SET, Elements :-

SET: List or collections of objects is called a set. and the object in the set is called its elements or members, write $x \in A$ if x is an element in the set.

If every element of A also belongs to a set B, i.e. if $x \in A$ implies $x \in B$, then A is called a subset of B (or A contained in B), denoted by

$$A \subset B \quad \text{or} \quad B \supset A$$

Two sets are equal if each is contained in the other, that is $A = B$ if and only if (iff, $\Leftrightarrow$, اذا واذا فقط) $A \subset B$ and $B \subset A$.

The negations of $x \in A$, $A \subset B$ and $A = B$ are written $x \notin A$, $A \not\subset B$ and $A \neq B$ respectively.

يمكن ان نعين المجموعة بذكر عناصرها ونضعها بين الاقواس او الاشارة لها من خلال وصفها نصيا

Ex. $A = \{1, 3, 5, 7, 9\}$
and $B = \{x : x \text{ is a prime number}, x < 15\}$

ملاحظة: إلا في حالة ما يتم ذكره فان كل مجموعة هي مجموعة جزئية من مجموعة تسمى المجموعة الشاملة Universal ويرمز لها بالرمز U ، وكذلك نستخدم مجموعة خالية ϕ لا تحتوي على اي عنصر وتكون

$\phi \subset A \subset U$ لكل مجموعة A.

Ex. N = The set of positive integers $1, 2, 3, \ldots$
Z = The set of integers $\ldots, -2, -1, 0, 1, 2, \ldots$
R = The set of real numbers

Thus we have $N \subset Z \subset R$

Ex. Let $C = \{x : x^2 = 4, x \text{ is odd}\}$

Then $C = \phi$ (an empty set)

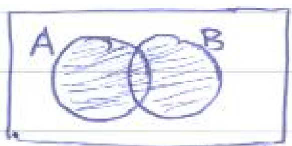
Theorem 1. Let A, B and C be any sets then
 i) $A \subset A$
 ii) if $A \subset B$ and $B \subset A$ then $A = B$
 iii) if $A \subset B$ and $B \subset C$ then $A \subset C$

Note : If $A \subset B$ but $A \neq B$ then we say that A is a proper subset of B, denoted $A \subseteq B$

Set Operations:-

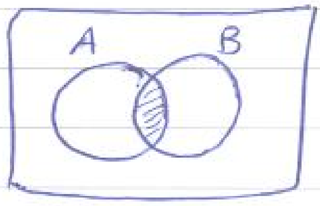
If A and B any sets, then

$A \cup B = \{x : x \in A \text{ or } x \in B\}$
union



$A \cup B$ is shaded

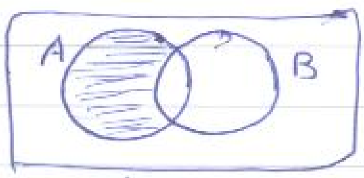
$A \cap B = \{x : x \in A \text{ and } x \in B\}$
intersection



$A \cap B$

The difference of A and B, denoted by $A \setminus B$ or $A - B$ is

$A \setminus B = \{x : x \in A, x \notin B\}$



$A - B$

A^c means the complement of A is

$A^c = \{x \mid x \in U, x \notin A\}$

Ex: Let $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$, where $U = \{1,2,3,\ldots\}$

$A \cup B =$ $\qquad$ $A \setminus B =$

$A \cap B =$ $\qquad$ $A^c =$

Laws of the algebra of sets:-

1. Idempotent laws a) $A \cup A = A$ b) $A \cap A = A$
2. Associative Laws a) $(A \cup B) \cup C = A \cup (B \cup C)$
   b) $(A \cap B) \cap C = A \cap (B \cap C)$
3. Commutative Laws:
   a) $A \cup B = B \cup A$
   b) $A \cap B = B \cap A$
4. Distributive laws
   a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. Identity Laws a) $A \cup \phi = A$
   $A \cup U = U$
   b) $A \cap U = A$
   $A \cap \phi = \phi$
6. Complement Laws
   a) $A \cup A^c = U$
   $(A^c)^c = A$
   b) $A \cap A^c = \phi$
   $U^c = \phi$, $\phi^c = U$
7. De Morgan's Laws
   a) $(A \cup B)^c = A^c \cap B^c$
   b) $(A \cap B)^c = A^c \cup B^c$

Theorem 2. Each of the following ~~are~~ conditions is equivalent to $A \subset B$

i) $A \cap B = A$ ii) $A \cup B = B$ iii) $B^c \subset A^c$

Introduction to Probability :-

Probability is the study of random or nondeterministic experiments. If a die is tossed in the air, then it is certain that the die will come down, but it is not certain that, say, a 6 will appear. However, suppose we repeat this experiment of tossing a die; let s be the number of successes, i.e. the number of times a 6 appears, and let n be the number of tosses. Then it has been empirically observed that the ratio $f = s/n$, called the relative frequency, becomes stable in the long run, i.e. approaches a limit. This stability is the basis of probability theory.

Historically, probability theory began with the study of games of chance, such as roulette and cards. The probability P of an event A was defined as follows: if A can occur in s ways out of total of n equally likely (i.e. with equal probability) ways, then

$$p = p(A) = s/n$$

For example, in tossing a die an even number can occur in 3 ways out of 6 "equally likely" ways, hence $p = \frac{3}{6} = \frac{1}{2}$

Sample Space and Events :-

The set S of all possible outcomes of some given experiment is called the sample space.

فضاء العينة: هي المجموعة التي عناصرها جميع النتائج الممكنة للتجربة أو الاختبار

A particular outcome, i.e. an element in S, is called a sample point or sample.

An event A is a set of outcomes or, in other words, a subset of the sample space S.

The event $\{a\}$ consisting of a single sample $a \in S$ is called an elementary event.

The empty set ϕ and S itself are event; ϕ is sometimes called the impossible event and S the certain or sure event.

we can combine events to form new events.

i) $A \cup B$ is the event that occurs iff A occurs or B occurs (or both).

ii) $A \cap B$ is the event that occurs iff A occurs and B occurs.

iii) A^c, is the event that occurs iff A does not occur.

Two events A and B are called mutually exclusive if they are disjoint, i.e. if $A \cap B = \phi$

اي ان حدث A و B لايمكن حدوثهما معاً (simultaneously)

Ex.1. Toss a die and observe the number that appears on top. Then the sample space consists of the six possible numbers;

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let A be the event that an even number occurs

$$A = \{2, 4, 6\}$$

Let B be the event that an odd number occurs

$$B = \{1, 3, 5\}$$

Let C be the event that a prime number occurs

$$C = \{2, 3, 5\}$$

$A \cup C =$

$B \cap C =$

$C^c =$

$A \cap B =$

Ex: 2: Toss a coin 3 times and observe the sequence of heads <H> and tails <T> that appears.
The sample space S consists $2^3 = 8$ elements.

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$A =$ The event that two or more heads appears consecutively (تعاقب، تراكي)

$A = \{HHH, HHT, THH\}$

Let B that all the tosses are the same.

$B = \{HHH, TTT\}$

$A \cap B = \{HHH\}$ elementary event.

The event that 5 heads appear $= \phi$

Ex.3. Toss a coin until a head appears and then count the number of times the coin was tossed.

$S = \{1, 2, 3, ---, \infty\}$,

Here ∞ refers to the case when a head never appears,
sample space S which is countably infinite.
مثال 3 يوضح ان فضاء العينة غير محدود العد.

Remark: ملاحظة
If the sample space S is finite or countably infinite, then every subset of S is an event.

Axioms of Probability: (بديهيات) $P(A)$ is called the probability of the event A. if the following axioms hold.

1) For every event A, $0 \le P(A) \le 1$.
2) $P(S) = 1$.
3) If A and B are mutually exclusive events, then
$$P(A \cup B) = P(A) + P(B)$$
هنا يكون الحدثين A , B منفصلان اي لا يمكن حدوثها في نفس اللحظة.

4) If $A_1, A_2, \ldots$ is a sequence (متتابعة) of mutually exclusive events, then
$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

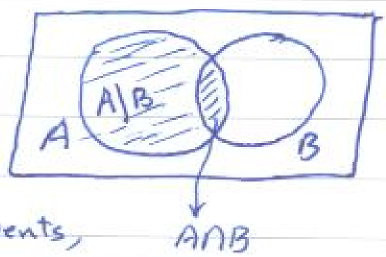
Theorem: If ϕ is the empty set, then
$$P(\phi) = 0.$$

Theorem: If A^c is the complement of an event A, then $P(A^c) = 1 - P(A)$.

Theorem: If $A \subset B$ then $P(A) \le P(B)$.

Theorem: If A and B are any two events, then
$$P(A \setminus B) = P(A) - P(A \cap B)$$

A is shaded



Theorem: If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Corollary نتيجة: For any events A, B and C,
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Finite probability spaces:

Let S be a finite sample space, say $S = \{a_1, a_2, \dots, a_n\}$. A finite probability space is obtained by assigning to each point $a_i \in S$ a real number p_i called the probability of a_i satisfying

i) each p_i, is nonnegative, $p_i \geq 0$

ii) the sum of the p_i is one,

$$p_1 + p_2 + \cdots + p_n = 1$$

Examples: 1) Let three coins be tossed and the number of heads observed;

عند رمي ثلاث قطع نقدية ولاحظنا عدد الصور (الرأس).

$S = \{0, 1, 2, 3\}$ فيكون فضاء العينة

$p(0) = \frac{1}{8}$ لذا يكون

$p(1) = \frac{3}{8}$, $p(2) = \frac{3}{8}$ and $p(3) = \frac{1}{8}$

$$P(S) = P(0) + P(1) + P(2) + P(3)$$
$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1.$$

لأن الاحتمالات غير سالبة ومجموعها = ١.

والآن اذا كانت A ظهور الصورة (الرأس) على الأقل مرة واحدة
ولتكن B هي الحدث لظهور كافة الصور او كل الكتابة.

$A = \{1, 2, 3\}$

$B = \{0, 3\}$

Then, by definition,

$$P(A) = P(1) + P(2) + P(3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

and

$$= \frac{7}{8}$$

$$P(B) = P(0) + P(3)$$
$$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

2) Three horses A, B and C are in a race (سباق); A is twice as likely to win as B and B is twice as likely to win as C. What are their respective probabilities of winning ح.ت.ع, i.e. $P(A)$, $P(B)$ and $P(C)$?

لنفرض أن احتمالية فوز C هي $P(C) = p$

وبما أن B حظه في الفوز مرتين عن حظ C.

فإن $P(B) = 2p$

وبما أن A حظه في الفوز مرتين أكثر من B.

يكون $P(A) = 2P(B)$

$$= 2(2p) = 4p$$

Now the sum of the probabilities must be 1, hence $p + 2p + 4p = 1$ or

$$7p = 1 \quad \text{or}$$

$$p = \frac{1}{7}$$

Accordingly,

$$P(A) = 4p = \frac{4}{7}$$

$$P(B) = 2p = \frac{2}{7}$$

$$P(C) = p = \frac{1}{7}$$

Question سؤال: What is the probability that B or C wins, i.e. $P(\{B, C\})$?

By definition

$$P(\{B, C\}) = P(B) + P(C)$$

$$= \frac{2}{7} + \frac{1}{7} = \frac{3}{7}$$

3) A class contain 10 men and 20 women of which half the men and half the women have brown eyes.

«أي أن نصف الذكور ونصف الإناث لهم عيون جوزية»

Find the probability P that a persion chosen at random عشوائي is a man or has brown eyes.

Let $A = \{\text{person is a man}\}$ and

$B = \{\text{person has brown eyes}\}$.

We seek $P(A \cup B)$.

Then $P(A) = \frac{10}{30} = \frac{1}{3}$

$P(B) = \frac{15}{30} = \frac{1}{2}$

$P(A \cap B) = \frac{5}{30} = \frac{1}{6}$

لسنة ثانية

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$$

4) Let A and B be events with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$ find,

i) $P(A \cup B)$

ii) $P(A^c)$ and $P(B^c)$

iii) $P(A^c \cap B^c)$

iv) $P(A^c \cup B^c)$

v) $P(A \cap B^c)$

vi) $P(B \cap A^c)$

5) Let A and B be events with $P(A \cup B) = \frac{3}{4}$, $P(A^c) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, Find,

i) $P(A)$

ii) $P(B)$

iii) $P(A \cap B^c)$

6) Let a coin and a die be tossed; let the sample space S consist of the twelve elements:

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

i) Express explicitly the following events:

$A = \{$heads and an even number appear$\}$,

$B = \{$a prime number appears$\}$,

$C = \{$tails and an odd number appear$\}$.

ii) Express explicitly the event that

a) A or B occurs,

b) B and C occurs,

c) only B occurs.

iii) Which of the events A, B and C are mutually exclusive?