1. Dispersion in Optical Glasses

The refractive index of any glass is a function of wavelength, and this leads to a variation of aberrations with wavelength - chromatic aberration. The change of index with wavelength is called dispersion and the dispersion curves for two common glasses are shown in Figure 14.1.

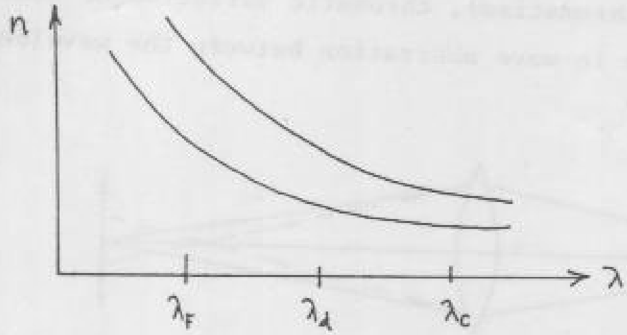

Figure 14.1 : Dispersion

The dispersion of a glass is commonly characterised by a "V-value" (or "Abbe number" or "ν-value") which is an inverse measure:

$$\left.\begin{aligned} V_{dFc} &= \frac{n_d - 1}{n_F - n_c} \\ V_{eF'c'} &= \frac{n_e - 1}{n_{F'} - n_{c'}} \\ V_{egc} &= \frac{n_e - 1}{n_g - n_c} \end{aligned}\right\} \qquad (14.1)$$

where λ_d = 587.56 nm, λ_F = 486.13 nm, λ_c = 656.27 nm, λ_e = 546.07 nm, $\lambda_{F'}$ = 479.99 nm, $\lambda_{c'}$ = 643.85 nm, λ_g = 435.84 nm.

A V-value is a function of three wavelengths - a central or design wavelength in the yellow/green and two extreme wavelengths, one red and one blue.

3. Conrady's Formula

Conrady showed that the chromatic difference in wave aberration for a finite ray over a certain wavelength range could be approximated by a formula which did not require the explicit tracing of the ray at different wavelengths. This is

$$W_{\lambda+\partial\lambda} - W_{\lambda} \simeq \Sigma(\overline{D} - D)\partial n \qquad (14.3)$$

where the sum extends over all dispersive media in the optical system, where $\overline{D}$ is the path length of the pupil ray at wavelength λ in a given medium, D is the corresponding path length at wavelength λ of the aperture ray under consideration, and ∂n is the dispersion $(n_{\lambda+\partial\lambda} - n_{\lambda})$ in the medium.

4. C_L and C_T for a Single Surface and for a System of Surfaces

Just as the monochromatic Seidel aberrations for a single surface can be calculated from paraxial raytrace data so can the wave aberration difference terms corresponding to chromatic variations of focus and magnification. The calculation is based on an approximation to Conrady's formula and gives

$$\partial_0 W_{20} = \tfrac{1}{2}C_L \qquad \partial_1 W_{11} = C_T \qquad (14.4)$$

where

$$\left.\begin{aligned} C_L &= Ah\Delta\left(\frac{\partial n}{n}\right) \\ C_T &= Bh\Delta\left(\frac{\partial n}{n}\right) \end{aligned}\right\} \qquad (14.5)$$

where A, B, h, n, n, and $\Delta(\ )$ have the usual meanings.

For a system of several surfaces the total C_L and C_T are simply the sums of the individual surface contributions (as with the monochromatic Seidel aberrations). Thus

$$\left.\begin{aligned} C_L &= \Sigma Ah\Delta\left(\frac{\partial n}{n}\right) \\ C_T &= \Sigma Bh\Delta\left(\frac{\partial n}{n}\right) \end{aligned}\right\} \qquad (14.6)$$

and (14.4) converts these sums into actual polynomial coefficients.

For the vast majority of glasses ('normal' glasses) this quantity varies linearly with V-value and this greatly hinders the attainment of apochromatism.

6. Thin Lens in Air; C_L, C_T, S_{IV} For Such a Lens

The thin-lens concept is very useful in aberration theory; it refers to a singlet lens whose thickness is negligible compared to its diameter so that a paraxial ray height can be assumed to be the same at both surfaces. As one might expect, the primary aberrations of such a lens can be expressed much more simply than those for a thick lens. The idea can also be extended to thin doublets and triplets. For example, for a single surface,

$$C_L = Ah\Delta\left(\frac{\partial n}{n}\right)$$

so that for a thin lens of index μ in air

$$\Sigma C_L = A_1 h\left(\frac{\partial \mu}{\mu}\right) + A_2 h\left(-\frac{\partial \mu}{\mu}\right)$$

Noting that $A_1 = hc_1 - u_1$ and $A_2 = hc_2 - u_2'$
and $hK = u_2' - u_1$ and $K = (\mu - 1)(c_1 - c_2)$
(the latter being the expression for the power of a thin lens) soon gives

$$\Sigma C_L = \frac{h^2 K}{V} \tag{14.7}$$

where $V = \frac{\mu - 1}{\partial \mu}$.

In a similar way

$$\Sigma C_T = \frac{h\bar{h}K}{V} \tag{14.8}$$

for a thin lens in air, and

$$\Sigma S_{IV} = \frac{H^2 K}{\mu} \tag{14.9}$$

- all very simple formulae.

It turns out that the corresponding results for the remaining four primary aberrations are more complicated. This is

$$Y = \frac{u' + u}{u' - u}$$

or

$$Y = \left[\frac{1}{hK}\right](u' + u) \qquad (14.12)$$

or

$$Y = \frac{1 + M}{1 - M}$$

where M is the Gaussian magnification and these can be inverted to give

$$u = hK\left(\frac{Y - 1}{2}\right)$$

$$u' = hK\left(\frac{Y + 1}{2}\right) \qquad (14.13)$$

$$M = \frac{Y - 1}{Y + 1}$$

Figure 14.5 shows how Y depends on various conjugate combinations

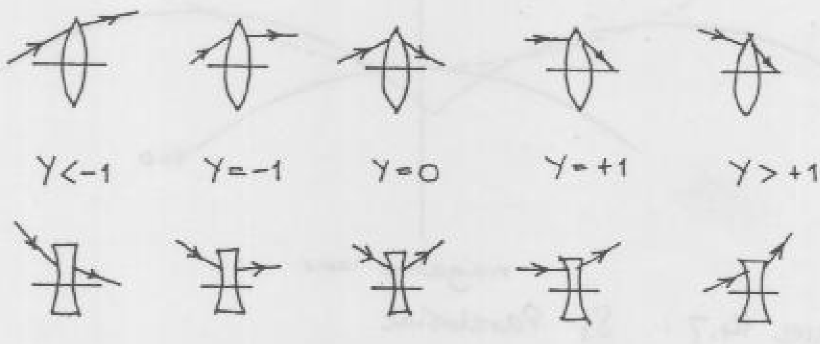

Figure 14.6 : Conjugate

8. S_I For a Thin Lens in Air

Using (14.11) to substitute for curvatures in terms of X and (14.13) to substitute for convergence angles in terms of Y it is possible in a few lines to deduce an expression for S_I of a thin lens:

$$S_I = \frac{h^4K^3}{4}\left\{\frac{\mu + 2}{\mu(\mu - 1)^2}\left[X - \frac{2(\mu^2 - 1)Y}{\mu + 2}\right]^2 + \left[\left(\frac{\mu}{\mu - 1}\right)^2 - \frac{\mu}{\mu + 2}Y^2\right]\right\}$$

(14.14)

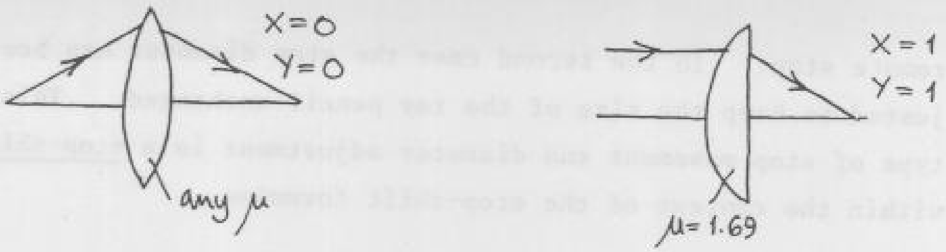

Figure 14.8 : Unit Magnification / Infinite Object

9. Central Aberrations

The remaining thin-lens aberrations (S_{II}, S_{III}, S_V) are more complex functions of X and Y because the stop position is also significant. For this reason it is useful to first consider the central aberrations when the stop is located at the thin lens, and then consider the case of a remote stop via the stop-shift formulae.

When the stop is at the lens, B reduces to

$$B = \frac{H}{h} \qquad (14.15)$$

and a calculation then gives

$$(S_{II})_c = \tfrac{1}{2}Hh^2K^2\left\{\frac{(\mu+1)X}{\mu(\mu-1)} - \frac{2\mu+1}{\mu}Y\right\} \qquad (14.16)$$

$$(S_{III})_c = H^2K \qquad (14.17)$$

$$(S_V)_c = 0 \qquad (14.18)$$

where $(\ )_c$ indicates a central aberration.

It is seen that central coma is a linear function of X and Y, while central astigmatism is constant. As far as earlier results are concerned, S_I, C_L, and S_{IV} are independent of stop position, while $(C_T)_c$ is zero.

10. Stop Shift; Seidel Eccentricity Ratio, E

Figure 14.9 shows a lens (a) at the stop, and (b) with a

(ii) a single thin lens located at the stop with primary aberrations S_I, $(S_{II})_c$, $(S_{III})_c$, S_{IV}, $(S_V)_c = 0$, C_L, $(C_T)_c = 0$; now the stop is moved so that the eccentricity ratio at the lens is E and the non-central thin-lens aberrations become S_I, S_{II}, S_{III}, S_{IV}, S_V, C_L, C_T.

11. Stop Shift Formulae

Applying (14.21) to the single surface primary aberration formulae gives the results for the two types of system as:

$$\left.\begin{aligned} S_I^* &= S_I \\ S_{II}^* &= S_{II} + \delta E S_I \\ S_{III}^* &= S_{III} + 2\delta E S_{II} + \delta E^2 S_I \\ S_{IV}^* &= S_{IV} \\ S_V^* &= S_V + \delta E(3S_{III} + S_{IV}) + 3\delta E^2 S_{II} + \delta E^3 S_I \\ C_L^* &= C_L \\ C_T^* &= C_T + \delta E C_L \end{aligned}\right\} \quad (14.22)$$

and

$$\left.\begin{aligned} S_I &= S_I \\ S_{II} &= (S_{II})c + E S_I \\ S_{III} &= (S_{III})_c + 2E(S_{II})_c + E^2 S_I \\ S_{IV} &= S_{IV} \\ S_V &= E(3S_{III} + S_{IV}) + 3E^2 S_{II} + E^3 S_I \\ C_L &= C_L \\ C_T &= E C_L \end{aligned}\right\} \quad (14.23)$$

These powerful formulae enable us to calculate the effect of stop shift on any system and the noncentral aberrations of a thin lens. Applications are studied in Lectures 16 and 17. We can conclude that the noncentral versions of S_{II}, S_{III}, S_V all show parabolic dependences on shape and conjugate. It is possible to deduce various useful rules regarding the relative positions and sizes of the different parabolae.