



University of Technology
Department of Applied Sciences
Final Examination 2016/2017



Subject : Ordinary D.E.
Division : Math. & Computer Applications
Examiner : Assist. Prof. Ikram A. Saed

year: 2-nd year
Time : 3 hours
Date :

Answer five Questions only

Q1: Solve the following D.E. :

(a) $y^2 dx + (3xy - 4y^3) dy = 0$ (7 marks)

(b) $(x + \sin y) dx + (x \cos y - 2y) dy = 0$ (7 marks)

Q2: (a) If $y_1(x)$ and $y_2(x)$ are two linearly dependent functions , prove that
 $W(y_1, y_2) = 0$ (7 marks)

(b) Find the orthogonal trajectories of the following family of curves :
 $x^3 = 3(y - c)$. (c is a parameter) (7 marks)

Q3: (a) Find the general solution of the following D. E. :

$y''' + y' = \sec x$. (9 marks)

(b) Prove that : If $L\{f(x)\} = F(s)$, then (5 marks)

$L\{e^{ax} f(x)\} = F(s - a)$, a is a constant .

Q4: Use the power - series method to find the general solution near $x_0 = 0$ of the following D . E. :

$8x^2 y'' + 10xy' + (x - 1)y = 0$ (14 marks)

Q5: (a) Solve the following D. E. by using Laplace transform .

$y'' - 3y' + 4y = 0$, $y(0) = 1$, $y'(0) = 5$ (9 marks)

(b) Find : $L\{e^{-x} x \cos 2x\}$. (5 marks)

Q6: (a) Expand $f(x) = \sin x$, $0 < x < \pi$ in a Fourier cosine series . (7 marks)

(b) Find : $L^{-1}\left\{\frac{1}{(s-1)^2}\right\}$ by using convolution theorem . (7 marks)



Note: Answer seven questions(10 marks for each one)

Q1) What kind of solution that, the indirect methods for system of linear equations, give? What the Gauss-Seidal dose? Before solving these systems? Use this method to solve $\{13X_1 + 5X_2 - 3X_3 + X_4 = 18; 2X_1 + 12X_2 + X_3 - 4X_4 = 13; 3X_1 - 4X_2 + 10X_3 + X_4 = 29; 2X_1 + X_2 - 3X_3 + 9X_4 = 31\}$? Take $X^0 = (0,0,0,0)^T$; find X^2 and stop.

Q2) Find the best polynomial which enter through all the following points $\{(-1,2),(0,-1), (2,5)\}$? Calculate the approximate value of $f(1)$; $f'(1)$ and $f''(1)$?

Q3) Show that, this nonlinear equation $\{x = e^{-x}\}$ has root at the interval $[0,1]$? Use the Newton-Raphson method to find the approximate value of this root? Take $x^0 = 0$, then stop when you get absolute error bounded by $E=0.01$?

Q4) What is the order of error in the fourth order Runge-Kutta method? Use this formula to solve the initial value problem $\{y' = xy^{1/3}; y(1)=1\}$ at $[1,1.4]$, take size-step = 0.2?

Q5) What kind of solutions that the direct methods for system of linear equations, give? What the Gauss-Elimination dose? Before solving this system? Use this method to solve $\{5X_1 + 7X_2 + 6X_3 + 5X_4 = 23; 7X_1 + 10X_2 + 8X_3 + 7X_4 = 32; 6X_1 + 8X_2 + 10X_3 + 9X_4 = 33; 5X_1 + 7X_2 + 9X_3 + 10X_4 = 31\}$.

Q6) Calculate the approximate area under the curve $\{f(x) = e^x\}$ between the two lines $\{x = 0$ and $x = 0.5\}$? Find the error at this formula? Take the length of subintervals equal $(1/6)$ then keep the order of accuracy less than $O(h^4)$?

Q7) Prove that the iterative methods (Jacobi ; Gauss-Seidal and SOR) have the same form $\vec{X}^{k+1} = M\vec{X}^k + \vec{Y}$? Where M is a matrix and Y is a vector, both have different form for each method.

Q8) Use fixed point iterative method to find the approximate solution for the system of non-linear equations $\{X^3 + Y^3 - 6X + 3 = 0; X^3 - Y^3 - 6Y + 2 = 0\}$? Show that, the iterative relations will be used converges to the exact solution? Take $(x_0, y_0) = (0.5, 0.5)$, stop after two iteration?

Good Luck