Electromagnetic

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Electromagnetic (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied. Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion. It entails the analysis, synthesis, physical interpretation, and application of electric and magnetic fields.

EM principles find applications in various allied disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiber optics, electromagnetic interference and compatibility, electromechanical energy conversion, radar meteorology, and remote sensing.

EM devices include transformers, electric relays, radio/TV, telephone, electric motors, transmission lines, waveguides, antennas, optical fibers, radars, and lasers. The design of these devices requires thorough knowledge of the laws and principles of EM

VECTOR ANALYSIS

- A field is a function that specifies a quantity in space. For example, $A(x, y, z)$ is a vector field where as $V(x, y, z)$ is a scalar field.
- A vector A is uniquely specified by its magnitude and a unit vector along it, that is:
  $\mathbf{A} = A\mathbf{a}_A$
- Multiplying two vectors A and B results in either a scalar $A \cdot B = AB \cos \theta_{AB}$ or a vector $A \times B = AB \sin \theta_{AB}\, a_n$. Multiplying three vectors A, B, and C yields a scalar A • (B X C) or a vector A X (B X C).

- The scalar projection (or component) of vector A onto B is $A_B = A \cdot a_B$ whereas vector projection of A onto B is $A_B = A_B a_B$.

COORDINATE SYSTEMS AND TRANSFORMATION

- A point P is represented as $P(x, y, z)$, $P(\rho, \phi, z)$, and $P(r, \theta, \phi)$ in the Cartesian, cylindrical, and spherical systems respectively. A vector field A is represented as (A_x, A_y, A_z) or $A_x a_x + A_y a_y + A_z a_z$ in the Cartesian system, as (A_ρ, A_ϕ, A_z) or $A_\rho a_\rho + A_\phi a_\phi + A_z a_z$ in the cylindrical system, and as (A_r, A_θ, A_ϕ) or $A_r a_r + A_\theta a_\theta + A_\phi a_\phi$ in the spherical system.
- Fixing one space variable defines a surface; fixing two defines a line; fixing three defines a point.
- A unit normal vector to surface n = constant is $\pm a_n$.

VECTOR CALCULUS

- The differential displacements in the Cartesian, cylindrical, and spherical systems are respectively

$$dl = dx\, a_x + dy\, a_y + dz\, a_z$$
$$dl = d\rho a_\rho + d\varphi a_\varphi + dz\, a_z$$
$$dl = dr a_r + r\, d\Theta\, a_\Theta + r \sin\Theta\, d\varphi\, a_\varphi$$

- The differential normal areas in the three systems are respectively

$$dS = dydz\, a_x$$
$$dxdz\, a_y$$
$$dxdy\, a_z$$

$$dS = \rho\, d\varphi dz\, a_x$$
$$d\rho\, dz\, a_\varphi$$
$$\rho\, d\rho\, d\varphi\, a_z$$

$$dS = r^2 \sin\Theta\, d\Theta d\varphi\, a_r$$
$$r \sin\Theta\, dr\, d\varphi\, a_\Theta$$
$$r\, dr\, d\Theta\, a_\varphi$$

- The differential volumes in the three systems are

$$dv = dx\, dy\, dz$$
$$dv = \rho\, d\rho\, d\varphi\, dz$$
$$dv = r^2 \sin\varphi\, dr\, d\Theta\, d\varphi$$

- The line integral of vector A along a path L is given by $\int_L \boldsymbol{A.dl}$. If the path is closed, the line integral becomes the circulation of A around L; that is, $\S_L\, \boldsymbol{A.dl}$
- The flux or surface integral of a vector A across a surface S is defined as $\int_s \boldsymbol{A.ds}$. When the surface S is closed, the surface integral becomes the net outward flux of A across S; that is, $\S_s\, \boldsymbol{A.ds}$.
- The volume integral of a scalar ρ_v over a volume v is defined as $\int_v \rho_v\, dv$.
- Vector differentiation is performed using the vector differential operator ∇. The gradient of a scalar field V is denoted by ∇V, the divergence of a vector field A by $\nabla . A$, the curl of A by $\nabla \times A$, and the Laplacian of V by $\nabla^2 V$.
- The divergence theorem, ($\S_s\, \boldsymbol{A.ds} = \int_v \nabla . \boldsymbol{A}\, dv$, relates a surface integral over a closed surface to a volume integral.
- Stokes's theorem, $\S_L\, \boldsymbol{A.dl} = \int_s (\nabla \times \boldsymbol{A}) \bullet d\boldsymbol{S}$, relates a line integral over a closed path to a surface integral.
- If Laplace's equation, $\nabla^2 V = 0$, is satisfied by a scalar field V in a given region, V is said to be harmonic in that region.

ELECTROSTATIC FIELDS

- Coulomb's law of force states that:

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\,\mathbf{a}_R$$

- Based on Coulomb's law, we define the electric field intensity E as the force per unit charge; that is,

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o R^2}\,\mathbf{a}_R = \frac{Q\,\mathbf{R}}{4\pi\varepsilon R^3} \qquad \text{(point charge only)}$$

- For a continuous charge distribution, the total charge is given by

$$Q = \int \rho_L \, dl \qquad \text{for line charge}$$

$$Q = \int \rho_S \, dS \qquad \text{for surface charge}$$

$$Q = \int \rho_v \, dv \qquad \text{for volume charge}$$

- For an infinite line charge,

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o \rho}\,\mathbf{a}_\rho$$

For an infinite line charge,

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\,\mathbf{a}_n$$

- The electric flux density D is related to the electric field intensity (in free space) as

$$\mathbf{D} = \varepsilon_o \mathbf{E}$$

The electric flux through a surface *S* is

$$\Psi = \int_S \mathbf{D} \cdot d\mathbf{S}$$

- Gauss's law states that the net electric flux penetrating a closed surface is equal to the total charge enclosed, that is,

$$\Psi = Q_{enc}$$

$$\Psi = \oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc} = \int \rho_v \, dv$$

or

$$\rho_v = \nabla \cdot \mathbf{D} \qquad \text{(first Maxwell's equation)}$$

When charge distribution is symmetric so that a Gaussian surface (where D = $D_n\, a_n$ is constant) can be found, Gauss's law is useful in determining D; that is,

$$D_n \oint dS = Q_{enc} \qquad \text{or} \qquad D_n = \frac{Q_{enc}}{S}$$

- The total work done, or the electric potential energy, to move a point charge *Q* from point *A* to *B* in an electric field *E* is

$$W = -Q \int_A^B \mathbf{E} \cdot d\mathbf{l}$$

- The potential at r due to a point charge *Q* at r' is

$$V(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|} + C$$

- If the charge distribution is not known, but the field intensity E is given, we find the potential using

$$V = -\int \mathbf{E} \cdot d\mathbf{l} + C$$

- The potential difference V_{AB}, the potential at *B* with reference to *A*, is

$$V_{AB} = -\int_A^B \mathbf{E} \cdot d\mathbf{l} = \frac{W}{Q} = V_B - V_A$$

- Since an electrostatic field is conservative (the net work done along a closed path in a static E field is zero),

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

or

$$\nabla \times \mathbf{E} = 0 \qquad \text{(second Maxwell's equation}$$

- Given the potential field, the corresponding electric field is found using

$$\mathbf{E} = -\nabla V$$

- For an electric dipole centered at r' with dipole moment p, the potential at r is given by

$$V(\mathbf{r}) = \frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|^3}$$

- The electrostatic energy due to *n* point charges is

$$W_E = \frac{1}{2} \sum_{k=1}^{n} Q_k V_k$$

For a continuous volume charge distribution,

$$W_E = \frac{1}{2} \int_v \mathbf{D} \cdot \mathbf{E}\, dv = \frac{1}{2} \int \varepsilon_o |\mathbf{E}|^2\, dv$$

ELECTRIC FIELDS IN MATERIAL SPACE

- Materials can be classified roughly as conductors ($\sigma \gg 1$, $\epsilon_r = 1$) and dielectrics ($\sigma \ll 1$, $\epsilon_r \geq 1$).
- Electric current is the flux of electric current density through a surface; that is, $I = \int \mathbf{J} \cdot d\mathbf{S}$
- The resistance of a conductor of uniform cross section is

$$R = \frac{\ell}{\sigma S}$$

- The macroscopic effect of polarization on a given volume of a dielectric material is to "paint" its surface with a bound charge $Q_b = \oint_S \rho_{ps}\, dS$ and leave within it an accumulation of bound cha$Q_b = \int_v \rho_{pv}\, dv$ where $\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n$ and $\rho_{pv} = -\nabla \cdot \mathbf{P}$.
- In a dielectric medium, the D and E fields are related as $D = \epsilon E$, where $\varepsilon = \varepsilon_o \varepsilon_r$ is the permittivity of the medium.
- The electric susceptibility $\chi_e (= \varepsilon_r - 1)$ of a dielectric measures the sensitivity of the material to an electric field.
- A dielectric material is linear if D = ϵ E holds, that is, if ϵ is independent of E. It is homogeneous if ϵ is independent of position. It is isotropic if ϵ is a scalar.
- The principle of charge conservation, the basis of Kirchhoff's current law, is stated in the continuity equation

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho_v}{\partial t} = 0$$

- The relaxation time, $T_r = \varepsilon/\sigma$, of a material is the time taken by a charge placed in its interior to decrease by a factor of $\varepsilon^{-1} \simeq 37$ percent.
- Boundary conditions must be satisfied by an electric field existing in two different media separated by an interface. For a dielectric-dielectric interface

$$E_{1t} = E_{2t}$$

$$D_{1n} - D_{2n} = \rho_S \quad \text{or} \quad D_{1n} = D_{2n} \quad \text{if} \quad \rho_S = 0$$

For a dielectric-conductor interface,

$$E_t = 0 \qquad D_n = \varepsilon E_n = \rho_S$$

because E = 0 inside the conductor.

ELECTROSTATIC BOUNDARYVALUE PROBLEMS

- Boundary-value problems are those in which the potentials at the boundaries of a region are specified and we are to determine the potential field within the region. They are solved using Poisson's equation if $\rho_v \neq 0$ or Laplace's equation if $\rho_v = 0$.
- In a nonhomogeneous region, Poisson's equation is

$$\nabla \cdot \varepsilon \nabla V = -\rho_v$$

For a homogeneous region, ϵ is independent of space variables. Poisson's equation becomes:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

In a charge-free region ($p_v = 0$), Poisson's equation becomes Laplace's equation;
that is,

$$\nabla^2 V = 0$$

MAGNETOSTATICS

MAGNETOSTATIC FIELDS

- Biot-Savart's law, which is similar to Coulomb's law, states that the magnetic

field intensity *dH* at r due to current element I dI' at r' is

$$d\mathbf{H} = \frac{I\, d\mathbf{l} \times \mathbf{R}}{4\pi R^3} \qquad \text{(in A/m)}$$

where R = r — r' and R = |R|. For surface or volume current distribution, we replace I *dI* with K *dS* or J *dv* respectively; that is,

$$I\, d\mathbf{l} = \mathbf{K}\, dS = \mathbf{J}\, dv$$

- Ampere's circuit law, which is similar to Gauss's law, states that the circulation of H around a closed path is equal to the current enclosed by the path; that is,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{S}$$

or

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \text{(Third Maxwell's equation)}$$

When current distribution is symmetric so that an Amperian path (on which H = H_ϕ a_ϕ is constant) can be found, Ampere's law is useful in determining H; that is,

$$H_\phi \oint dl = I_{enc} \qquad \text{or} \qquad H_\phi = \frac{I_{enc}}{\ell}$$

- The magnetic flux through a surface *S* is given by

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S} \qquad \text{(in Wb)}$$

where B is the magnetic flux density in Wb/m^2. In free space,

$$\mathbf{B} = \mu_o \mathbf{H}$$

- Since an isolated or free magnetic monopole does not exist, the net magnetic flux through a closed surface is zero;

$$\Psi = \oint \mathbf{B} \cdot d\mathbf{S} = 0$$

Or $\nabla \cdot \mathbf{B} = 0$ (fourth Maxwell's equation)

- At this point, all four Maxwell's equations for static EM fields have been derived, namely:

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

- The magnetic scalar potential V_m is defined as

$$\mathbf{H} = -\nabla V_m \qquad \text{if } \mathbf{J} = 0$$

and the magnetic vector potential A as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

the magnetic flux through a surface *S* can be found from

$$\Psi = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

where L is the closed path defining surface S . Rather than using Biot-Savart's law, the magnetic field due to a current distribution may be found using A, a powerful approach that is particularly useful in antenna theory. For a current element $I\, dI$ at r', the magnetic vector potential at r is

$$\mathbf{A} = \int \frac{\mu_o I\, d\mathbf{l}}{4\pi R}, \qquad R = |\mathbf{r} - \mathbf{r}'|$$

- Corresponding to Poisson's equation $\nabla^2 V = -\rho_v/\varepsilon$, in electrostatics. Poisson's equation in magnetostatic is:

$$\nabla^2 \mathbf{A} = -\mu_o \mathbf{J}$$

MAGNETIC FORCES, MATERIALS, AND DEVICES

- The Lorentz force equation

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) = m\frac{d\mathbf{u}}{dt}$$

relates the force acting on a particle with charge Q in the presence of EM fields. It expresses the fundamental law relating EM to mechanics.

- Based on the Lorentz force law, the force experienced by a current element ***Idl*** in a magnetic field B is

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}$$

From this, the magnetic field B is defined as the force per unit current element.

- The torque on a current loop with magnetic moment m in a uniform magnetic field B is

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} = IS\mathbf{a}_n \times \mathbf{B}$$

- A magnetic dipole is a bar magnet or a small filament current loop; it is so called due to the fact that its B field lines are similar to the E field lines of an electric dipole.
- When a material is subjected to a magnetic field, it becomes magnetized. The magnetization M is the magnetic dipole moment per unit volume of the material. For linear material,

$$\mathbf{M} = \chi_m \mathbf{H}$$

- In terms of their magnetic properties, materials are either linear (diamagnetic or paramagnetic) or nonlinear (ferromagnetic). For linear materials,

$$\mathbf{B} = \mu\mathbf{H} = \mu_o\mu_r\mathbf{H} = \mu_o(1 + \chi_m)\mathbf{H} = \mu_o(\mathbf{H} + \mathbf{M})$$

where μ = permeability and $\mu r = \mu/\mu 0$ = relative permeability of the material. For nonlinear material, $B = \mu(H)\, H$, that is, μ does not have a fixed value; the relationship between B and H is usually represented by a magnetization curve.

- The boundary conditions that H or B must satisfy at the interface between two different media are

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K} \qquad \text{or} \qquad \mathbf{H}_{1t} = \mathbf{H}_{2t} \qquad \text{if } \mathbf{K} = 0$$

where a_{nl2} is a unit vector directed from medium 1 to medium 2.

- Energy in a magnetostatic field is given by

$$W_m = \frac{1}{2}\int \mathbf{B} \cdot \mathbf{H}\, dv$$

For an inductor carrying current I

$$W_m = \frac{1}{2} LI^2$$

Thus the inductance L can be found using

$$L = \frac{\int \mathbf{B} \cdot \mathbf{H}\, dv}{I^2}$$

- The inductance L of an inductor can also be determined from its basic definition: the ratio of the magnetic flux linkage to the current through the inductor, that is,

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

Thus by assuming current I, we determine B and $\Psi = \int \mathbf{B} \cdot d\mathbf{S}$, and finally find

$$L = N\Psi/I.$$

- The magnetic pressure (or force per unit surface area) on a piece of magnetic material is

$$P = \frac{F}{S} = \frac{1}{2} BH = \frac{B^2}{2\mu_o}$$

where B is the magnetic field at the surface of the material

WAVES

MAXWELL'S EQUATIONS

- Faraday's law states that the induced emf is given by *(N* = 1)

$$V_{emf} = -\frac{\partial \Psi}{\partial t}$$

For transformer emf, $V_{emf} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$

and for motional emf, $V_{emf} = \int (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$.

- The displacement current

$$I_d = \int \mathbf{J}_d \cdot d\mathbf{S}$$

where $\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$ (displacement current density), is a modification to Ampere's circuit law.

- In differential form, Maxwell's equations for dynamic fields are:

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

ELECTROMAGNETIC WAVE PROPAGATION

- The wave equation is of the form

$$\frac{\partial^2 \Phi}{\partial t^2} - u^2 \frac{\partial^2 \Phi}{\partial z^2} = 0$$

with the solution

$$\Phi = A \sin (\omega t - \beta z)$$

- In a lossy, charge-free medium, the wave equation based on Maxwell's equations is of the form

$$\nabla^2 \mathbf{A}_s - \gamma^2 \mathbf{A}_s = 0$$

where A_s is either E_s or H_s and $\gamma = \alpha + j\beta$ is the propagation constant. If we assume $E_s = E_{xs}(z)\, a_x$, we obtain EM waves of the form

$$\mathbf{E}(z, t) = E_o e^{-\alpha z} \cos (\omega t - \beta z)\, \mathbf{a}_x$$
$$\mathbf{H}(z, t) = H_o e^{-\alpha z} \cos (\omega t - \beta z - \theta_\eta)\, \mathbf{a}_y$$

- Wave propagation in other types of media can be derived from that for lossy media as special cases. For free space, set $\sigma = 0,\ \varepsilon = \varepsilon_o,\ \mu = \mu_o$; for lossless dielectric media, set $\sigma = 0,\ \varepsilon = \varepsilon_o \varepsilon_r,$ and $\mu = \mu_o \mu_r$; and for good conductors, set $\sigma \simeq \infty,\ \varepsilon = \varepsilon_o,$ $\mu = \mu_o,$ or $\sigma/\omega\varepsilon \to 0.$

- A medium is classified as lossy dielectric, lossless dielectric or good conductor depending on its loss tangent given by

$$\tan \theta = \frac{|\mathbf{J}_s|}{|\mathbf{J}_{d_s}|} = \frac{\sigma}{\omega \varepsilon} = \frac{\varepsilon''}{\varepsilon'}$$

where $\varepsilon_c = \varepsilon' - j\varepsilon''$ is the complex permittivity of the medium. For lossless dielectrics $\tan \theta \ll 1$, for good conductors $\tan \theta \gg 1$, and for lossy dielectrics tan θ is of the order of unity.

- In a good conductor, the fields tend to concentrate within the initial distance δ from the conductor surface. This phenomenon is called skin effect. For a conductor of width *w* and length ℓ the effective or ac resistance is

$$R_{ac} = \frac{\ell}{\sigma w \delta}$$

where δ is the skin depth.

- The Poynting vector, $\mathcal{P}$, is the power-flow vector whose direction is the same as the direction of wave propagation and magnitude the same as the amount of power flowing through a unit area normal to its direction.

$$\mathcal{P} = \mathbf{E} \times \mathbf{H}, \qquad \mathcal{P}_{ave} = 1/2 \text{ Re } (\mathbf{E}_s \times \mathbf{H}_s^*)$$

- If a plane wave is incident normally from medium 1 to medium 2, the reflection coefficient Γ and transmission coefficient *τ* are given by

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \qquad \tau = \frac{E_{to}}{E_{io}} = 1 + \Gamma$$

The standing wave ratio, *s*, is defined as

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

- For oblique incidence from lossless medium 1 to lossless medium 2, we have the

Fresnel coefficients as

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}, \qquad \tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

for parallel polarization and

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad \tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

for perpendicular polarization. As in optics,

$$\theta_r = \theta_i$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$$

Total transmission or no reflection (Γ = 0) occurs when the angle of incidence θ_i, is

equal to the Brewster angle.