University of Technology
Department of Applied Sciences
Final Examination 2014/2015

Subject : *Abstract Algebra* | **Class :** 3^{rd} *year*
Branch : *Math.* | **Time :** *3 hours*
Examiner: *Associ.prof.Dr.Anwar Khaleel* | **Date :** 31/*5/2015*

ملاحظة: الاجابه عن سبعة اسئلة فقط (لكل سؤال 10 درجات)

Q₁: Define and give an example for each of the following

1. Normal subgroup 2. Endomorphism of group 3. Field 4. permutation.

Q₂: **A.** Explain the relationship between the following

1. Maximal ideal and prime ideal. 2. Cyclic group and abelain group.

B. Show that (H,.) is a subgroup of (G,.), where $H=\{2^n: n\in Z\}$ and $G=R-\{0\}$.

Q₃: Prove or disprove any **four** of the following

1.Any abelain group is simple. 2. Any field is a division ring.

3. The union of two cyclic groups is also cyclic. 4. Every group of index 2 is normal.

5.A multiplication group can't be isomorphic to an additive group.

Q₄: prove that

1. Let R be a commutative ring with identity and I be a maximal ideal of R. Then the quotient ring R/I is a field.
2. Every group of order less than or equal to 5 is abelain.

Q₅: **A.** Define the symmetric group S_3 . Then find

1. Cent(S_3) 2. All normal subgroups of S_3 3. A_3 4. S_3 / A_3

B. Let G be a group of order 60. Is there exist a subgroup of G of order 24? Explain your answer.

Q₆: Answer any **one** of the following

A. State and prove the first isomorphism theorem.

B. Prove that, if I is an ideal of the ring Z of integer numbers then I=<n>, for some $n\in Z^+\cup\{0\}$.

Q₇: **A.** Let G be a group. Show that G is abelain iff the mapping $f:G\to G$ defined by $f(x)=x^{-1}$, $\forall x\in G$ is an automorphism.

B. Let $R=(Z_9,+_9,\cdot_9)$. Find 1. Char(R) 2. Nilpotent ideals of R 3. Prime ideals of R.

Q₈: Define the Boolean ring and prove that every Boolean ring is commutative. Is the converse true? Explain your answer.

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Subject: Mathematical Analysis
Branch: Applied Mathematics
Examiner: Dr. Jabbar Abbas

Class: Third year
Time: 3 hours
Date:

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Note: Answer **only five** of the following questions.

Q1: Define and give example on each of the following

a) Metric space b) Open cover
c) Derived set d) Cauchy sequence.

(14 marks)

Q2:

a) Define the uncountable set, and prove the set of all real numbers is uncountable set.
b) Prove that, a set S is closed iff its complement is open set.

(14 marks)

Q3: In any metric space (X, d), prove or disprove the following

a) If $A \subseteq X$, then $\overline{A} \subseteq B$, for every closed set $B \subseteq X$ such that $A \subseteq B$.
b) $A \cap B$ is compact set, whenever A is closed set and B is compact set.

(14 marks)

Q4:

a) Prove that, if $\sum_{n=1}^{\infty} a_n$ is a series of nonnegative numbers with $S_n = a_1 + a_2 + \ldots + a_n$, then $\sum_{n=1}^{\infty} a_n$ converges iff its partial sums $\{S_n\}$ form a bounded sequence.
b) Show that, if $f(x)$ has a limit at p, then this limit is unique.

(14 marks)

Q5:

a) Compare the continuity with uniform continuity, and explain that by examples.
b) Show by example that if f is continuous function at a point then f is not differentiable at the same point.

(14 marks)

Q6:

Suppose f is continuous on $[a,\ b]$, $f'(x)$ exists at some point $x \in (a,\ b)$, g is defined on I which contains the range of f, and g is differentiable at $f(x)$. Then $h(t) = g(f(t)),\quad a \leq t \leq b$ is differentiable at x, and $h'(x) = g'(f(x))\ f'(x)$.

(14 marks)

.... Good Luck. ....