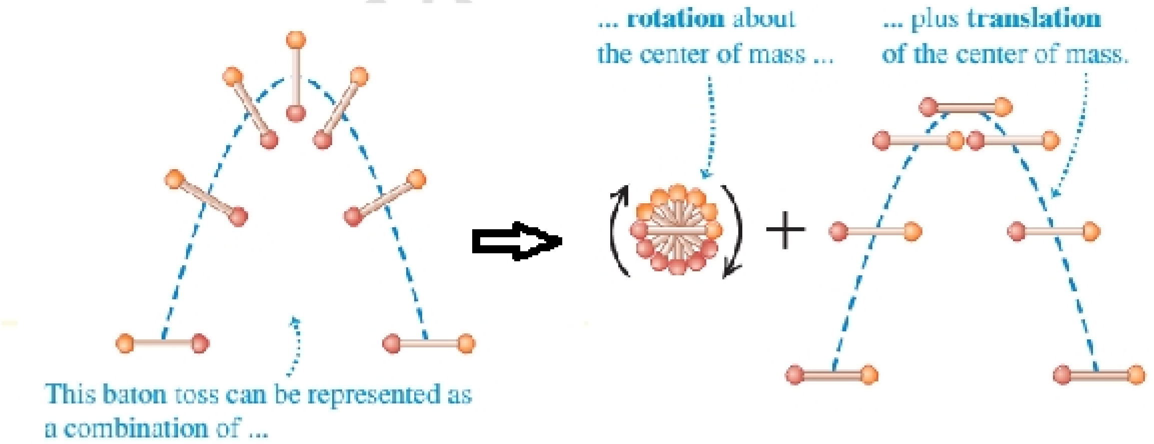
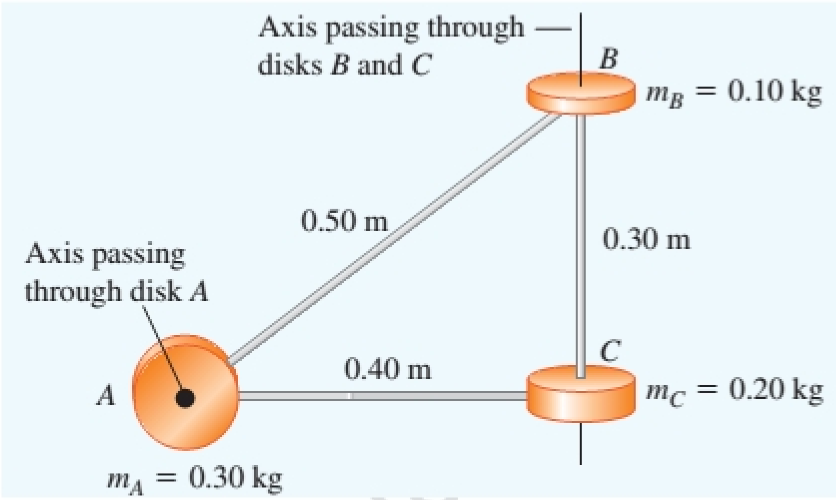


Figure The motion of a rigid body is a combination of translational motion of the center of mass and rotation around the center of mass.

Example

This design is the part consists of three massive (هائلة) disks connected by light supporting rods in figure that can be considered massless. Find the moment of inertia about an axis passing through disks B and C and the moment of inertia about an axis that passes through disk A and is perpendicular to the plane of that disk. If the object rotates about the axis through disk A with angular velocity 4rad/sec, find its kinetic energy.



SOLVE The moment of inertia is defined as $I = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$, where each r is measured from the chosen axis of rotation. For axis BC, disks B and C both lie *on* the axis $(r_B = r_C = 0)$, so neither contributes to the moment of inertia. Only disk A contributes; $m_A = 0.30$ kg, $r_A = 0.40$ m, and we find that

$$I_{BC} = m_A r_A^2 = (0.30\ \text{kg})(0.40\ \text{m})^2 = 0.048\ \text{kg}\cdot\text{m}^2.$$

For the axis through point A perpendicular to the plane of the diagram, disk A lies on the axis, so $r_A = 0$. The masses are the same as before, but now

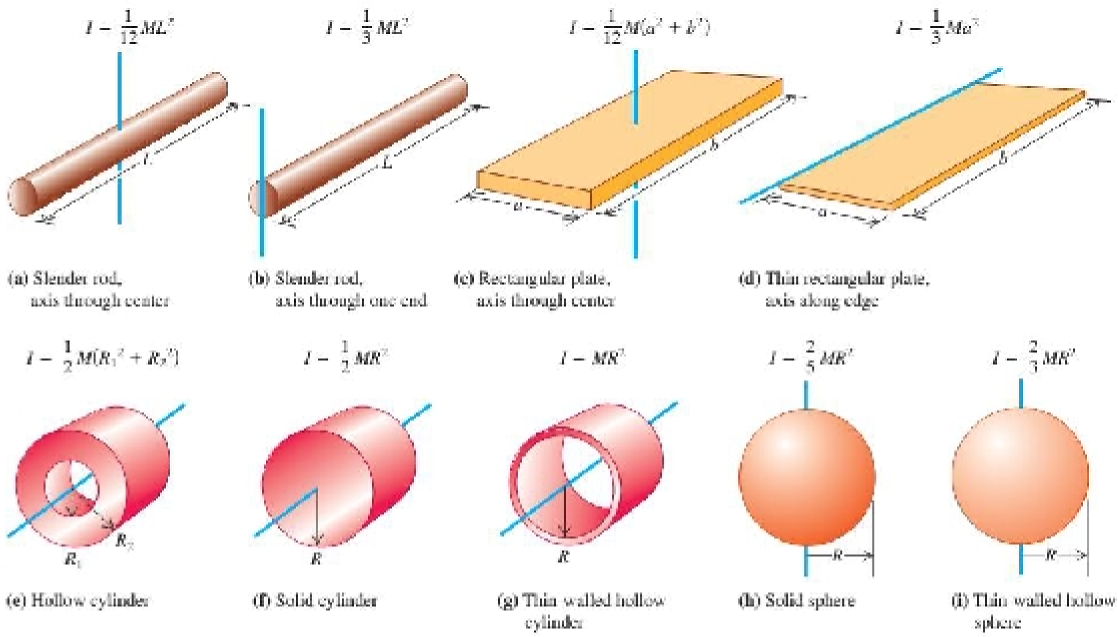
$$r_B = 0.50\ \text{m} \qquad \text{and} \qquad r_C = 0.40\ \text{m}.$$

$$I_A = m_B r_B^2 + m_C r_C^2$$
$$= (0.10\ \text{kg})(0.50\ \text{m})^2 + (0.20\ \text{kg})(0.40\ \text{m})^2$$
$$= 0.057\ \text{kg} \cdot \text{m}^2.$$

If the object rotates about this axis with angular velocity $\omega = 4.0$ rad/s, its kinetic energy is

$$K = \tfrac{1}{2} I_A \omega^2 = \tfrac{1}{2}(0.057\ \text{kg} \cdot \text{m}^2)(4.0\ \text{rad/s})^2 = 0.46\ \text{J}.$$

In Example up, we represented the body as several point masses and we evaluated the sum in Equation of a body's moment of inertia directly. When the body is a continuous distribution of matter, such as a solid cylinder or plate, we need methods of integral calculus to evaluate the sum. The table gives moments of inertia for several familiar shapes in terms of the masses and dimensions. In the formulas in the table, note that dimensions measured parallel to the axis of rotation never appear in the expressions.



Example

Test devices was spin testing a sample of a solid steel rotor (a disk) of mass 272 kg and radius R=38.0 cm, and angular speed ω of 14000 rev/min. how much energy of the rotor?

KEY IDEA

The released energy was equal to the rotational kinetic energy K of the rotor just as it reached the angular speed of 14 000 rev/min.

Calculations

We can find K with Eq., $K = \frac{1}{2}IW^2$, but first we need an expression for the rotational inertia *I*. Because the rotor was a disk that rotated like a merry-go-round, *I* is given by the expression in up Table, ($I = \frac{1}{2}MR^2$). Thus we have

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(272kg)(0.38m)^2 = 19.64\ kg.m^2$$

The angular speed of the rotor was

$$\omega = (14000\,rev/\text{min})(2\pi\ rad/rev)(\frac{1\,min}{60\,sec})$$

$$=1.466 \times 10^7\ \text{J}.$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(19.64\,kg.m^2)(1.466 \times 10^3\,\frac{rad}{sec})^2$$

$$=2.1 \times 10^7\ \text{J}. \qquad \text{(Answer)}$$

Periodic motion or Oscillation

Any kinds of motion repeat themselves over and over: the vibration of a quartz crystal in a watch, the swinging pendulum of a grandfather clock, the sound vibrations produced by a clarinet or an organ pipe, and the back-and-forth motion of the pistons in a car engine. This kind of that motion is called **Periodic motion or Oscillation**.

Simple Harmonic Motion

Figure shows a sequence of "snapshots" of a simple oscillating system, a particle moving repeatedly back and forth about the origin of an x axis. In this section, we simply describe the motion

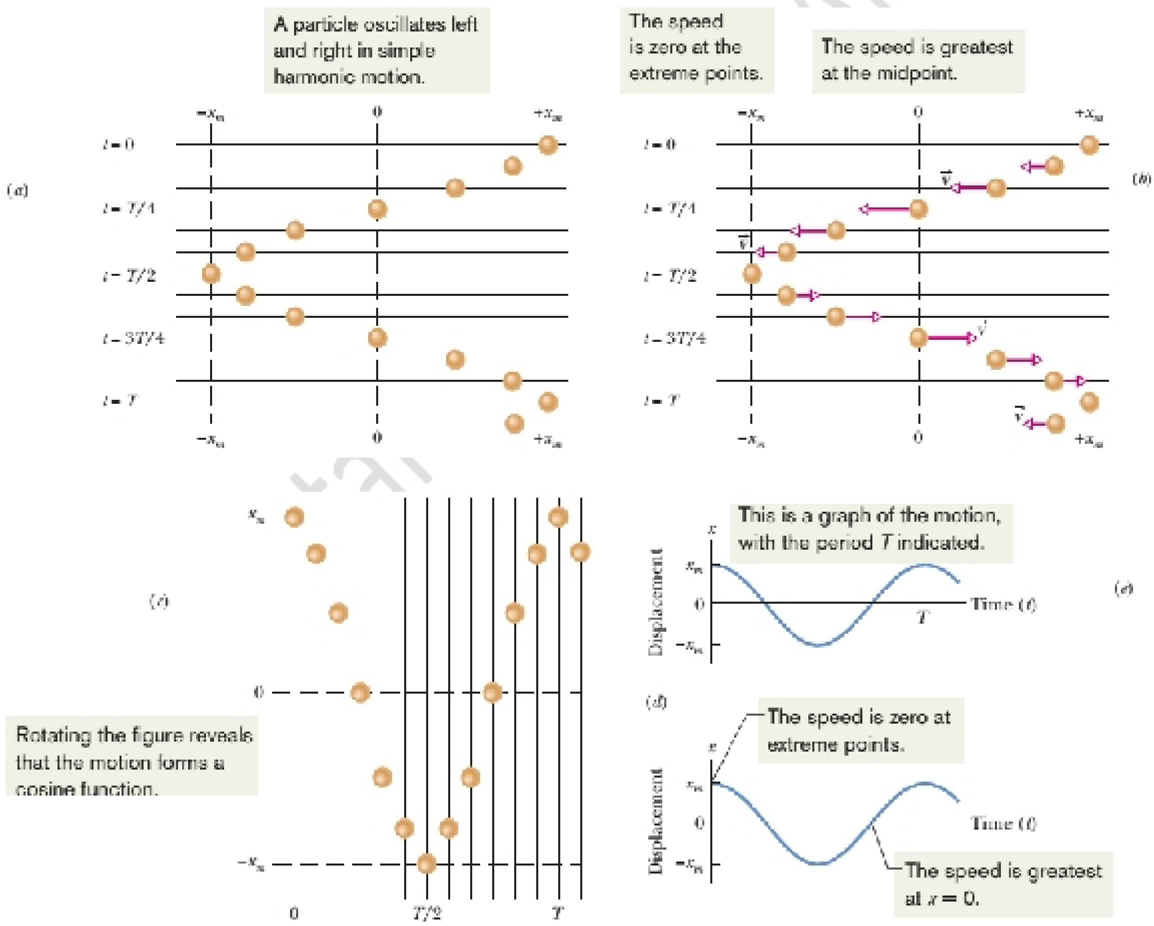


Figure (a) A sequence of "snapshots" (taken at equal time intervals) showing the position of a particle as it oscillates back and forth about the origin of an x axis, between the limits + x $_m$ and - x_m (b) The vector arrows are scaled to

indicate the speed of the particle. The speed is maximum when the particle is at the origin and zero when it is at $\pm x_m$. If the time t is chosen to be zero when the particle is at $+ x_m$, then the particle returns to $+ x_m$ at t=T, where Tis the period of the motion. The motion is then repeated. (c) Rotating the figure reveals the motion forms a cosine function of time, as shown in (d). (e) The speed (the slope) changes

One important property of oscillatory motion is its frequency, or number of oscillations that are completed each second. The symbol for frequency is f, and its SI unit is the **hertz** (Hz), where

$$1\,hertz = 1Hz = 1\ oscillation\ per\ second = 1\ s^{-1}$$

Related to the frequency is the period T of the motion, which is the time for one complete oscillation (or cycle); that is,

$$T = \frac{1}{f}.$$

Any motion that repeats itself at regular intervals is called **periodic motion or harmonic motion**.

The displacement x of the motion particle of the particle from the origin is given as a function of time by

$$x(t) = x_{m\cos(\omega t+\ \ )} \qquad (DISPLACEMENT)$$

in which x_m, ω, and Φ are constants. This motion is called simple harmonic motion (SHM). a term that means the periodic motion is a **sinusoidal function** of time. in which the sinusoidal function is a cosine function, is graphed in Fig. (d). (We get that graph by rotating Fig. (a) counterclockwise by 90°.) The quantities that determine the shape of the graph are displayed in Fig. down with their names. We now shall define those quantities.

The quantity x_m, called the amplitude of the motion, is a positive constant whose value depends on how the motion was started. The subscript m stands for maximum because the amplitude is the magnitude of the maximum displacement of the particle in either direction. The cosine function in Eq.

varies between the limits ±1; so the displacement x(t) varies between the limits ± x_m.

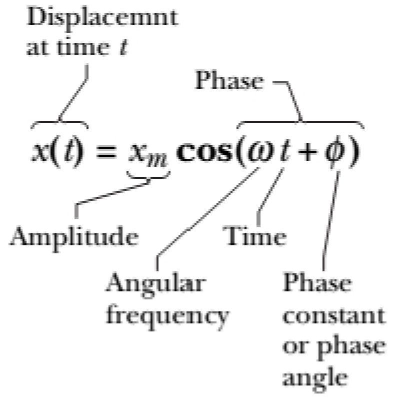


Fig. A handy reference to the quantities in Eq. for simple harmonic motion.

The time-varying quantity ($\omega t + \phi$) in Eq. 15-3 is called the **phase** of the motion, and the constant ϕ is called the **phase constant** (or **phase angle**). The value of ϕ depends on the displacement and velocity of the particle at time $t = 0$. For the $x(t)$ plots of Fig. a, the phase constant ϕ is zero.

To interpret the constant ω, called the **angular frequency** of the motion, we first note that the displacement $x(t)$ must return to its initial value after one period T of the motion; that is, $x(t)$ must equal $x(t + T)$ for all t. To simplify this analysis, let us put $\phi = 0$ in Eq. From that equation we then can write

$$x_m \cos \omega t = x_m \cos \omega(t + T).$$

The cosine function first repeats itself when its argument (the phase) has increased by 2π rad; so Eq. gives us

$$\omega(t + T) = \omega t + 2\pi$$

or

$$\omega T = 2\pi.$$

the angular frequency is

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

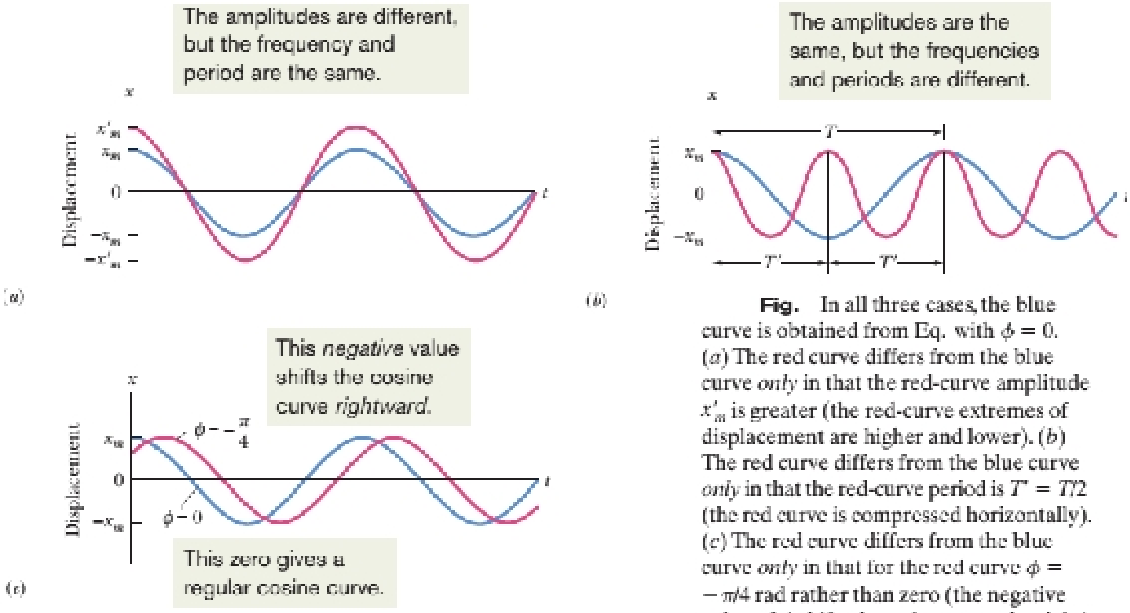


Fig. In all three cases, the blue curve is obtained from Eq. with $\phi = 0$. (*a*) The red curve differs from the blue curve *only* in that the red-curve amplitude x'_m is greater (the red-curve extremes of displacement are higher and lower). (*b*) The red curve differs from the blue curve *only* in that the red-curve period is $T' = T/2$ (the red curve is compressed horizontally). (*c*) The red curve differs from the blue curve *only* in that for the red curve $\phi = -\pi/4$ rad rather than zero (the negative value of ϕ shifts the red curve to the right).

The Velocity of SHM

The velocity of a particle moving with simple harmonic motion; that is,

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi) \qquad \text{(velocity).}$$

The positive quantity ωx_{min} of up Eq. is called the velocity amplitude v_m. When the magnitude of the displacement is greatest (that is,$x(t)=x_m$), the magnitude of the velocity is least (that is, $v(t)=0$).When the magnitude of the displacement is least (that is, zero), the magnitude of the velocity is greatest (that is,$v_m=\omega x_m$).

The Acceleration of SHM

Knowing the velocity υ(t) for simple harmonic motion, we can find an expression for the acceleration of the oscillating particle by differentiating once more. Thus, we have, from

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \qquad \text{(acceleration).}$$

The positive quantity $\omega^2 x_{min}$ as the up Eq. is called the acceleration amplitudeam.

the **acceleration amplitude** a_m; that is, the acceleration of the particle varies between the limits $\pm a_m = \pm\omega^2 x_m$, as Fig. c shows. Note also that the acceleration curve $a(t)$ is shifted (to the left) by $\frac{1}{4}T$ relative to the velocity curve $v(t)$.

We can combine to yield

$$a(t) = -\omega^2 x(t),$$

In SHM, the acceleration is proportional to the displacement but opposite in sign, and the two quantities are related by the square of the angular frequency

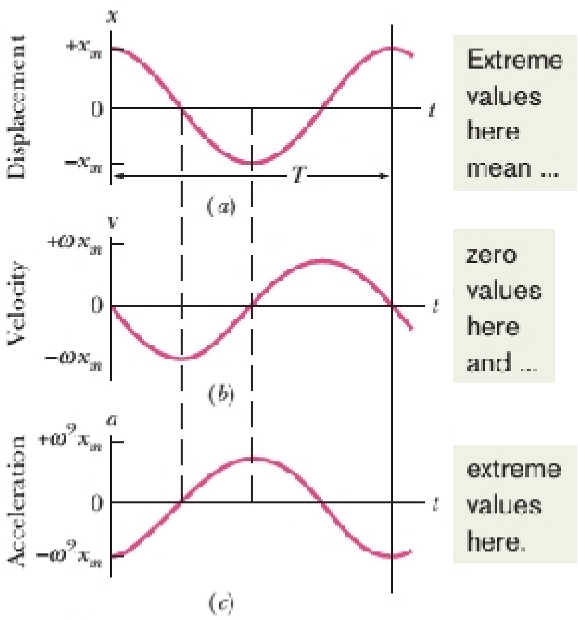


Fig. (a) The displacement $x(t)$ of a particle oscillating in SHM with phase angle ϕ equal to zero. The period T marks one complete oscillation. (b) The velocity $v(t)$ of the particle. (c) The acceleration $a(t)$ of the particle.

A HOOKEAN SYSTEM (a spring, wire, rod, etc.) when such a system is stretched a distance x (for compression, x is negative), the restoring force exerted by the spring is given by **Hooke's Law:**

$$F = -kx$$

The minus sign indicates that the restoring force is always opposite in direction to the displacement. The *spring constant k* has units of N/m and is a measure of the stiffness of the spring. Most springs obey Hooke's Law for small distortions.

THE ELASTIC POTENTIAL ENERGY stored in a Hookean spring (PE_e) that is distorted a distance (x) is: $1/2kx^2$. If the amplitude of motion is x_0 for a mass at the end of a spring, then the energy of the vibrating system is $1/2kx^2_0$ at all times. However, this energy is completely stored in the spring only when $x = \pm x_0$, that is, when the mass has its maximum displacement.

ENERGY INTERCHANGE between kinetic and potential energy occurs constantly in a vibrating system. When the system passes through its equilibrium position, KE = maximum and $PE_e = 0$. When the system has its maximum displacement, then KE = 0 and PE_e = maximum. From the law of conservation of energy, in the absence of friction,

$$KE + PE_e = \text{constant}$$

For a mass m at the end of a spring (whose own mass is negligible), this becomes

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_0^2$$

Where x_0 is the amplitude of the motion. And via the above energy equation the velocity is:

$$|v| = \sqrt{(x_0^2 - x^2)\frac{k}{m}}$$

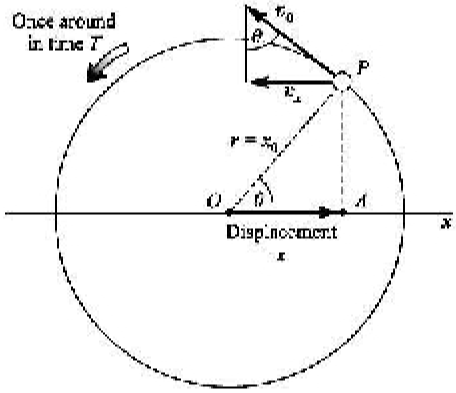
F= - kx, and F= ma $\longrightarrow$ $a = -\dfrac{k}{m}x$

The minus sign indicates that the direction of **a** (and **F**) is always opposite to the direction of the displacement **x**.

REFERENCE CIRCLE: Suppose that a point P moves with constant speed v_0 around a circle, as shown in Fig. This circle is called the *reference circle* for SHM. Point A is the projection of point P on the x-axis, which coincides with the horizontal diameter of the circle. The motion of point A back and forth about point O as center is SHM. The amplitude of the motion is x_0, the radius of the circle. The time taken for P to go around the circle once is the period T of the motion. The velocity, $\vec{v}_0$, of point A has a scalar x-component of

$$v_x = -v_0 \sin\theta$$

When this quantity is positive $\vec{v}_x$ points in the positive x-direction, when it's negative $\vec{v}_x$ points in the negative x-direction.



PERIOD IN SHM: The period T of a SHM is the time taken for point P to go once around the reference circle in Fig. Therefore,

$$T = \frac{2\pi r}{v_0} = \frac{2\pi x_0}{v_0}$$

But v_0 is the maximum speed of point A in Fig. that is, v_0 is the value of $|v_x|$ in SHM when $x = 0$:

$$|v_x| = \sqrt{(x_0^2 - x^2)\frac{k}{m}} \qquad \text{gives} \qquad v_0 = x_0\sqrt{\frac{k}{m}}$$

This then gives the period of SHM to be

$$T = 2\pi\sqrt{\frac{m}{k}}$$

for a Hookean spring system.

ACCELERATION IN TERMS OF T: Eliminating the quantity k/m between the two equations $a = -(k/m)x$ and $T = 2\pi\sqrt{m/k}$, we find

$$a = -\frac{4\pi^2}{T^2}x$$

THE SIMPLE PENDULUM very nearly undergoes SHM if its angle of swing is not too large. The period of vibration for a pendulum of length L at a location where the gravitational acceleration is g is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

SHM can be expressed in analytic form by reference to Fig. 11-2 where we see that the horizontal displacement of point P is given by $x = x_0 \cos\theta$. Since $\theta = \omega t = 2\pi f t$, where the *angular frequency* $\omega = 2\pi f$ is the angular velocity of the reference point on the circle, we have

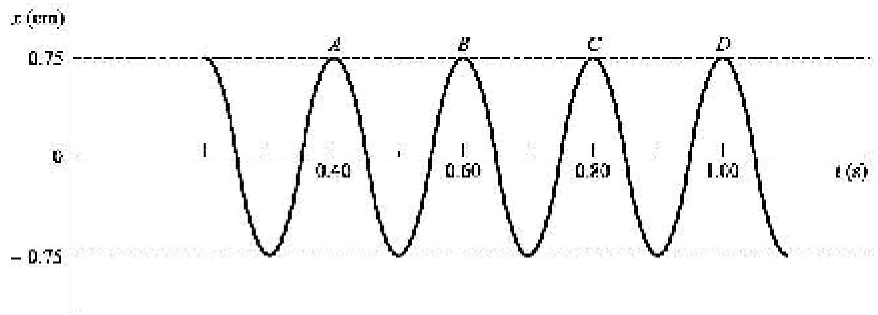
$$x = x_0 \cos 2\pi f t = x_0 \cos \omega t$$

Similarly, the vertical component of the motion of point P is given by

$$y = x_0 \sin 2\pi f t = x_0 \sin \omega t$$

Also from the figure, $v_x = v_0 \sin 2\pi f t$.

(1) For the motion shown in Fig. , what are the amplitude, period, and frequency?



The amplitude is the maximum displacement from the equilibrium position and so is 0.75 cm. The period is the time for one complete cycle, the time from A to B, for example. Therefore the period is 0.20 s. The frequency is

$$f = \frac{1}{T} = \frac{1}{0.20\ \text{s}} = 5.0\ \text{cycles/s} = 5.0\ \text{Hz}$$

(2) A 200-g mass vibrates horizontally without friction at the end of a horizontal spring for which $k = 7.0$ N/m. The mass is displaced 5.0 cm from equilibrium and released. Find (*a*) its maximum speed and (*b*) its speed when it is 3.0 cm from equilibrium. (*c*) What is its acceleration in each of these cases?

From the conservation of energy,

$$\tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

where $k = 7.0$ N/m, $x_0 = 0.050$ m, and $m = 0.200$ kg. Solving for v gives

$$v = \sqrt{\frac{k}{m}(x_0^2 - x^2)}$$

(*a*) The speed is a maximum when $x = 0$; that is, when the mass is passing through the equilibrium position:

$$v = x_0\sqrt{\frac{k}{m}} = (0.050\text{ m})\sqrt{\frac{7.0\text{ N/m}}{0.200\text{ kg}}} = 0.30\text{ m/s}$$

(*b*) When $x = 0.030$ m,

$$v = \sqrt{\frac{7.0\text{ N/m}}{0.200\text{ kg}}[(0.050)^2 - (0.030)^2]\text{ m}^2} = 0.24\text{ m/s}$$

(*c*) By use of $F = ma$ and $F = kx$, we have

$$a = \frac{k}{m}x = (35\text{ s}^{-2})(x)$$

which yields $a = 0$ when the mass is at $x = 0$ and $a = 1.1$ m/s² when $x = 0.030$ m.

Finding SHM phase constant from displacement and velocity

At $t = 0$, the displacement $x(0)$ of the block in a linear oscillator like that of Fig. 15-5 is -8.50 cm. (Read $x(0)$ as "x at time zero.") The block's velocity $v(0)$ then is -0.920 m/s, and its acceleration $a(0)$ is $+47.0$ m/s².

(a) What is the angular frequency ω of this system?

KEY IDEA

With the block in SHM, Eqs. 15-3, 15-6, and 15-7 give its displacement, velocity, and acceleration, respectively, and each contains ω.

Calculations: Let's substitute $t = 0$ into each to see whether we can solve any one of them for ω. We find

$$x(0) = x_m \cos\phi, \qquad (15\text{-}15)$$

$$v(0) = -\omega x_m \sin\phi, \qquad (15\text{-}16)$$

and

$$a(0) = -\omega^2 x_m \cos\phi. \qquad (15\text{-}17)$$

In Eq. 15-15, ω has disappeared. In Eqs. 15-16 and 15-17, we know values for the left sides, but we do not know x_m and ϕ. However, if we divide Eq. 15-17 by Eq. 15-15, we neatly eliminate both x_m and ϕ and can then solve for ω as

$$\omega = \sqrt{-\frac{a(0)}{x(0)}} = \sqrt{-\frac{47.0\text{ m/s}^2}{-0.0850\text{ m}}}$$

$$= 23.5\text{ rad/s}. \qquad \text{(Answer)}$$

(b) What are the phase constant ϕ and amplitude x_m?

Calculations: We know ω and want ϕ and x_m. If we divide Eq. 15-16 by Eq. 15-15, we eliminate one of those unknowns and reduce the other to a single trig function:

$$\frac{v(0)}{x(0)} = \frac{-\omega x_m \sin\phi}{x_m \cos\phi} = -\omega\tan\phi.$$

Solving for $\tan\phi$, we find

$$\tan\phi = -\frac{v(0)}{\omega x(0)} = -\frac{-0.920\text{ m/s}}{(23.5\text{ rad/s})(-0.0850\text{ m})}$$

$$= -0.461.$$

This equation has two solutions:

$$\phi = -25^\circ \quad\text{and}\quad \phi = 180^\circ + (-25^\circ) = 155^\circ.$$

Normally only the first solution here is displayed by a calculator, but it may not be the physically possible solution. To choose the proper solution, we test them both by using them to compute values for the amplitude x_m. From Eq. 15-15, we find that if $\phi = -25^\circ$, then

$$x_m = \frac{x(0)}{\cos\phi} = \frac{-0.0850\text{ m}}{\cos(-25^\circ)} = -0.094\text{ m}.$$

We find similarly that if $\phi = 155^\circ$, then $x_m = 0.094$ m. Because the amplitude of SHM must be a positive constant, the correct phase constant and amplitude here are

$$\phi = 155^\circ \quad\text{and}\quad x_m = 0.094\text{ m} = 9.4\text{ cm}. \qquad \text{(Answer)}$$

Block–spring SHM, amplitude, acceleration, phase constant

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA

The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\text{ N/m}}{0.68\text{ kg}}} = 9.78\text{ rad/s}$$
$$\approx 9.8\text{ rad/s.} \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

$$f = \frac{\omega}{2\pi} = \frac{9.78\text{ rad/s}}{2\pi\text{ rad}} = 1.56\text{ Hz} \approx 1.6\text{ Hz.} \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56\text{ Hz}} = 0.64\text{ s} = 640\text{ ms.} \qquad \text{(Answer)}$$

(b) What is the amplitude of the oscillation?

KEY IDEA

With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11\text{ cm.} \qquad \text{(Answer)}$$

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

KEY IDEA

The maximum speed v_m is the velocity amplitude ωx_m in Eq. 15-6.

Calculation: Thus, we have

$$v_m = \omega x_m = (9.78\text{ rad/s})(0.11\text{ m})$$
$$= 1.1\text{ m/s.} \qquad \text{(Answer)}$$

This maximum speed occurs when the oscillating block is rushing through the origin; compare Figs. 15-4a and 15-4b, where you can see that the speed is a maximum whenever $x = 0$.

(d) What is the magnitude a_m of the maximum acceleration of the block?

KEY IDEA

The magnitude a_m of the maximum acceleration is the acceleration amplitude $\omega^2 x_m$ in Eq. 15-7.

Calculation: So, we have

$$a_m = \omega^2 x_m = (9.78\text{ rad/s})^2(0.11\text{ m})$$
$$= 11\text{ m/s}^2. \qquad \text{(Answer)}$$

This maximum acceleration occurs when the block is at the ends of its path. At those points, the force acting on the block has its maximum magnitude; compare Figs. 15-4a and 15-4c, where you can see that the magnitudes of the displacement and acceleration are maximum at the same times.

(e) What is the phase constant ϕ for the motion?

Calculations: Equation 15-3 gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions*, as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos\phi. \qquad (15\text{-}14)$$

Taking the inverse cosine then yields

$$\phi = 0\text{ rad.} \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies Eq. 15-14; we chose the smallest angle.)

(f) What is the displacement function $x(t)$ for the spring–block system?

Calculation: The function $x(t)$ is given in general form by Eq. 15-3. Substituting known quantities into that equation gives us

$$x(t) = x_m\cos(\omega t + \phi)$$
$$= (0.11\text{ m})\cos[(9.8\text{ rad/s})t + 0]$$
$$= 0.11\cos(9.8t), \qquad \text{(Answer)}$$

where x is in meters and t is in seconds.

Example

An ultrasonic transducer (a kind of ultrasonic loudspeaker) used for medical diagnosis is oscillating at a frequency of 6.7 MHz. How much time does each oscillation take, and what is the angular frequency?

$$T = \frac{1}{f} = \frac{1}{6.7 \times 10^6 Hz} = 1.5 \times 10^{-7} sec = 0.15\ \mu s$$

$$\omega = 2\pi f = 2\pi(6.7 \times 10^6 Hz) = 4.2 \times 10^7 rad/s$$

Energy in Simple Harmonic Motion

$$E = U + K$$

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

$$= \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi) + \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

$$= \tfrac{1}{2}kx_m^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)].$$

For any angle α,

$$\cos^2 \alpha + \sin^2 \alpha = 1.$$

$$E = U + K = \tfrac{1}{2}kx_m^2.$$

Example

Many tall buildings have *mass dampers*, which are anti-sway devices to prevent them from oscillating in a wind. The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say, eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.

Suppose the block has mass $m = 2.72 \times 10^5$ kg and is designed to oscillate at frequency $f = 10.0$ Hz and with amplitude $x_m = 20.0$ cm.

(a) What is the total mechanical energy E of the spring–block system?

KEY IDEA

The mechanical energy E (the sum of the kinetic energy $K = \frac{1}{2}mv^2$ of the block and the potential energy $U = \frac{1}{2}kx^2$ of the spring) is constant throughout the motion of the oscillator. Thus, we can evaluate E at any point during the motion.

Calculations: Because we are given amplitude x_m of the oscillations, let's evaluate E when the block is at position $x = x_m$, where it has velocity $v = 0$. However, to evaluate U at that point, we first need to find the spring constant k. From Eq. 15-12 ($\omega = \sqrt{k/m}$) and Eq. 15-5 ($\omega = 2\pi f$), we find

$$\begin{aligned} k &= m\omega^2 = m(2\pi f)^2 \\ &= (2.72 \times 10^5 \text{ kg})(2\pi)^2(10.0 \text{ Hz})^2 \\ &= 1.073 \times 10^9 \text{ N/m}. \end{aligned}$$

We can now evaluate E as

$$\begin{aligned} E &= K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \\ &= 0 + \tfrac{1}{2}(1.073 \times 10^9 \text{ N/m})(0.20 \text{ m})^2 \\ &= 2.147 \times 10^7 \text{ J} \approx 2.1 \times 10^7 \text{ J}. \qquad \text{(Answer)} \end{aligned}$$

(b) What is the block's speed as it passes through the equilibrium point?

Calculations: We want the speed at $x = 0$, where the potential energy is $U = \frac{1}{2}kx^2 = 0$ and the mechanical energy is entirely kinetic energy. So, we can write

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

$$2.147 \times 10^7 \text{ J} = \tfrac{1}{2}(2.72 \times 10^5 \text{ kg})v^2 + 0,$$

or

$$v = 12.6 \text{ m/s}. \qquad \text{(Answer)}$$

Because E is entirely kinetic energy, this is the maximum speed v_m.

WAVES

Types of Waves

Waves are of three main types:

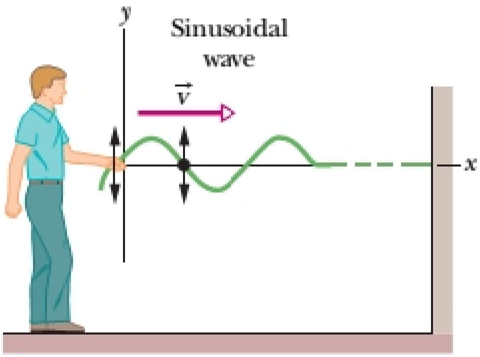
1. **Mechanical waves.** These waves are most familiar (مالوفة) because we encounter them almost constantly; common examples include water waves, sound waves, and seismic waves. All these waves have two central features: They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock.
2. **Electromagnetic waves**. These waves are less familiar, but you use them constantly; common examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar waves. These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed c299 792 458 m/s.
3. **Matter waves**. Although these waves are commonly used in modern technology, they are probably very unfamiliar to you. These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

wave has a sinusoidal shape: is, the wave has the shape of a sine curve or a cosine curve

Transverse and Longitudinal Waves

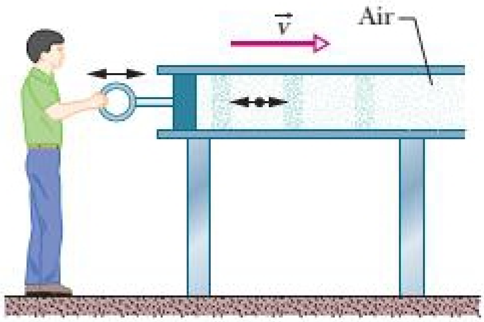
The figure, if you move your hand of sting, up and down in continuous simple harmonic motion, a continuous wave travels along the string at velocity $\vec{v}$. For forming the motion is a sinusoidal function of time.

The wave forms are moving to the right with oscillates up and down, we would find that the displacement of every such oscillating string element is perpendicular to the direction of travel of the wave. This wave is said to be a **transverse wave**.



The figure, a sinusoidal wave is sent along the string. A typical string element moves up and down continuously as the wave passes. This is a transverse wave.

In the other figure in down shown. If you push and pull on the piston in simple harmonic motion, as is being done a sinusoidal wave travels along the pipe. Because the motion of the elements of air is parallel to the direction of the wave's travel, the wave is said to be a **longitudinal wave**.



The figure, a sound wave is set up in an air-filled pipe by moving a piston back and forth. Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a longitudinal wave.

Both a transverse wave and a longitudinal wave are said to be traveling waves

Wavelength and Frequency

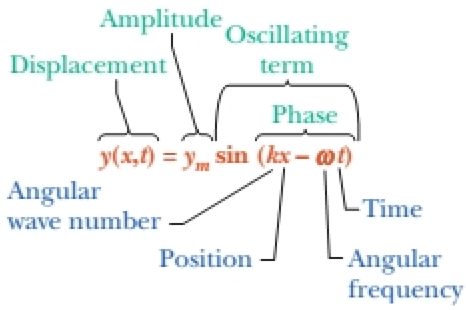
We need a function that gives the shape of the wave. This means that we need a relation in the form

$$y = h(x, 0)$$

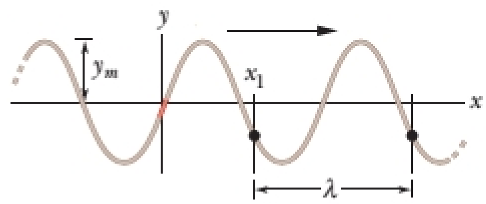
Which **y** is the transverse displacement of any string element as a function **h** of the time **t** and the position **x** of the element along the string.

h=sinusoidal shape function (sine or cosine function)

$$y(x,t) = y_m \sin(kx - \omega t)$$



This equation is written in terms of position x, it can be used to find the displacements of all the elements of the string as a function of time. Thus, it can tell us the shape of the wave at any given time and how that shape changes as the wave moves along the string.



The amplitude y_m of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them.

The phase of the wave is the argument $kx - vt$ of the sine in up Eq., as the wave sweeps through a string element at a particular position x, the phase

changes linearly with time t. This means that the sine also changes, oscillating between +1 and -1. Its extreme positive value (+1) corresponds to a peak of the wave moving through the element; at that instant the value of y at position x is y_m. Its extreme negative value (-1) corresponds to a valley of the wave moving through the element; at that instant the value of y at position x is $-y_m$. Thus, the sine function and the time-dependent phase of a wave correspond to the oscillation of a string element, and the amplitude of the wave determines the extremes of the element's displacement.

The wavelength λ of a wave is the distance (parallel to the direction of the wave's travel) between repetitions of the shape of the wave.

the description of the wave shape,

$$y(x,0) = y_m \sin kx$$

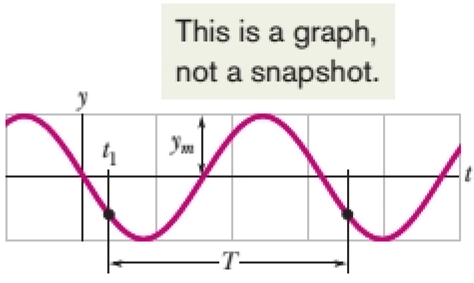
At $x = x_1$, $x = \lambda$

$$y(x_1,\lambda) = y_m \sin k(x_1 + \lambda)$$

$$k = \frac{2\pi}{\lambda}$$

The angular wave number k of the wave; its SI unit is the radian per meter, or the inverse meter. (Note that the symbol k here does not represent a spring constant as previously.)

Period, Angular Frequency, and Frequency



The figure is a graph of the displacement of the string element at x=0 as a function of time.

In simple harmonic motion, we toke Period, Angular Frequency, and Frequency. Therefore, the equation is

$$y(0,t) = y_m \sin(wt)$$

The period of oscillation T of a wave to be the time any string element takes to move through one full oscillation.

The angular frequency ω of the wave; its SI unit is the radian per second.

$$\omega = \frac{2\pi}{T}$$

The frequency *f* of a wave is defined as 1/T and is related to the angular frequency ω by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad (frequency)$$

We can generalize equation by inserting a phase constant Φ in the wave function

$$y = y_m \sin(kx - \omega t + \quad)$$

The Speed of a Traveling Wave

The wave speed is derivative $\frac{dx}{dt}$

To find the wave speed v, we take the derivative

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}.$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{(wave speed)}$$

Sound Waves

Sound Waves

Mechanical waves are waves that require a material medium to exist. There are two types of mechanical waves: Transverse waves involve oscillations perpendicular to the direction in which the wave travels; longitudinal waves involve oscillations parallel to the direction of wave travel. Sound wave is defined roughly as any longitudinal wave as the down figure.

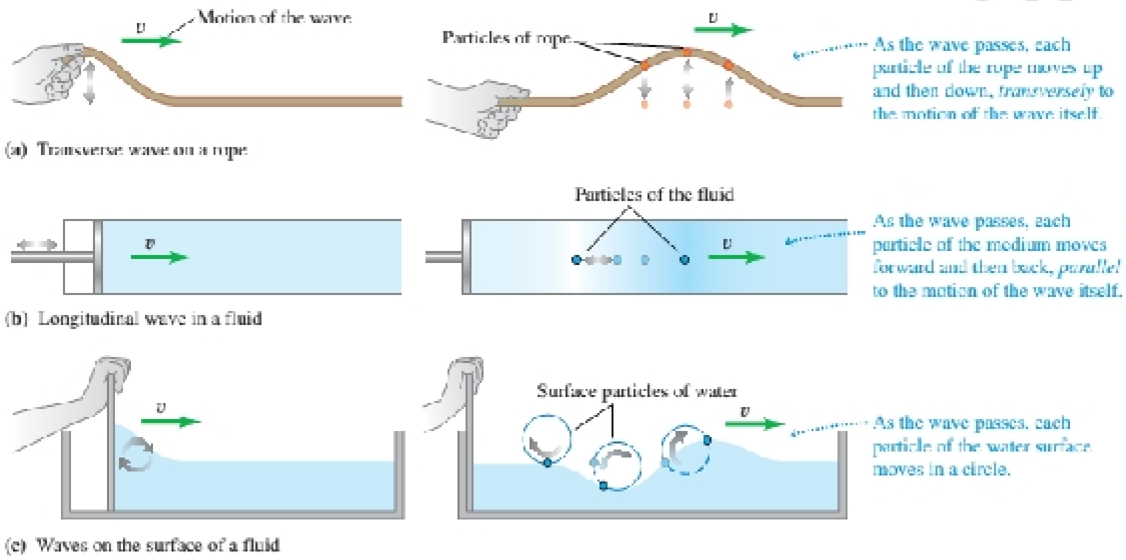


FIGURE Examples of (a) a transverse wave on a rope, (b) a longitudinal wave in a cylinder containing a gas or liquid, and (c) a surface wave with transverse and longitudinal components.

We said previously, when the piston in the left moving to rightward the tube (air-filled), will generate pressure the element of air associate with sinusoidal sound wave in inside the tube, traveling through a long tube. And when pull the piston to the left the tube that will generate pressure decrease. As each element of air pushes on the next element in turn, the right – left motion of the air and the change in its pressure travel along the tube due to expansion of the air. That pressure travel along the tube as a sound wave.

Consider the thin element of air of thickness Δx, and we could write their displacements on x-axis via $s(x, t)$ instead from $y(x, t)$.

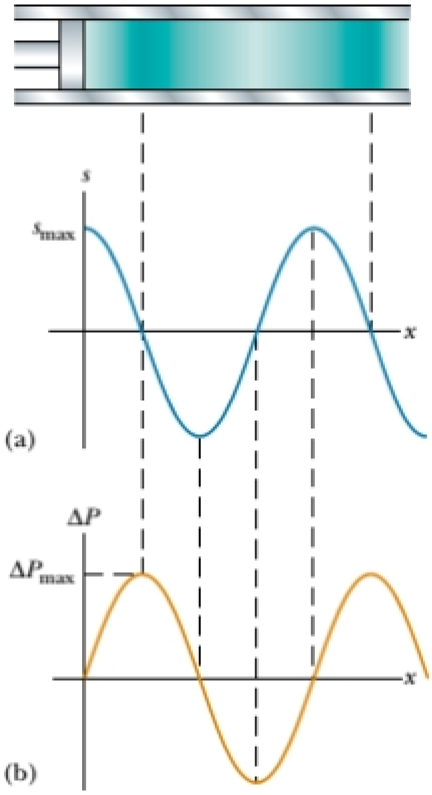


Figure 17.3 (a) Displacement amplitude versus position and (b) pressure amplitude versus position for a sinusoidal longitudinal wave. The displacement wave is 90° out of phase with the pressure wave.

To show that the displacements s(x, t) are sinusoidal functions (sin or cosine) of x and t, we can use either a sine function or a cosine function. We will write the displacement function, as

$$s_{(x,\ \ t)} = s_m \cos(kx - wt)$$

s_m (is the displacement amplitude) that is, the maximum displacement of the air element to either side (لجانبين الانبوب) of its equilibrium position. **k** is The angular wave number. **ω** angular frequency, f frequency, υ speed, T and period for a sound (longitudinal) wave, λ wavelength except here is represent the distance (again along the direction of travel) of compression and expansion due to the wave begins to repeat itself (see Fig.). (We assume s_m is much less than λ.)

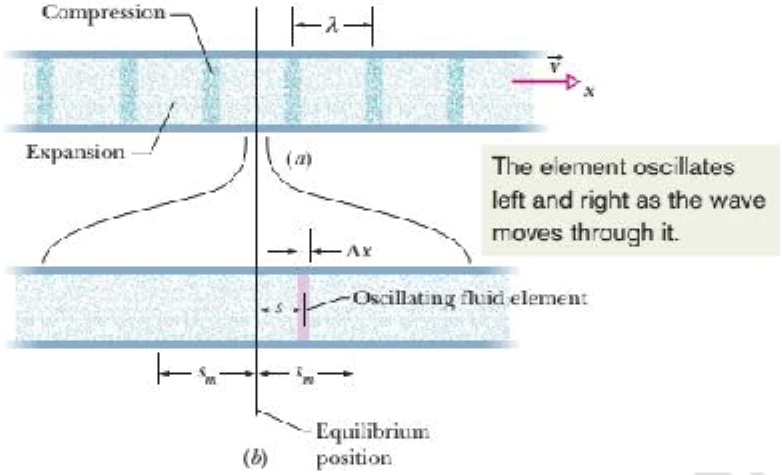


Fig. (a) A sound wave, traveling through a long air-filled tube with speed υ, consists of a moving, periodic pattern of expansions and compressions of the air. The wave is shown at an arbitrary instant. (b) A horizontally expanded view of a short piece of the tube. As the wave passes, an air element of thickness Δx oscillates left and right in simple harmonic motion about its equilibrium position. At the instant shown in (b), the element happens to be displaced a distance s to the right of its equilibrium position. Its maximum displacement, either right or left, is s_m.

The air pressure equation inside the tube is

$$\Delta P_{(X,\ t)} = \Delta P_m \sin(kx - wt)$$

A negative value of Δp in equation corresponds to an expansion of the air, and a positive value to a compression. Here Δp_m is the pressure amplitude, which is the maximum increase or decrease in pressure due to the wave; Δpm is normally very much less than the pressure p.

Let us apply Newton's second law to the element. During Δt, when compression the piston for generator pulse in pipe, the force is generator PA, and pressure in the pipe will be (P + ΔP) A, Therefore the value is always positive. Then, the average net force will be

$$F = PA - (P + \Delta P)A$$

$$F = -PA \qquad (NET\ FORCE)$$

The minus sign indicates that the net force on the air element is directed to the left.

The time interval is

$$\Delta t = \frac{\Delta x}{\upsilon}$$

The volume of the element is A Δx, we can write its mass as

$$\Delta m = \rho \Delta V = \rho A \Delta x = \rho A \upsilon \Delta t$$

The average acceleration of the element during Δt is

$$a = \frac{\Delta \upsilon}{\Delta t}$$

Substituting of all equation

$$-PA = \rho A \upsilon \Delta t \frac{\Delta \upsilon}{\Delta t}$$

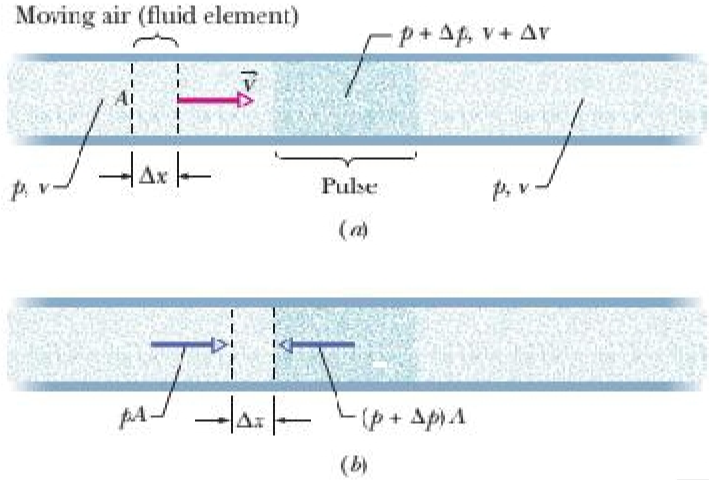
Product by $\frac{\upsilon}{\upsilon}$

$$\rho \upsilon^2 = -\frac{P}{\Delta \upsilon / \upsilon}$$

The air that occupies a volume V(=A υ Δt) outside the pulse is compressed by an amount ΔV(=A Δυ Δt) as it enters the pulse. Thus,

$$\frac{\Delta V}{V} = \frac{A \Delta v \Delta t}{A v \Delta t} = \frac{\Delta v}{v}$$

$$\rho v^2 = -\frac{\Delta P}{\Delta v / v} = -\frac{\Delta P}{\Delta V / V} = B$$



The figure, a compression pulse is sent from right to left down a long air-filled tube. The reference frame of the figure is chosen so that the pulse is at rest and the air moves from left to right. (a) An element of air of width Δx moves toward the pulse with speed v. (b) The leading face of the element enters the pulse. The forces acting on the leading and trailing faces (due to air pressure) are shown.

The pressure amplitude Δp_m is related to the displacement amplitude s_m by.

$$\Delta P = B\frac{\Delta V}{V}$$

the volume of the element, given by

$$V = A\Delta x$$

The quantity ΔV is the change in volume that occurs when the element is displaced. This volume change comes about because the displacements of the two faces of the element are not quite the same, differing by some amount Δs. Thus, we can write the change in volume as

$$\Delta V = A\Delta s$$

Substituting in original equation and passing to the differential limit yield.

$$\Delta P = -B\frac{\Delta s}{\Delta x} = -B\frac{\partial s}{\partial x}$$

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}[s_m \cos(kx - wt)] = -ks_m \sin(kx - wt)$$

$$\therefore \Delta P = -B(-ks_m \sin(kx - wt) = Bks_m \sin(kx - wt)$$

This pressure as a sinusoidal function of time, and the amplitude ΔP_m is

$$\Delta P_m = (Bk)s_m = (\rho v^2 k)s_m = (\rho w v)s_m$$

$$\Delta P_m = (\rho \omega v)s_m$$

The other down figure illustrates several ideas that we shall use in our discussions. Point S represents a tiny sound source, called a point source, that emits sound waves in all directions. The wave fronts and rays indicate the direction of travel and the spread of the sound waves. Wave fronts are surfaces over which the oscillations due to the sound wave have the same value; such surfaces are represented by whole or partial circles in a two-dimensional drawing for a point source. Rays are directed lines perpendicular to the wave fronts that indicate the direction of travel of the wave fronts. The short double arrows superimposed on the rays of the Fig. indicate that the longitudinal oscillations of the air are parallel to the rays.

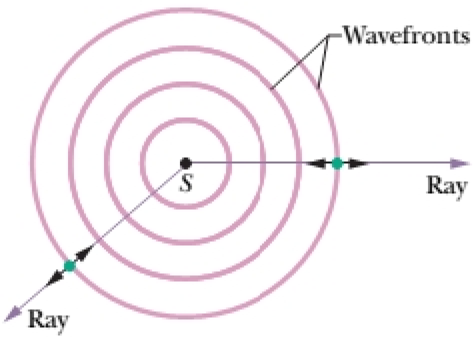


Fig. A sound wave travels from a point source *S* through a three-dimensional medium. The wavefronts form spheres centered on *S*; the rays are radial to *S*. The short, double-headed arrows indicate that elements of the medium oscillate parallel to the rays.

The Speed of Sound

The speed of any mechanical wave, transverse or longitudinal, depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy). The equation down gives the speed of a transverse wave along a stretched string, by writing.

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

Where (for transverse waves) tis the tension in the string and μ is the string's linear density. If the medium is air and the wave is longitudinal, we can guess that the inertial property, corresponding to μ, is the volume density ρ of air.

In a stretched string, potential energy is associated with the periodic stretching of the string elements as the wave passes through them. As a sound wave passes through air, potential energy is associated with periodic compressions and expansions of small volume elements of the air. The property that determines the extent to which an element of a medium changes in volume when the pressure (force per unit area) on it changes is the bulk modulus B, defined a

$$B = -\frac{\Delta p}{\Delta V/V} \qquad \text{(definition of bulk modulus).}$$

Here ΔV/Vis the fractional change in volume produced by a change in pressure Δp. the unit for B is also the Pascal. The signs of Δp and ΔV are always opposite: When we increase the pressure on an element (Δp is positive), its volume decreases (ΔV is negative). so that B is always a positive quantity. Now substituting B for τ and ρ for μ yields

$$v = \sqrt{\frac{B}{\rho}} \qquad \text{(speed of sound)}$$

As the speed of sound in a medium with bulk modulus B and density ρ.

Also, the speed of sound depends on the temperature of the medium. For sound traveling through air, the relationship between wave speed and air temperature is:

$$v = 331\sqrt{1 + \frac{T_C}{273}}$$

Where v is in meters/second, 331 m/s is the speed of sound in air at 0 °C, and T_C is the air temperature in degrees Celsius.

Sound Level in Decibels

Example

A tuning fork oscillates at 284 Hz in air. Compute the wavelength of the tone emitted at 25°C?

$$v = 331\sqrt{1 + \frac{T_c}{273}} = 331\sqrt{1 + \frac{25}{273}} = 345.8\ m/sec$$

$$v = \lambda f$$

$$\lambda = \frac{v}{f} = \frac{345.8m/sec}{284H_z} = 1.217m$$

Example

The maximum pressure amplitude Δp_m that the human ear can tolerate in loud sounds is about 28 Pa (which is very much less than the normal air pressure of about 10^5 Pa). What is the displacement amplitude s_m for such a sound in air of density ρ = 1.21 kg/m3, at a frequency of 1000 Hz and a speed of 343 m/s?

Solution

The displacement amplitude s_m of a sound wave is related to the pressure amplitude Δp_m of the wave according to Eq.

$$s = \frac{p_m}{\rho\omega\upsilon} = \frac{\Delta P_m}{\upsilon\rho(2\pi f)} = \frac{2p_a}{343m/sec \times 1.21kg/m^3 \times (2\pi 1000H_z)} = 11\mu m$$

Example

The maximum pressure amplitude ΔP_m that the human ear can tolerate in loud sounds is about 28 Pa. What is the displacement amplitude s_m for such a sound in air of density ρ=1.21 kg/m^3, at a frequency of 1000 Hz and a speed of 343 m/s?

Calculations:

$$s_m = \frac{\Delta P_m}{\upsilon\rho\omega} = \frac{\Delta P_m}{\upsilon\rho(2\pi f)}$$

$$s_m = \frac{28\,P_a}{(343\,m/sec)(1.21\,kg/m^3)(2\pi)(1000H_z)} = 1.1 \times 10^{-5}m = 11\mu m$$

Example

If a solid bar is struck at one end with a hammer, a longitudinal pulse propagates down the bar with a speed , $\upsilon = \sqrt{\gamma/\rho}$ where $\gamma = 7 \times 10^{10}\,N/m^2$ is the Young's modulus for the material, $\rho = 2.7 \times 10^3\,kg/m^3$ Find the speed of sound in an aluminum bar?

Calculation:

$$\upsilon = \sqrt{\frac{7 \times 10^{10}\,N/m^2}{2.7 \times 10^3\,kg/m^3}} = 5.1\,km/sec$$

This typical value for the speed of sound in solids is much greater than the speed of sound in gases. This difference in speeds makes sense because the molecules of a solid are bound together into a much more rigid structure than those in a gas and hence respond more rapidly to a disturbance.

Example

a) Find the speed of sound in water, which has a bulk modulus of $2.1 \times 10^9 N/m^2$ and a density of $1.00 \times 10^3 kg/m^3$?

$$v_{water} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \times 10^9\, N/m^2}{1 \times 10^3\, kg/m^3}} = 1.4\, km/sec$$

b) Dolphins use sound waves to locate food. Experiments have shown the dolphin can detected a bit of "dinner" at distance 110 m away, even in murky water (ماء مظلم). How much time passes between the moment the dolphin emits a sound pulse and the moment (لحظة) the dolphin hears (سمع) its wave reflection (موجة الانعكاس) from a bit of dinner?

Answer:

The total distance covered by the sound wave as it travels from dolphin to target and back is $2 \times 110\, m = 220$

$$\Delta t = \frac{\Delta x}{v_x} = \frac{220\, m}{1400\ m/sec} = 0.16\, sec$$

Intensity and Sound Level

Intensity is a ratio between the **powers** of surface sound wave with **area**.

$$I = \frac{P}{A}$$

Where P is the time rate of energy transfer (the power) of the sound wave and, A is the area of the surface intercepting the sound. As we shall derive shortly, the intensity I is related to the displacement amplitude s_m of the sound wave by

$$I = \frac{1}{2}\rho v w^2 s_m^2$$

Consider, in Fig. 17-4a, a thin slice of air of thickness dx, area A, and mass dm, oscillating back and forth as the sound wave of Eq. 17-12 passes through it. The kinetic energy dK of the slice of air is

$$dK = \tfrac{1}{2} dm\, v_s^2.$$

$$v_s = \frac{\partial s}{\partial t} = -\omega s_m \sin(kx - \omega t)$$

$$dK = \tfrac{1}{2}(\rho A\, dx)(-\omega s_m)^2 \sin^2(kx - \omega t).$$

Divided on $\frac{1}{dt}$

$$\frac{dK}{dt} = \tfrac{1}{2}\rho A v \omega^2 s_m^2 \sin^2(kx - \omega t)$$

The average rate at which kinetic energy is transported is

$$\left(\frac{dK}{dt}\right)_{\text{avg}} = \tfrac{1}{2}\rho A v \omega^2 s_m^2 [\sin^2(kx - \omega t)]_{\text{avg}}$$

$$= \tfrac{1}{4}\rho A v \omega^2 s_m^2.$$

$$I = \frac{2(dK/dt)_{\text{avg}}}{A} = \tfrac{1}{2}\rho v \omega^2 s_m^2$$

The intensity Iat the sphere must then be

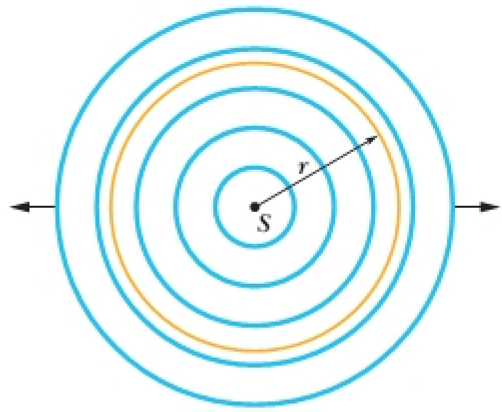


Fig. A point source S emits sound waves uniformly in all directions. The waves pass through an imaginary sphere of radius r that is centered on S.

$$I = \frac{\text{‘}P_S}{4\pi r^2}$$

The Doppler Effect

A police car is parked by the side of the highway (طريق عام), sounding its 1000 Hz siren (صفارة). **If** you are also parked by the highway, you will hear that same frequency. **If** there is relative motion between you and the police car, either toward or away from each other, you will hear a different frequency. For example, if you are driving toward the police car at 120 km/h (about 75 mi/h), you will hear a higher frequency (1096 Hz, an increase of 96 Hz). If you are driving away from the police car at that same speed, you will hear a lower frequency (904 Hz, a decrease of 96 Hz).

We shall assume the speeds of a source S of sound waves and a detector D move either directly toward or directly away from each other, at speeds less than the speed of sound. Therefore, if either the detector or the source is moving, or both are moving, the emitted frequency f and the detected frequency f' are related by

$$f' = f\frac{v \pm v_D}{v \pm v_S} \quad \text{(general Doppler effect)}.$$

Where υ is the speed of sound through the air, v_D is the detector's speed relative to the air, and v_S is the source's speed relative to the air. The choice of plus or minus signs is set by this rule:

When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from the other, the sign on its speed must give a downward shift in frequency.

Example Doppler effect I: Wavelengths

A police car's siren emits a sinusoidal wave with frequency $f_S = 300$ Hz. The speed of sound is 340 m/s and the air is still. (a) Find the wavelength of the waves if the siren is at rest. (b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.

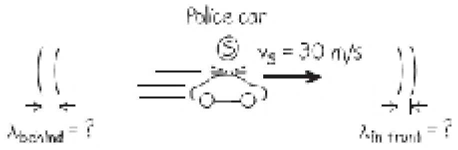
SOLUTION

IDENTIFY and SET UP: In part (a) there is no Doppler effect because neither source nor listener is moving with respect to the air; $v = \lambda f$ gives the wavelength. shows the situation in part (b): The source is in motion, so we find the wavelengths for the Doppler effect.

EXECUTE: (a) When the source is at rest,

$$\lambda = \frac{v}{f_S} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

Our sketch for this problem.



(b) in front of the siren

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S} = \frac{340 \text{ m/s} - 30 \text{ m/s}}{300 \text{ Hz}} = 1.03 \text{ m}$$

From Eq. (16.28), behind the siren

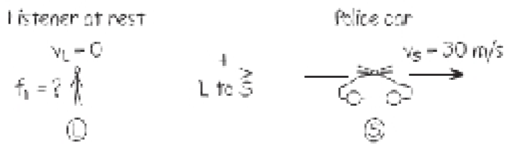
$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} = \frac{340 \text{ m/s} + 30 \text{ m/s}}{300 \text{ Hz}} = 1.23 \text{ m}$$

Example Doppler effect II: Frequencies

If a listener L is at rest and the siren in Example 16.14 is moving away from L at 30 m/s, what frequency does the listener hear?

SOLUTION

IDENTIFY and SET UP: Our target variable is the frequency f_L heard by a listener behind the moving source. Figure shows the situation. We have $v_L = 0$ and $v_S = +30$ m/s (positive, since the velocity of the source is in the direction from listener to source).



EXECUTE:

$$f_L = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + 30 \text{ m/s}}(300 \text{ Hz}) = 276 \text{ Hz}$$

EVALUATE: The source and listener are moving apart, so $f_L < f_S$. Here's a check on our numerical result.

the wavelength behind the source (where the listener in is located) is 1.23 m. The wave speed relative to the stationary listener is $v = 340$ m/s even though the source is moving, so

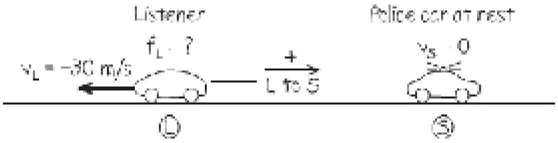
$$f_L = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.23 \text{ m}} = 276 \text{ Hz}$$

Example Doppler effect III: A moving listener

If the siren is at rest and the listener is moving away from it at 30 m/s, what frequency does the listener hear?

SOLUTION

IDENTIFY and SET UP: Again our target variable is f_L, but now L is in motion and S is at rest. Figure shows the situation. The velocity of the listener is $v_L = -30$ m/s (negative, since the motion is in the direction from source to listener).



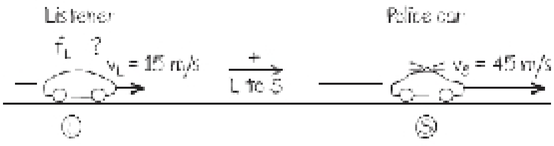
EXECUTE:

$$f_L = \frac{v + v_L}{v} f_S = \frac{340 \text{ m/s} + (-30 \text{ m/s})}{340 \text{ m/s}}(300 \text{ Hz}) = 274 \text{ Hz}$$

EVALUATE: Again the source and listener are moving apart, so $f_L < f_S$. Note that the *relative velocity* of source and listener is the same but the Doppler shift is different because v_S and v_L are different.

Example **Doppler effect IV: Moving source, moving listener**

The siren is moving away from the listener with a speed of 45 m/s relative to the air, and the listener is moving toward the siren with a speed of 15 m/s relative to the air. What frequency does the listener hear?



SOLUTION

IDENTIFY and SET UP: Now *both* L and S are in motion. Again our target variable is f_L. Both the source velocity $v_S = +45$ m/s and the listener's velocity $v_L = +15$ m/s are positive because both velocities are in the direction from listener to source.

EXECUTE:

$$f_L = \frac{v + v_L}{v + v_S} f_S = \frac{340 \text{ m/s} + 15 \text{ m/s}}{340 \text{ m/s} + 45 \text{ m/s}}(300 \text{ Hz}) = 277 \text{ Hz}$$

EVALUATE: the source and listener again move away from each other at 30 m/s, so again $f_L < f_S$. But f_L is different in all three cases because the Doppler effect for sound depends on how the source and listener are moving relative to the *air*, not simply on how they move relative to each other.

Fluid Mechanics

Fluid

A fluid, in contrast ("مغايرة، مقارنة) to a solid state, is a substance that can flow. Fluids conform (يتوافق، يتماشى) with the boundaries of any container in which we put them. They do so because a fluid hasn't sustained a force (قوة اسناد او قوة تعزيز) that is tangential to its surface. Also, the fluid is a substance that flows because it cannot withstand (يقاوم) a shearing stress (اجهاد القص) . It can, however, exert a force in the direction perpendicular to its surface.

The fluid also, has properties is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are consider a fluids.

Density

To find the density $\boldsymbol{\rho}$ of a fluid at any point, we isolate a small volume element ΔV around that point and measure the mass Δm of the fluid contained within that element. The density is then

$$\rho = \frac{\Delta V}{\Delta m}$$

To homogeneous fluid is

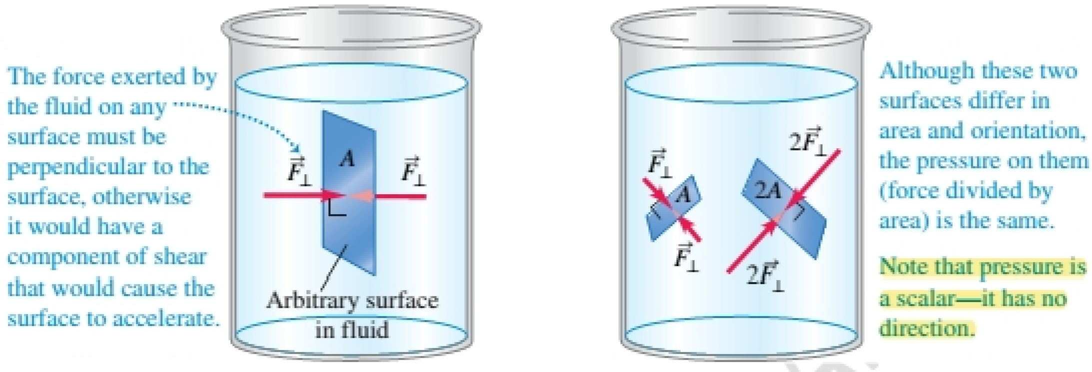
$$\rho = \frac{V}{m}$$

Units: The SI unit of density is the kilogram per cubic meter kg/m^3, the gram per cubic centimeter g/cm^3

The **specific gravity** of a material is the ratio of its density to the density of water; it is a pure number. For example, the specific gravity of aluminum is 2.7; this means that the density of aluminum is 2.7 times as great as that of water. ("Specific gravity" is a poor term, since it has nothing to do with gravity; "**relative density**" would be better.)

Pressure in a Fluid

When a fluid (either a liquid or a gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as the wall of a container or the surface of a body immersed in it, as the figure.



(a) Why forces due to fluid pressure must act normal to a

(b) Pressure equals force divided by area.

The figure. The forces due to pressure on arbitrary surfaces within a fluid.

That the forces acting on the two sides must be equal in magnitude and opposite in direction, and they must be oriented perpendicular to the surface. Similarly, any surface of an object immersed in the fluid is acted upon by a force perpendicular to the surface.

The pressure p at a point in a fluid is the ratio of the normal force $F_\perp$ on a small area A around that point to the area. Units: The SI unit of pressure is 1 N/m^2 = 1 Pa

$$P = \frac{F_\perp}{A}$$

$$1\,atm = 1.01 \times 10^5 P_a = 14.7\,psi = 1.01 bars = 760\,torr = 14.7\ 1b/in^2$$

P_a is special name of pressure unit

atm is the unit of pressure of atmosphere at sea

torr (named for Evangelista Torricelli, who invented the mercury barometer in 1674) was formerly called the millimeter of mercury(mm Hg).

$1b/in^2$ is often abbreviated (مختصر) psi. It shows some pressure.

Example

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.

(a) What does the air in the room weigh when the air pressure is 1.0 atm?

KEY IDEAS

(1) The air's weight is equal to mg, where m is its mass. (2) Mass m is related to the air density ρ and the air volume V by ($\rho = m/V$).

Calculation: Putting the two ideas together and taking the density of air at 1.0 atm from Table 14-1, we find

$$mg = (\rho V)g$$
$$= (1.21\ \text{kg/m}^3)(3.5\ \text{m} \times 4.2\ \text{m} \times 2.4\ \text{m})(9.8\ \text{m/s}^2)$$
$$= 418\ \text{N} \approx 420\ \text{N}. \quad \text{(Answer)}$$

(b) What is the magnitude of the atmosphere's downward force on the top of your head, which we take to have an area of 0.040 m^2?

KEY IDEA

When the fluid pressure p on a surface of area A is uniform, the fluid force on the surface can be obtained from ($p = F/A$).

Calculation: Although air pressure varies daily, we can approximate that $p = 1.0$ atm.

$$F = pA = (1.0\ \text{atm})\left(\frac{1.01 \times 10^5\ \text{N/m}^2}{1.0\ \text{atm}}\right)(0.040\ \text{m}^2)$$
$$= 4.0 \times 10^3\ \text{N}. \quad \text{(Answer)}$$

This large force is equal to the weight of the air column from the top of your head to the top of the atmosphere.

Example

The mattress (فرشة) of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep. (a) Find the weight of the water in the mattress, and the volume is 1.2 m^3.

Solution The density of water is 1 000 kg/m^3 and so the mass of the water is

$$m = \rho V = \left(\frac{1000kg}{m^3}\right)(1.2m^3) = 1.2 \times 10^3 kg$$

$$F = mg = (1.2 \times 10^3 kg)\left(9.8\frac{m}{sec^2}\right) = 1.18 \times 10^4 N$$

(b) Find the pressure exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

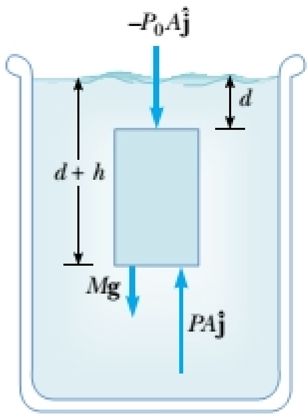
Solution When the bed is in its normal position, the cross sectional area is 4.00 m^2

$$P = \frac{F}{A} = \frac{1.18 \times 10^4 N}{4m^2} = 2.95\ P_a$$

Variation of Pressure with Depth

The water pressure increases with depth. Likewise (بطريقة مماثلة), atmospheric pressure decreases with increasing altitude, and show how the pressure in a liquid increases with depth. The density of a substance is defined as its mass per unit volume.

Now consider a liquid of density rat rest as shown in Figure. The density of liquid is ρ, this liquid contained within a cylinder of cross-sectional area A extending from depth d to depth $d + h$. The pressure exerted by the liquid on the bottom face of the sample is P, and the pressure on the top face is P0. Therefore, the upward force exerted by the outside fluid on the bottom of the cylinder has a magnitude PA, and the downward force exerted on the top has a magnitude P_0A. The mass of liquid in the cylinder is $m = \rho V = \rho Ah$; therefore, the weight of the liquid in the cylinder is $mg = \rho Ahg$. Because the cylinder is in equilibrium, the net force acting on it must be zero.



The figure. The net force exerted on the parcel of fluid must be zero because it is in equilibrium

$$\sum \vec{F}_y = PA\hat{j} - P_OA\hat{j} - mg\hat{j} = 0$$

$$PA - P_OA - \rho Ahg = 0$$

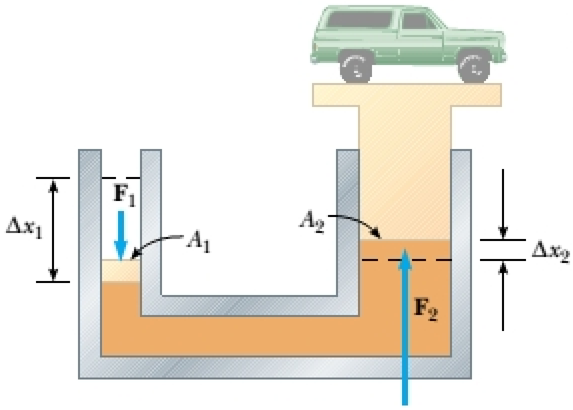
$$P = P_O + \rho gh$$

Note: The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not depends on any horizontal dimension of the fluid or its container.

Pascal's law

Pascal's law: a change in the pressure applied to a fluid is transmitted (ينقل) undiminished (بدون نقصان) to every point of the fluid and to the walls of the container.

Therefore, the Pascal's low is applied in the hydraulic press, as figure



Figure, Diagram of a hydraulic press

The diagram of a hydraulic press show, the increase in pressure is the same on the two sides; a small force F_1 at the left produces a much greater force F_2 at the right.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{And} \qquad F_2 = \frac{A_2}{A_1} F_1$$

Example

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. (A) What force must the compressed air exert to lift a car weighing 13300 N?

Solution: $F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi(5\times10^{-2}m)^2}{\pi(15\times10^{-2}m)^2} 1.33 \times 10^4 N = 1.48 \times 10^3 N$

(B) What air pressure produces this force?

Solution: $P_1 = \frac{F_1}{A_1} = \frac{1.48\times10^3 N}{\pi(5\times10^{-2} m)^2} = 1.8 \times 10^5 P_a$

Pressure Measurements

For measurement pressure of the atmosphere by used the barometric pressure, we say

$$F = P_O A$$

$$ma = P_O A$$

$$\rho_{Hg} A g h = P_O A$$

$$\therefore P_O = \rho_{Hg} g h$$

$$h = \frac{P_O}{\rho_{Hg} g} = \frac{1.013\times10^5 P_a}{(13.6\times\frac{10^3\,kg}{m^3})(\frac{9.8m}{\sec^2})} = 0.760m$$

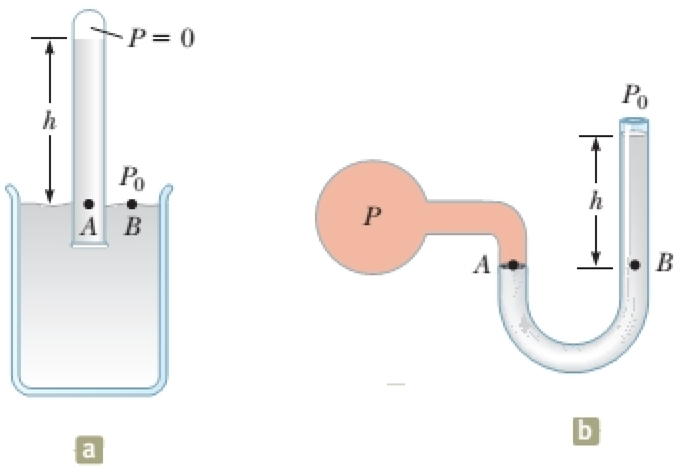


Figure. Two devices for measuring pressure: (a) a mercury barometer and (b) an open-tube manometer

The Equation of Continuity

You may have noticed that you can increase the speed of the water emerging from a garden hose (لخرطوم الحديقة) by partially closing (غلق جزئي) the hose opening (خرطوم مفتوح) with your thumb (ابهام). Apparently the speed v of the water depends on the cross-sectional area A through which the water flows.

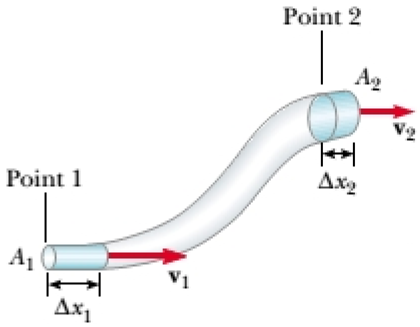


Figure. A fluid is moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1in a time interval Δt must equal the volume flowing through area A_2in the same time interval. Therefore, $A1v1 = A2v2$..

Now. The mass of fluid contained in the blue portion in Figure (a) is given by $m_1 = \rho A_1 \Delta X_1 = \rho A_1 v_1 \Delta t$

Where ρ is density of the ideal fluid. The fluid in the blue portion in Figure (b) has a mass $m_2 = \rho A_2 \Delta X_2 = \rho A_2 v_2 \Delta t$.

However, the mass of fluid that passes point 1 in a time interval Δt must equal the mass that passes point 2 in the same time interval.

$$m_1 = m_2$$

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 = constant \qquad \text{(Equation of continuity)}$$

EXAMPLE **Niagara Falls**

Each second, 5 525 m^3 of water flows over the 670-m-wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. What is its speed at that instant?

Solution The cross-sectional area of the water as it reaches the edge of the cliff is $A = (670\text{ m})(2\text{ m}) = 1\,340\text{ m}^2$. The flow rate of 5 525 m^3/s is equal to Av. This gives

$$v = \frac{5\,525\text{ m}^3/\text{s}}{A} = \frac{5\,525\text{ m}^3/\text{s}}{1\,340\text{ m}^2} = \boxed{4\text{ m/s}}$$

Note that we have kept only one significant figure because our value for the depth has only one significant figure.

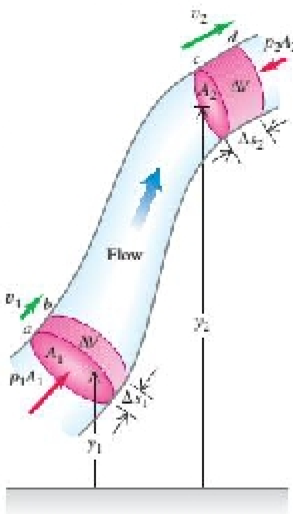
Exercise A barrel floating along in the river plunges over the Falls. How far from the base of the cliff is the barrel when it reaches the water 49 m below?

Answer 13 m ≈ 10 m.

Bernoulli's Equation

According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid particles. The pressure can also vary; it depends on height, as in the static situation, and it also depends on the speed of flow. We can now use the concepts with the work–energy theorem, to derive an important relationship called Bernoulli's equation.

The volume at two final of the tube considers equally, because the exchanging is same at any point in tube.



Then,

$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V \qquad W_2 = F_2\Delta x_2 = P_2A_2\Delta x_2 = P_2V$$

To derive the Bernoulli equation, we use the work–energy theorem,

$$W = W_1 - W_2$$

$$W = (P_1 - P_2)V$$

$$\Delta K = \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}$$

$$\Delta U = mgy_2 - mgy_1$$

$$(P_1 - P_2)V = \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2} + mgy_2 - mgy_1$$

$$P_1 - P_2 = \tfrac{1}{2}\rho v_2^{\,2} - \tfrac{1}{2}\rho v_1^{\,2} + \rho gy_2 - \rho gy_1$$

$$P_1 + \tfrac{1}{2}\rho v_1^{\,2} + \rho gy_1 = P_2 + \tfrac{1}{2}\rho v_2^{\,2} + \rho gy_2$$

$$P + \tfrac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

Viscosity

The term viscosity is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the fluid's kinetic energy to be transformed to internal energy. This mechanism is similar to the one by which the kinetic energy of an object sliding over a rough.

THE VISCOSITY (η) of a fluid is a measure of how large a shear stress is required to produce unit shear rate. Its unit is that of stress per unit shear rate, or Pa·s in the SI. Another SI unit is the $N \cdot s/m^2$ (or kg/m·s), called the *poiseuille* (Pl): 1 Pl = 1 kg/m·s = 1 Pa·s. Other units used are the *poise* (P), where 1 P = 0.1 , and the *centipoise* (cP), where 1 cP = 10^{-3} Pl. A viscous fluid, such as tar, has large η.

$$J = \frac{\pi R^4 (P_i - P_o)}{8\eta L}$$

$$J = \frac{\Delta V}{\Delta t}$$

The pipe's radius R.

One more useful relation in viscous fluid flow is the expression for the magnitude F of force exerted on a sphere of radius r that moves with speed v through a fluid with viscosity η. When v is small enough that there is no turbulence, the relationship is simple:

$$F = 6\pi\eta r v.$$

This relation is called Stokes's law.