Chapter Six

Non-Newtonian Liquid
For many fluids a plot of shear stress against shear rate does not give a straight line. These are so-called “Non-Newtonian Fluids”. Plots of shear stress against shear rate are experimentally determined using viscometer. The term viscosity has no meaning for a non-Newtonian fluid unless it is related to a particular shear rate. An *apparent viscosity* ($\mu_a$) can be defined as follows:

$$\mu_a = \frac{\tau}{\gamma}$$
Types of Non-Newtonian Fluids

There are two types of non-Newtonian fluids: -

- Time-independent.
- Time-dependent.

- **Time-independent liquid**
  - In this type of liquid, the current value of the rate of shear at a point in the fluid is determined only by the corresponding current value of the shear stress and vice versa.
  - Conversely, one can say that such fluids have no memory of their past history. i.e., the apparent viscosity depends only on the rate of shear at any particular moment and not on the time for which the shear rate is applied.
  - The relationship between shear stress and shear rate is more complex and this type can be written as: -
    \[ \tau = k (-\dot{\gamma})^n \]
There are three types of it (as shown in figure):

1. Shear-thinning or pseudoplastic behavior. The apparent viscosity of the fluid decreases with an increase in shear rate. \([\text{power-law } n < 1]\) Ex. Polymer solution, detergent.

2. Visco-plastic behavior with or without shear-thinning behavior. It required \(\tau^o\) for initial flow. Ex. Chocolate mixture, soap, sewage sludge, toothpaste.

3. Shear-thickening or dilatant behavior. The apparent viscosity of the fluid increases with an increase in shear rate. \([\text{power-law } n > 1]\) Ex. Wet beach sand, starch in water.
• **Time-Dependent Non-Newtonian Fluids**
  
  For this type, the curves of shear stress versus shear rate depend on how long the shear has been active. This type is classified into:

  1. **Thixotropic Fluids**
     
     Which exhibit a reversible decrease in shear stress and apparent viscosity with time at a constant shear rate. Ex. Paints.

  2. **Rheopectic Fluids**
     
     Which exhibit a reversible increase in shear stress and apparent viscosity with time at a constant shear rate. Ex. Gypsum suspensions, bentonite clay.
With respect to non-newtonian fluids, we need to answer the following questions:

• What is the pressure drop across a pipe?
• What is the average velocity?
• What is the velocity profile?
• How can I determine the friction factor for these fluids?
Flow Characteristic [8u/d] for Newtonian fluid of laminar flow through a circular pipe

\[ \tau_w = \frac{\Delta P}{\frac{4L}{d}} = \mu \frac{8u}{d} \]

For non-newtonian time independent fluid

\[ \tau_w = \frac{\Delta P}{\frac{4L}{d}} = K_p \left( \frac{8u}{d} \right)^{n'} \]

Or

\[ \tau_w = \frac{\Delta P}{\frac{4L}{d}} = (\mu_a)_P \left( \frac{8u}{d} \right) \]

where, \( K_p' \) and \( n' \) are point values for a particular value of the flow characteristic (8u/d), \((\mu a)_P\) is apparent viscosity for pipe flow.

This equation gives a point value for the apparent viscosity of non-Newtonian fluid flow through a pipe.
Reynolds number for the of non-Newtonian fluids can be written as

\[ Re = \frac{\rho u^{2-n'}d^{n'}}{m} \]

These equations give a point value for \( Re \) at a particular flow characteristic \((\delta u / d)\).

**Pressure Drop**

The pressure drop due to skin friction can be calculated in the same way as for Newtonian fluids,

\[ -\Delta P_{fs} = 4f (L/d) (\rho u^2 / 2) \]

This is used for laminar and turbulent flow.

**Friction Factor**

A point value of the basic friction factor \((\Phi \text{ or } J_f)\) or fanning friction factor \((f)\) for laminar flow can be obtained from:

\[ \Phi = J_f = 8 / Re \text{ or } f = 16 / Re \]
the fanning friction factor ($f$) for turbulent flow of general time independent non-Newtonian fluids in smooth cylindrical pipes can be calculated from:

$$f = \frac{a}{Re} = \frac{4}{(n')^{0.75}} \log[Re f^{(1-n')/2}] - \frac{0.4}{(n')^{1.2}}$$

where, $a$, and $b$ are function of the flow behavior index ($n'$).

<table>
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<th>$n'$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
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<td>0.307</td>
<td>0.281</td>
<td>0.263</td>
<td>0.25</td>
<td>0.231</td>
<td>0.213</td>
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</table>

There is another equation to calculate ($f$) for turbulent flow of time-independent non-Newtonian fluids in smooth cylindrical pipes:

$$\frac{1}{f^{1/2}} = \frac{4}{(n')^{0.75}} \log[Re f^{(1-n')/2}] - \frac{0.4}{(n')^{1.2}}$$
Flow of Power-Law Fluids in Pipes

Power-law fluids are those in which the shear stress ($\tau$) is related to the shear rate ($\dot{\gamma}$) by this equation:

$$\tau = k(\dot{\gamma})^n$$

for power-law fluids the parameters $K_p'$ and $n'$ are no longer point values but remain constant over a range of $(8u/d)$, so that for power-law fluids the shear stress at wall can be written as:

$$\tau_w = \frac{\Delta P}{4L/d} = K_p\left(\frac{8u}{d}\right)^n$$

where,

- $K_p$: is the consistency coefficient for pipe flow.
- $n$: is the power-law index.
The shear rate at pipe wall for general time-independent non-Newtonian fluids is:

\[ \dot{\gamma}_w = \frac{8u}{d} \left( \frac{3n' + 1}{4n'} \right) \]

and for power-law fluids:

\[ \dot{\gamma}_w = \frac{8u}{d} \left( \frac{3n + 1}{4n} \right) \]

the relationship between the general consistency coefficient (K) and the consistency coefficient for pipe flow (K_p).

\[ K_p = \frac{8u}{d} \left( \frac{3n + 1}{4n} \right)^n \]

The apparent viscosity for power-law fluids in pipe flow:

\[ (\mu_a) = K_p \left( \frac{8u}{d} \right)^{n-1} \]

The Reynolds number for non-Newtonian fluids flow in pipe:

\[ Re = \frac{\rho ud}{(\mu_a)_p} \]

For power-law fluids flow in pipes the Re can be written either as:

\[ Re = \frac{\rho ud}{K_p \left( \frac{8u}{d} \right)^{n-1}} \]
Laminar flow of power-law non-newtonian liquid

\[ \Delta P = \frac{K'4L}{D} \left( \frac{8V}{D} \right)^{n'} = 4f \rho \frac{L}{D} \frac{V^2}{2g_c} \]

Pressure drop across pipe

\[ V = \frac{D}{8} \left( \frac{\Delta PD}{K'4L} \right)^{n'} \]

Average velocity in pipe

\[ f = \frac{16}{N_{Ra_{gen}}} \]

Friction factor

\[ N_{Ra_{gen}} = \frac{D^{n'}V^{2-n'} \rho}{K'8^{n'-1}} \]

Reynolds number in pipe

\[ v_x = v_{x,max} \left[ 1 - \left( \frac{r}{R_o} \right)^{(n+1)/n} \right] \]

Velocity profile in pipe

\[ n' = n \]
Friction Losses Due to Form Friction in Laminar Flow

Kinetic Energy in Laminar Flow
Average kinetic energy per unit mass = \( \frac{u^2}{2\alpha} \) [m^2/s^2 or J/kg]
\( \alpha = 1.0 \) \( \text{in turbulent flow} \)
\( \alpha = \frac{(2n + 1)(5n + 3)}{3(3n + 1)^2} \) \( \text{in laminar flow} \)
- For Newtonian fluids (n = 1.0) \( \Rightarrow \alpha = \frac{1}{2} \) in laminar flow
- For power-law non-Newtonian fluids (n < 1.0 or n > 1.0)

Losses in Contraction and Fittings
The frictional pressure losses for non-Newtonian fluids are very similar to those for Newtonian fluids at the same generalized Reynolds number in laminar and turbulent flow for contractions and also for fittings and valves.

Losses in Sudden Expansion
For a non-Newtonian power-law fluid flow in laminar flow through a sudden expansion from a smaller inside diameter \( d_1 \) to a larger inside diameter \( d_2 \) of circular cross-sectional area, then the energy losses is

\[
F_e = \left[ \frac{n + 3}{2(5n + 3)} \left( \frac{d_1}{d_2} \right)^4 - \left( \frac{d_1}{d_2} \right)^4 + \frac{3(3n + 1)}{2(5n + 3)} \right] \frac{3n + 1}{2n + 1} \alpha_1^2
\]
Friction Factor of power-law non-newtonian liquid

The fanning friction factor is plotted versus the generalized Reynolds number. Since many non-Newtonian power-law fluids have high effective viscosities, they are often in laminar flow. The correction for smooth tube also holds for a rough pipe in laminar flow.

For rough pipes with various values of roughness ratio (e/d), this figure cannot be used for turbulent flow, since it is derived for smooth pipes.
Turbulent Flow

Turbulent flow modeling for liquids is very difficult. Turbulence creates turbulent eddies which are difficult to predict and model. The velocity distributions for turbulent flow are a lot flatter than for laminar flow.
Example 1
A general time-independent non-Newtonian liquid of density 961 kg/m³ flows steadily with an average velocity of 1.523 m/s through a tube 3.048 m long with an inside diameter of 0.0762 m. For these conditions, the pipe flow consistency coefficient $K_p'$ has a value of 1.48 Pa·s⁰.³ [or 1.48 (kg / m·s²) s⁰.³] and $n'$ a value of 0.3. Calculate the values of the apparent viscosity for pipe flow ($\mu_a$)P, the Reynolds number Re and the pressure drop across the tube, neglecting end effects.

Example 2
A Power-law liquid of density 961 kg/m³ flows in steady state with an average velocity of 1.523 m/s through a tube 2.67 m length with an inside diameter of 0.0762 m. For a pipe consistency coefficient of 4.46 Pa·sⁿ [or 4.46 (kg / m·s²) s⁰.³], calculate the values of the apparent viscosity for pipe flow ($\mu_a$)P in Pa·s, the Reynolds number Re, and the pressure drop across the tube for power-law indices $n = 0.3$, 1.0, and 1.5 respectively.

Example 3
A pseudoplastic fluid that follows the power-law, having a density of 961 kg/m³ is flowing in steady state through a smooth circular tube having an inside diameter of 0.0508 m at an average velocity of 6.1 m/s. the flow properties of the fluid are $n' = 0.3$, $K_p = 2.744$ Pa·sⁿ. Calculate the frictional pressure drop across the tubing of 30.5 m long.