Simulation of Aircraft Control Using Bond Graph Techniques and Matlab/Simulink Software

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Abstract

to develop the nonlinear transformer that is required for particular implementation of a complex aircraft control system. Aircraft longitudinal and lateral motions are presented by studying and analyzing the aircraft dynamics. The equations of motion are covered both forces and moments exerted on the aircraft using Newton's second law with the assumption that at each equation the aircraft is regarded as a rigid body. Six equations were classified, three equations for the longitudinal motion and three equations for the lateral motion. The matrix for the differential equations vectors for each force and moment were found with the aid of the computer aided modern bond graph program (CAMP-G) including symbolic manipulation. MATLAB/SIMULINK offered additional simulation capability.

Keywords: aircraft Control, Bond Graph, lateral motion, longitudinal motion, simulation

1. Introduction

When the aircraft rotates about an axis through the pilots feet in the forwards direction gives (rolling) or turn right or left, but when the aircraft rotates about a vertical axis through its body gives (yawing), or when topples forwards or backwards about an axis through its feet it will be going sideways (pitching)[1]. These are three degrees of freedom of

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rotational motion which when combined with the three degrees of translational freedom produce the complete set of freedoms. Any motion of any complete body can be resolved into these six degrees of freedom.

To control and steer the aircraft, destabilizes must be fitted and these, for present purposes, can be considered to take the form of hinged flaps which effectively cause moments about the principal axis and hence persuade the aircraft into the required flight path. These primary controls are capable of movement relative to the aircraft as a whole and thus, in a way, the concept of rigidity is lost. This will enable certain fundamentals to be studied without the introduction of too many complications. The first assumption, then, is that the aircraft can be treated as a rigid body subject to six degrees of freedom [1].

2. Aircraft Axes
The set of three mutually perpendicular axes of a body are so complicated geometrically as an airplane can be defined in almost any way relative to the body, but it is usual to restrict the number of three axes. These three are sufficient to produce convenient systems for most problems.

They all have as origin (O) the center of gravity of the aircraft (CG) and all take a roughly forward, sideway (to the right), and downward direction as the three positive axes of linear motion OX, OY, and OZ respectively. When the aircraft is in normal motion they are usually called body axes [1] figure (1).

3. Aircraft Control System
The motion of the airplane is described by a set of Euler's equations for translation and rotation. Bond graphs can represent rigid body motion so that rigid body airplane can be modeled with this technique [2]. The Bond Graphs technique is an explicit graphical tool for capturing the common energy structure of systems. It increases one's insight into systems behavior.

4. Simulink
The Aerospace Block set brings the full power SIMULINK to aerospace system design, integration, and simulation by providing key aerospace subsystems and components in the adaptable block format. From environmental models to equations of motion, from gain scheduling to animation, the block set gives the core components to rapidly and efficiently assemble a broad range of large aerospace system architectures. One can use the aerospace block set together with real-time workshop to automatically generate code for real-time execution in rapid prototyping and for hardware-in-the-loop systems.

5. The Airplane as a Rigid Body
The airplane is considered as a rigid body flying through the airspace, moves along paths that are determined...
by the airplanes inertia characteristics, the attraction from the earth's gravitational field, the propulsive force generated by power plant, and aerodynamic force and moments created on it because of the reaction between it and these through which it moves [3].

The forces and moments created on the airplane are function of the velocity of the airplane, the density through which it flies the geometry of the airplane and finally the angle that the relative wind makes with the airplane (angle of attack). The paths along which the airplane can fly in the airspace are limited only by the aerodynamic characteristics of the airplane, its propulsive system and the structural strength of the air frame. These limitations indicate the maximum performance and maneuverability of the airplane. If the airplane is to realize maximum utility, it must be safely controllable by the pilot to these limits without exceeding its strength and without requiring acrobatic ability.

6. Aircraft Motion

The motion of an aircraft is particularly complex because the rotations and translations are coupled to gather; a rotation affects the magnitude and direction of the forces which affect translations. Objects move in two ways. An object translates, or changes location, from one point to another and an object rotates, or changes its attitude. In general, the motion of any object involves both translation and rotation. The translations are in direct response to external forces. The rotations are in direct response to external torques or moments [4]. During the flight, any aircraft will rotate about its center of gravity, a point which is the average location of the mass of the aircraft. The aircraft flies in all directions. A three dimensional world coordinate system through the center of gravity are mutually perpendicular. The orientation of the aircraft is measured by the amount of rotation of the parts of the aircraft along these principal axes.

1: The yaw axis is perpendicular to the plane of the wings.
2: The pitch axis is perpendicular to the yaw axis and lies in the plane of the wings.
3: The roll axis is perpendicular to the other two axes and lies in the fuselage of the aircraft pointing towards the nose [5].

7. Aircraft Equation of Motion

The aircraft during flight is considered as a dynamic system, depending on the velocity – change maneuvers during unsteady flight, these or the equations of its motion have considerable values during the analyses of stability or control. The system of these equations is so complex, that it might be simple by some assumption to enable the designers to investigate the type of stability and response. The solutions of these equations are either exact solution, which depends on the level of the study and type of the aircraft and its flight [6].

In general the summation of the external forces may be used according to Newton second law and the equations of linear motion are obtained as follows [7]:

...
\[ \sum \Delta F_x = m (U + WQ - VR) \]  \hspace{1cm} (1) \\
\[ \sum \Delta F_y = m (V' + UR - WP) \]  \hspace{1cm} (2) \\
\[ \sum \Delta F_z = m (W' + VP - UQ) \]  \hspace{1cm} (3)

The summation of the external moments acting on a body must be equal to the time rate of change of its moment of momentum (angular momentum). The moment of aircraft can be classified to three types rolling, pitching and yawing moment as follows [7]:

\[ \sum \Delta L = P' I_x - R' J_{xz} + QR (I_z - I_y) - PQ J_{xz} \]  \hspace{1cm} (4)

\[ \sum \Delta M = Q' I_y + PR (I_x - I_z) + (P' - R') J_{xz} \]  \hspace{1cm} (5)

\[ \sum \Delta N = R' I_z - P' J_{xz} + PQ (I_y - I_x) + QR J_{xz}. \]  \hspace{1cm} (6)

8. Longitudinal Control

The six equations are firstly broken up into two sets of three simultaneous equations. To accomplish this the aircraft is considered to be in straight and level flight and then to be disturbed by deflection of the elevator. This deflection applies a pitching moment about the OY axis, causing a rotation about this axis which eventually causes a change in Fx and Fz, but does not cause a rolling or yawing moment or any change in Fy; thus \( P = R = V = 0 \).

The longitudinal equations of motion for the aircraft according to [8] are as follows:

\[ v (u / v) = v_x \alpha + \frac{z_q}{\sin \theta} \dot{\theta} + \frac{z_x}{\sin \theta} \dot{\theta} \]  \hspace{1cm} (7)

\[ \alpha = v_z \alpha + \frac{z_q}{\sin \theta} \dot{\theta} + \frac{z_x}{\sin \theta} \dot{\theta} \]  \hspace{1cm} (8)

\[ -M \alpha + q = M_q \alpha + M_\alpha \alpha + M_q q + M_\delta \dot{\delta} \]  \hspace{1cm} (9)

\[ \theta = q \]

9. Lateral Control

From rolling equation, Yawing equation and Force in Y axis equation, the lateral modes can be determined and the various transfer functions for both rudder and aileron input can be derived and analyzed.

The lateral equations of motion as given by [8] are as follows:

\[ v \dot{\theta} = Y_\beta \dot{\beta} + Y_\psi p + g \cos \theta \phi + (Y_r - v) r + Y_\delta \dot{\delta} \]  \hspace{1cm} (10)

\[ P - (I_{xz} / I_x) r = L_\beta \dot{\beta} + L_\psi P + L_\theta \theta + L_\delta \dot{\delta} \]  \hspace{1cm} (11)

\[ -(I_{xz} / I_x) P + r = N_\beta \dot{\beta} + N_\psi P + N_\theta \theta + N_\delta \dot{\delta} \]  \hspace{1cm} (12)

\[ \dot{\phi} = p \]

10 Aircraft Models

The aircraft L 39 is selected as the model being investigated in the present study. The aircraft suggested flying in straight and level flight at 12000m with a velocity of 180m per sec (355knots), and compressibility effects will be neglected. L 39 Aircraft characteristics are as given in tables (3) as given in [7].
characteristics given in table (3). The longitudinal and lateral stability derivatives are given in tables (1 and 2) were used to test the aircraft transfer function.

From longitudinal state equations two modes of oscillations are obtained, short period and phugoid.

From lateral state equations three modes of oscillations are obtained, the Dutch roll, Roll Subsidence and Spiral divergence.

11. Bond Graph Techniques

Bond graph techniques gave the nodes from bond pad area and draw these nodes to analyze the differential equations used to simulate the aircraft dynamic system.

Figure (2) shows the Bond graph nodes which form the equations of motion of the aircraft in state space, three of them are for forces and another three are for moments. Figure (3.a) shows the input forces acting on the aircraft.

Figure (3.b) shows the input moments acting on the aircraft.

As has been seen the Bond graph technique gave a compact graphical representation for modeling. It forms the equation of motion of the rigid body in the symbol figure. The results obtained for the model are very close to reality of the prototype and the simulation has been done quickly and precisely as found in [9].

12. Matlab/Simulink

Equations (2, 3, 4, 8, 9 and 10) are settled as setting expressions for the model. Therefore one can call the S-function wizard to generate a MATLAB S-function block from the current document. SYMBOLS-2000 produces a MATLAB Library (mdl) file that contains links to the (dll) SYMBOLS-2000 file internally. The following steps involved in creation of MATLAB S-function block from a Bond Graph model, i.e. Bond Graph and MATLAB Software Structure Figure (1) as [10].

Graph model, i.e. Bond Graph and MATLAB Software Structure.

13 Results and Discussion

Figure (4) shows the input forces and moments acting upon the aircraft. This figure represents the input of the configurable subsystem blocks for the rolling, yawing and pitching moments, constant blocks for forces in (X, Y and Z) axes and slider gain blocks for the forces and moments. Figure (5) shows the output response of the body axes (forces output).

Figure (6) represent the velocity, position and Euler's angles (pitch, roll and yaw angles).

Figure (7) shows the input forces (Fx) and (Fz) and pitching moment, acting upon the aircraft. Figure (8) shows the aircraft rotational rates and velocity

Figure (9) shows the output response of the longitudinal motion.
Figure (10) shows the input force (Fy) and moments (Mx) and (Mz) which represents the aircraft rolling and yawing moments. Figure (11) represent the body axes (forceoutput). While Figure (12) represents the inertia axes (momentoutput).

13. Conclusions

The major observations and conclusions from this study are the MATLAB / SIMULINK and Symbol 2000 program software were used successfully as coupled strategy to predict the aircraft dynamics response by substituting the data of the aircraft in the dialog box parameterize for aerospace block set. The responses founded from the aerospace blocks / SIMULINK in all cases of motion for the aircraft such as longitudinal and lateral motions. This technique, successfully estimate the body axes response (velocity, and rotational rates), inertia axes (velocity, position, and Euler’s angle).

14. Symbols

- $C_D$: Drag Coefficient.
- $C_L$: Lift Coefficient.
- $Cm\delta_e$: Elevator Moment Derivative Due to Elevator Deflection.
- $Cm\alpha$: Longitudinal Static Stability Derivative.
- $Cm\eta$: Pitching Moment Derivative Due to Rate of Pitch.
- $Cm\alpha'$: Change in Angle of Attack.
- $Cx$: Aerodynamic Force Coefficient In X Direction.
- $Cx\eta$: X-Force Derivative Due to Rate of Pitch.

15. References

[2] Jose J. Granda, Raymond, C. Montgomery, "Automated Modeling and Simulation using the Bond Graph Method for the
Simulation of Aircraft Control Using Bond Graph Techniques and Matlab/Simulink Software

Table (1) Aerodynamic Coefficient for longitudinal stability [7].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Origin</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cxu</td>
<td>(U/Sq) ( \frac{\partial Fx}{\partial u} )</td>
<td>Variation of drag and thrust with ( u )</td>
<td>2CD-U( \frac{\partial CD}{\partial u} )</td>
</tr>
<tr>
<td>Cxa</td>
<td>(1/Sq) ( \frac{\partial Fx}{\partial \alpha} )</td>
<td>Lift and drag variation along the X axis</td>
<td></td>
</tr>
<tr>
<td>CL-CD</td>
<td>( \partial CD/\partial \alpha )</td>
<td>Slope of the normal force curve</td>
<td></td>
</tr>
<tr>
<td>Cw</td>
<td>- ( \frac{mg}{Sq} )</td>
<td>Gravity</td>
<td>-</td>
</tr>
<tr>
<td>Cx'</td>
<td>(1/Sq)(2U/c) ( \partial Fx/\partial \alpha )</td>
<td>Downwash lag on Neglect ( \partial Fx/\partial \alpha ) drag</td>
<td></td>
</tr>
<tr>
<td>Cz'</td>
<td>(1/Sq)(2U/c) ( \partial Fz/\partial \alpha )</td>
<td>Slope of the normal force curve</td>
<td></td>
</tr>
<tr>
<td>CzU</td>
<td>(1/Sq)(2U/c) ( \partial Fz/\partial u )</td>
<td>Variation of normal force</td>
<td></td>
</tr>
<tr>
<td>Cz</td>
<td>(1/Sq)(2U/c) ( \partial Fz/\partial \alpha )</td>
<td>Variation along normal force curve</td>
<td></td>
</tr>
<tr>
<td>Cmm</td>
<td>(1/Sq)(2U/c) ( \partial M/\partial \alpha )</td>
<td>Downwash lag on moment ( d\epsilon/du )</td>
<td></td>
</tr>
<tr>
<td>Cmq</td>
<td>(1/Sq)(2U/c) ( \partial M/\partial \theta )</td>
<td>Damping in pitch downwash lag ( 2K\epsilon M/\epsilon \theta )</td>
<td></td>
</tr>
<tr>
<td>Cmu</td>
<td>(U/Sqc) ( \partial M/\partial u )</td>
<td>Effect of thrust, and neglect for slipstream, and slipstream, and usually neglected for jets</td>
<td></td>
</tr>
<tr>
<td>Cm</td>
<td>(1/Sqc) ( \partial M/\partial \alpha )</td>
<td>Static longitudinal stability</td>
<td></td>
</tr>
<tr>
<td>Mj</td>
<td>(dCL/du)</td>
<td>Downwash lag ( d\epsilon M/\epsilon \theta ) on moment ( (8/c) )</td>
<td></td>
</tr>
<tr>
<td>Cmq'</td>
<td>(1/Sqc)(2U/c) ( \partial M/\partial \alpha )</td>
<td>Damping in pitch downwash lag ( 2K\epsilon M/\epsilon \theta ) on moment ( (8/c) )</td>
<td></td>
</tr>
</tbody>
</table>

Table (2) Aerodynamic Coefficient for lateral stability [7]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Origin</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLβ</td>
<td>(1/Sqb) ( \partial L/\partial \beta )</td>
<td>Dihedral and Vertical tail</td>
<td>Ref.Ch 9</td>
</tr>
<tr>
<td>CLp</td>
<td>(1/Sqb)(2U/b) ( \partial L/\partial p )</td>
<td>Wing damping</td>
<td>Ref.1, Ch 9</td>
</tr>
<tr>
<td>CLr</td>
<td>(1/Sqb)(2U/b) ( \partial L/\partial r )</td>
<td>Differential wing normal force</td>
<td>CL*w/4</td>
</tr>
<tr>
<td>Cnβ</td>
<td>(1/Sqb) ( \partial N/\partial \beta )</td>
<td>Directional stability</td>
<td>Ref.1, Ch 8</td>
</tr>
<tr>
<td>Cnp</td>
<td>(1/Sqp)(2U/b) ( \partial N/\partial P )</td>
<td>Differential wing Chord force</td>
<td>-CL*w/8</td>
</tr>
<tr>
<td>Cnr</td>
<td>(1/Sqb)(2U/b) ( \partial N/\partial r )</td>
<td>Damping in yaw (CD<em>w/4)( \eta (Sv/S) ) (Lv/b) (dCL/dα)</em>( v )</td>
<td></td>
</tr>
<tr>
<td>Cyβ</td>
<td>(1/Sq) ( \partial Fy/\partial \beta )</td>
<td>Fuselage and vertical tail no eqs</td>
<td></td>
</tr>
<tr>
<td>CyØ</td>
<td>(1/Sq) ( \partial Fy/\partial \varnothing )</td>
<td>Gravity mg/Sq (cos ( \theta ))</td>
<td></td>
</tr>
<tr>
<td>Cyp</td>
<td>(1/Sq)(2U/b) ( \partial Fy/\partial p )</td>
<td>Vertical tail Neglect</td>
<td></td>
</tr>
<tr>
<td>Cyr</td>
<td>(1/Sq)(2U/b) ( \partial Fy/\partial \varrho )</td>
<td>Gravity mg/Sq (sin ( \theta ))</td>
<td></td>
</tr>
</tbody>
</table>

Tables (3) L 39 Aircraft characteristics [7]

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The mass of the aircraft.</td>
<td>4300 kg</td>
</tr>
<tr>
<td>The speed of the aircraft.</td>
<td>253 m/sec</td>
</tr>
<tr>
<td>The density of the air at standard condition</td>
<td>( kg/m^3 ) 1.225</td>
</tr>
<tr>
<td>The mean aerodynamic chord.</td>
<td>m 2.15</td>
</tr>
<tr>
<td>The wing area.</td>
<td>m(^2) 18.8</td>
</tr>
<tr>
<td>Moment of inertia in right wing (pitch).</td>
<td>( kg.m^3 ) 13018</td>
</tr>
<tr>
<td>Lift coefficient.</td>
<td>0.3</td>
</tr>
<tr>
<td>Drag coefficient.</td>
<td>0.0294</td>
</tr>
<tr>
<td>Neutral point position or aerodynamic</td>
<td>m 0.36</td>
</tr>
<tr>
<td>Center of gravity position.</td>
<td>0.304</td>
</tr>
</tbody>
</table>
Figure (1) Aircraft axis of rotation

Figure (2) Forces and Moments applied on the Aircraft (Pad Area)
Figure (3) Forces and Moments applied on Aircraft, form in Bond Pad Area
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Figure (4)
Six Degrees of Freedom Motion Simulation for L39 Aircraft

Figure (5) Forces Response for Aerospace Block of L39 Aircraft
Figure (6)  Moment Response for Aerospace Block
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Figure (7)
Six Degrees of Freedom Motion Simulation Longitudinal Control for L39

Figure (8) Forces Response for Aerospace Block
Figure (9) Moments response for Aerospace Block of L39 Aircraft
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Figure (10)
Six Degrees of Freedom Motion Simulation Lateral Control for L39 Aircraft

Figure (11) Forces response for Aerospace Block of L39 Aircraft
Figure (12) Moments response for Aerospace Block of L39 Aircraft