5-1 INTRODUCTION

We now wish to examine the methods of calculating convection heat transfer and, in particular, the ways of predicting the value of the convection heat-transfer coefficient $h$. Our discussion in this chapter will
- first consider some of the simple relations of fluid dynamics and boundary layer analysis that are important for a basic understanding of convection heat transfer.

- Next, we shall impose an energy balance on the flow system and determine the influence of the flow on the temperature gradients in the fluid.

- Finally, having obtained a knowledge of the temperature distribution, the heat-transfer rate from a heated surface to a fluid that is forced over it may be determined.

Our development in this chapter is
- primarily analytical in character and is concerned only with forced-convection flow systems.
- Subsequent chapters will present empirical relations for calculating forced-convection heat transfer and
- will also treat the subjects of natural convection.

5-2 VISCOS FLOW

Consider the flow over a flat plate with different temperature as shown in Figures 5-1 and 5-2. Beginning at the leading edge of the plate, a region develops where the influence of viscous forces is felt.
These viscous forces are described in terms of a shear stress $\tau$ between the fluid layers.

$$\tau = \mu \frac{du}{dy}$$  \[5-1\]

$\mu$ is called the dynamic viscosity (Newton-seconds per square meter).

The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the boundary layer.
At the position, where the velocity becomes 99 percent of the free-stream value, the boundary layer ends.

The flow can be classified in the boundary layer to

- Initially laminar flow
- but at some critical distance from the leading edge, depending on the flow field and fluid properties, small disturbances in the
- flow begin to become amplified, and a transition process takes place
- until the flow becomes
- turbulent.

The transition from laminar to turbulent flow occurs when

$$\frac{u_\infty x}{\nu} = \frac{\rho u_\infty x}{\mu} > 5 \times 10^5$$

at flow on flat plate.

where

- $u_\infty$ = free-stream velocity, m/s
- $x$ = distance from leading edge, m
- $\nu = \frac{\mu}{\rho}$ = kinematic viscosity, m$^2$/s

This particular grouping of terms is called the Reynolds number, and is dimensionless if a consistent set of units is used for all the properties:

$$Re_x = \frac{u_\infty x}{\nu}$$  \hspace{1cm} [5-2]

The relative shapes for the velocity profiles in laminar and turbulent flow are indicated in Figure 5-1. The laminar profile is approximately parabolic, while the turbulent profile has a portion near the wall that is very nearly linear. This linear portion is said to be due to a laminar sublayer that hugs the surface very closely. Outside this sublayer the velocity profile is relatively flat in comparison with the laminar profile.
**** Consider the flow in a tube as shown in Figure 5-3. A boundary layer develops at the entrance, as shown. Eventually the boundary layer fills the entire tube, and the flow is said to be fully developed.

If the flow is laminar, a parabolic velocity profile is experienced, as shown in Figure 5-3a. When the flow is turbulent, a somewhat blunter profile is observed, as in Figure 5-3b.

In a tube, the Reynolds number is again used as a criterion for laminar and turbulent flow. For

\[ \text{Re}_d = \frac{u_m d}{\nu} > 2300 \quad [5-3] \]

- \( d \) is the tube diameter.

The continuity relation for one-dimensional flow in a tube is

\[ \dot{m} = \rho u_m A \quad [5-4] \]

Where \( \dot{m} \) = mass rate of flow, \( u_m \) = mean velocity and \( A \) = cross-sectional area
Consider a fluid flow over a flat plate with different temperatures (Fig 5-1)

\[ q = -kA \ \frac{\partial T}{\partial x} = hA (T - T_\infty) \]

since \( T \) depends on velocity of the stream

\[ h = f(\text{fluid, flow pattern}) \]

We term the heat transfer depends on relative motions as **convection heat transfer**.

The problem is how to evaluate/predict/estimate the value of \( h \) for various flow pattern?

**Evaluation of convection heat transfer**

1. Analytical solution of the fluid temperature distribution
2. Analogy between heat & momentum transfer
3. Dimensional analysis + experimental data in terms of dimensionless No.

**5-4 LAMINAR BOUNDARY LAYER ON A FLAT PLATE**

Consider the elemental control volume shown in Figure 5-4. We derive the equation of motion for the boundary layer by making a force-and-momentum balance on this element. To simplify the analysis we assume:

Assumptions: 1 incompressible, steady flow, 2. \( dP/\ dy = 0 \), 3. constant physical properties.

![Elemental control volume for force balance on laminar boundary layer.](image)

For this system the force balance is then written

**\[ \sum F_x = \text{increase in momentum flux in x direction} \]**

The momentum flux in the x direction is the product of the mass flow through a particular side of the control volume and the x component of velocity at that point. The mass entering the left face of the element per unit time is \( \rho u \ dy \)
if we assume unit depth in the z direction. Thus the momentum flux entering the left face per unit time is

\[ \rho u \ dy \ u = \rho u^2 \ dy \]

The mass flow leaving the right face is

\[ \rho \left( u + \frac{\partial u}{\partial x} \ dx \right) \ dy \]

and the momentum flux leaving the right face is

\[ \rho \left( u + \frac{\partial u}{\partial x} \ dx \right)^2 \ dy \]

The mass flow entering the bottom face is

\[ \rho v \ dx \]

and the mass flow leaving the top face is

\[ \rho \left( v + \frac{\partial v}{\partial y} \ dy \right) \ dx \]

A mass balance on the element yields:

\[ \rho u \ dy + \rho v \ dx = \rho \left( u + \frac{\partial u}{\partial x} \ dx \right) \ dy + \rho \left( v + \frac{\partial v}{\partial y} \ dy \right) \ dx \]

or

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \[5-12\]

This is the mass continuity equation for the boundary layer.

Returning to the momentum-and-force analysis, the momentum flux in the x direction that enters the bottom face is

\[ \rho vu \ dx \]

and the momentum in the x direction that leaves the top face is

\[ \rho \left( v + \frac{\partial v}{\partial y} \ dy \right) \left( u + \frac{\partial u}{\partial y} \ dy \right) \ dx \]

We are interested only in the momentum in the x direction because the forces considered in the analysis are those in the x direction. These forces are those due to viscous shear and the pressure forces on the element. The pressure force on the left face is \( p \ dy \), and that on the right is \(-[p + (\partial p/\partial x) \ dx] \ dy\), so that the net pressure force in the direction of motion is

\[ \frac{\partial p}{\partial x} \ dx \ dy \]

The viscous-shear force on the bottom face is

\[ -\mu \frac{\partial u}{\partial y} \ dx \]

and the shear force on the top is

\[ \mu \ dx \left[ \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) \ dy \right] \]
This is the momentum equation of the laminar boundary layer with constant properties.

5-5 ENERGY EQUATION OF THE BOUNDARY LAYER

Conservation of Energy

Consider the elemental control volume shown in Figure 5-6. To simplify the analysis we assume

1. Incompressible steady flow
2. Constant viscosity, thermal conductivity, and specific heat
3. Negligible heat conduction in the direction of flow (x direction), i.e.,
   \[ \frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y} \]

Then, for the element shown, the energy balance may be written

Energy convected in left face + energy convected in bottom face + heat conducted in bottom face + net viscous work done on element = energy convected out right face + energy convected out top face + heat conducted out top face
Viscous shear force: $\mu \frac{\partial u}{\partial y} \, dx$

The distance which it moves per unit time in respect to the control volume is: $\frac{\partial u}{\partial y} \, dy$

the viscous energy is: $\mu \left( \frac{\partial u}{\partial y} \right)^2 \, dx \, dy$

Writing the energy balance corresponding to the quantities shown in Figure 5-6, assuming unit depth in the $z$ direction, and neglecting second-order differentials yields

$$\frac{\partial c_p}{\partial z}(u + \frac{\partial u}{\partial x} \, dx) \left( T + \frac{\partial T}{\partial y} \, dy \right) \, dx$$

$$-k \frac{\partial T}{\partial y} \, dx$$

This is the energy equation of the laminar boundary layer. The left side represents the net transport of energy into the control volume, and the right side represents the sum of the net heat conducted out of the control volume and the net viscous work done on the element.
The equation may be solved exactly for many boundary conditions, and we shall be satisfied with an approximate analysis that furnishes an easier solution without a loss in physical understanding of the processes involved. The approximate method is due to von Kármán.

**Approximate integral boundary layer analysis**

Consider the control volume in the B.L. Figure 5-5.

The equation may be solved exactly for many boundary conditions, and we shall be satisfied with an approximate analysis that furnishes an easier solution without a loss in physical understanding of the processes involved. The approximate method is due to von Kármán.

**Approximate integral boundary layer analysis**

Consider the control volume in the B.L. Figure 5-5.
The mass flow through plane 1 is
\[ \int_0^H \rho u \, dy \]  
and the momentum flow through plane 1 is
\[ \int_0^H \rho u^2 \, dy \]

The momentum flow through plane 2 is
\[ \int_0^H \rho u^2 \, dy + \frac{d}{dx} \left( \int_0^H \rho u^2 \, dy \right) \, dx \]
and the mass flow through plane 2 is
\[ \int_0^H \rho u \, dy + \frac{d}{dx} \left( \int_0^H \rho u \, dy \right) \, dx \]

Considering the conservation of mass and the fact that no mass can enter the control volume through the solid wall, the additional mass flow in expression (d) over that in (a) must enter through plane A-A. This mass flow carries with it a momentum in the x direction equal to
\[ u_\infty \frac{d}{dx} \left( \int_0^H \rho u \, dy \right) \, dx \]

The net momentum flow out of the control volume is therefore
\[ \frac{d}{dx} \left( \int_0^H \rho u^2 \, dy \right) \, dx - u_\infty \frac{d}{dx} \left( \int_0^H \rho u \, dy \right) \, dx \]

This expression may be put in a somewhat more useful form by recalling the product formula from the differential calculus:
\[ d(\eta \phi) = \eta d\phi + \phi d\eta \]
or
\[ \eta d\phi = d(\eta \phi) - \phi d\eta \]

In the momentum expression given above, the integral
\[ \int_0^H \rho u \, dy \]
is the \( \phi \) function and \( u_\infty \) is the \( \eta \) function. Thus
\[ u_\infty \frac{d}{dx} \left( \int_0^H \rho u \, dy \right) \, dx - \frac{d}{dx} \left( u_\infty \int_0^H \rho u \, dy \right) \, dx - \frac{d}{dx} \left( \rho u_\infty \int_0^H \rho u \, dy \right) \, dx \]

[5.14]

The \( u_\infty \) may be placed inside the integral since it is not a function of y and thus may be treated as a constant insofar as an integral with respect to y is concerned.

The net forces acting on the C.V is
\[ P \delta - (P + \frac{dP}{dx}) \delta - \tau_w \, dx = -\delta \frac{dP}{dx} \, dx - \tau_w \, dx \]

\[ \left[ \frac{dP}{dx} = 0 \text{ since } \frac{1}{2} \rho U^2 = 0 \& \frac{\partial P}{\partial y} = 0 \right] \]
\[ \frac{d}{dx} \int_0^\infty \rho u (U_x - u) \, dy = \tau_w = \mu \frac{dU}{dy} \]

... Integral momentum eq. for B.L.

**Evaluation of friction coefficient**
1. assume \( u(y) \) the form of polynomials
\[
u(y) = a + by + cy^2 + dy^3\]
The constants are evaluated by applying the boundary conditions
\[
y = 0 \quad u = 0, \\
y = \delta \quad u = U_0, \quad a = 0, b = \frac{3 U_0}{2 \delta}, c = 0, d = -\frac{U_0^2}{2 \delta^3} \\
y = \delta \quad \frac{\partial u}{\partial y} = 0, \\
y = 0 \quad \frac{\partial^2 u}{\partial y^2} = 0 \left[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \left( \frac{\partial^2 u}{\partial y^2} \right), u = v = 0, \frac{\partial v}{\partial x} = 0 \right] \\
\Rightarrow 
\frac{u}{U_0} = \frac{3 y}{2 \delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \]

2. Substituting the expression into the integral momentum eq. yields
\[
\frac{d}{dx} \left[ \frac{\rho U_y^2}{2} \left( \frac{y}{\delta} \right)^2 \left( \frac{y}{\delta} \right)^2 \right] - \left[ \frac{3 y}{2 \delta} + \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right] \frac{U_0^2}{\delta} = \frac{\tau_u}{\mu} \left. \left( \frac{du}{dy} \right) \right|_{y=0} \\
\frac{d}{dx} \left( \frac{\rho U_y^2}{2} \right) = \frac{3}{2} U_0^2 \\
\frac{\delta^2}{2} \frac{140v_x}{13U_0} + C \quad (\delta = 0, x = 0 \rightarrow C = 0) \\
\frac{\delta}{x} = \frac{4.64}{x} = \frac{4.64}{x} \\
\frac{\delta}{x} = \frac{4.64}{x} \quad \text{Re}_y^{1/3} \\
3. \text{To evaluate friction coefficient} \\
\tau_u = \frac{\mu}{\delta} \left. \left( \frac{du}{dy} \right) \right|_{y=0} - \frac{3}{2} \frac{U_0}{\delta} = \frac{3}{9.28} \frac{\mu U_0}{x} \text{Re}_y^{1/3} \\
C_\mu = \frac{\tau_u}{\frac{1}{2} \rho U_0^2} = \frac{0.647}{\text{Re}_y^{1/3}} \\

Similarly, the integral energy equation (Fig 5-8)
We neglects the kinetic energy term and shear work term

Figure 5-8 | Control volume for integral energy analysis of laminar boundary flow.

We wish to make the energy balance
Energy convected in + viscous work within element + heat transfer at wall
enthalpy enter across plane 1: \( \int_0^{\delta(x)} \rho c u T \, dy \)

enthalpy leaves across plane 2: \( \int_0^{\delta(x)} \rho c u T \, dy + \frac{d}{dx} \left[ \int_0^{\delta(x)} \rho c u T \, dy \right] \, dx \)

The enthalpy carried into the C. V. across the upper face is

\[
c I_0 \frac{d}{dx} \left[ \int_0^{\delta(x)} \rho u dy \right] \, dx
\]

Heat conducted across the interface between the fluid and the solid surface is

\[
-k dx \frac{dT}{dy} \bigg|_{y=0}
\]

The work done within the element is \( \mu \left[ \int_0^{\delta(x)} \left( \frac{du}{dy} \right)^2 \, dy \right] \, dx \)

Conservation of energy gives

\[
c I_0 \frac{d}{dx} \left[ \int_0^{\delta(x)} \rho u dy \right] \, dx - \frac{d}{dx} \left[ \int_0^{\delta(x)} \rho u T dy \right] \, dx - k dx \, \frac{dT}{dy} \bigg|_{y=0} - \frac{dT}{dy} \bigg|_{y=0}
\]

\[
\frac{d}{dx} \left[ \int_0^{\delta(x)} u(T_0 - T) dy + \mu \left( \frac{du}{dy} \right)^2 \right] \, dy - \alpha \frac{dT}{dy} \bigg|_{y=0} \quad \text{...integral energy eq. for B.L.}
\]

**Evaluation of heat transfer coefficient**

The plate under consideration need not be heated over its entire length. The situation that we shall analyze is shown in Figure 5-9, where the hydrodynamic boundary layer develops from the leading edge of the plate, while heating does not begin until \( x = x_0 \).

![Figure 5-9](image-url)

5. Substituting the expression into the integral energy eq. yields
5. Substituting the expression into the integral energy eq. yields

\[
\frac{d}{dx} \int_0^y (T_e - T) dy = \frac{d}{dx} \int_0^y (\theta_e - \theta) dy
\]

\[
= \theta_e U_e \frac{d}{dx} \int_0^y \left[ \left( \frac{3y}{2\delta} \right) \left( \frac{1}{2\delta} \right) + \frac{1}{2\delta} \right] dy = \alpha \frac{d\theta}{dy} \bigg|_{x \to -\infty} = \frac{3\alpha \theta_e}{2\delta}
\]

Assume \( \delta > \delta_e \), let \( \zeta = \delta / \delta_e \)

\[
\theta_e U_e \frac{d}{dx} (\delta_e^3) = \frac{3\alpha \theta_e}{2\delta_e}
\]

\[
\frac{1}{10} U_e (2\delta_e^2 \frac{d\zeta}{dx} + \zeta^2 \frac{d\delta_e}{dx}) = \alpha
\]

\[
\frac{1}{10} U_e (2\delta_e^2 \frac{d\zeta}{dx} + \zeta^2 \frac{d\delta_e}{dx}) = \alpha
\]

But \( \delta d\delta = \frac{140}{401} \frac{v}{U_e} dx \) & \( \delta^2 = \frac{280v}{13U_e} \)

\[
\zeta^3 + 4\delta_e^2 \frac{d\zeta}{dx} = \frac{13\alpha}{14v}
\]

\[
\zeta = Cx^{1/4} + \frac{13\alpha}{14v}
\]

with BCs \( \delta_e = 0 \) at \( x = x_0 \) or \( \zeta = 0 \) at \( x = x_0 \)

\[
\zeta = \frac{\delta_e}{\delta_e} \frac{1}{1.026} Pr^{-1/3} \left[ 1 - \left( \frac{x_0^{1/4} \zeta}{x} \right)^{1/3} \right]
\]

\[
\frac{h}{k} = \frac{\int_0^{x_0} h dx}{\int_0^{x_0} dx} = 2h_{x=L}
\]

\[
\overline{Nu}_{x=L} = \frac{\frac{hL}{k}}{2} = 2 Nu_{x,L} = 0.664 Pr^{1/3} Re^{1/3}
\]
5-10 HEAT TRANSFER IN LAMINAR TUBE FLOW

Consider the tube-flow system in Figure 5-15. We wish to calculate the heat transfer under developed flow conditions when the flow remains laminar. The wall temperature is $T_w$, the radius of the tube is $r_0$, and the velocity at the center of the tube is $u_0$. 

6. To evaluate the convection heat transfer coefficient

$$h_c = -k \frac{\partial T}{\partial y} \bigg|_{x=0} = \frac{3}{2} \frac{k}{\delta_i} = \frac{3}{2} \frac{k}{\delta_i}$$

$$h_i = 0.332k Pr^{1/3} \left( \frac{U}{x} \right)^{1/2} \left[ 1 - \left( \frac{x}{x_0} \right)^{3/4} \right]^{1/3}$$

$$Nu_x = \frac{h_x}{k} = 0.332 Pr^{1/3} Re_x^{1/2} \left[ 1 - \left( \frac{x}{x_0} \right)^{3/4} \right]^{1/3}$$

for $x_0 = 0$

$$Nu_x = \frac{h_x}{k} = 0.332 Pr^{1/3} Re_x^{1/2}$$

Average heat transfer coefficient

$$\overline{h} = \frac{\int_0^1 h \, dx}{\int_0^1 dx} = 2h_{x=1}$$

$$\overline{Nu}_x = \frac{\overline{h} L}{k} = 2Nu_x \times 0.664 Pr^{1/3} Re_x^{1/2}$$

The foregoing analysis, based on the assumption that the fluid properties were constant throughout the flow, when there is an appreciable variation between wall and free-stream condition, it is recommended that the properties be evaluated at the so-called film temperature, $T_f$

$$T_f = \frac{T_w + T_0}{2}$$

For constant heat flux wall

$$Nu_x = \frac{h_x}{k} = 0.453 Pr^{1/3} Re_x^{1/2}$$

$$Nu_x = \frac{q_x}{k(T_0 - T_w)}$$

$$\frac{T_u - T_w}{1} = \frac{1}{L} \int_0^L (T_u - T_w) \, dx = \frac{1}{L} \int_0^L q_x \, dx = -\frac{q_x L}{k Nu_x \overline{h} L / k}$$

$$or \quad q_x = \frac{3}{2} h_{x=1}(T_u - T_w)$$

Other relations

Churchill & Ozee:

$$Nu_x = C_1 Re_x^{1/2} Pr^{1/3} \quad \text{for } Re_x, Pr > 100$$

$$\left[ 1 + \left( \frac{C_2}{Pr} \right)^{2/3} \right]$$

Isothermal flat plate $C_1 = 0.3387, C_2 = 0.0468$

Constant heat flux plate $C_1 = 0.4637, C_2 = 0.0207$
The velocity distribution may be derived by considering the fluid element shown in Figure 5-16. The pressure forces are balanced by the viscous-shear forces so that

\[ \pi r^2 dp = 2\pi r dx = 2\pi r \mu dx \frac{\partial u}{\partial r} \]

\[ du = \frac{1}{2 \mu} \frac{dp}{dx} dr \]

\[ u = \frac{1}{4 \mu} \frac{dp}{dx} r^2 + C \]

\[ B.C. \ at \ r = r_0, \ u = 0 \]

\[ u(r) = \frac{1}{4 \mu} \frac{dp}{dx} (r^2 - r_0^2) \]

the velocity at the centerline

\[ u_o = -\frac{r_0^2}{4 \mu} \frac{dp}{dx} \]

\[ \frac{u}{u_o} = 1 - \frac{r^2}{r_0^2} \]

which is the familiar parabolic distribution for laminar tube flow

**Heat transfer:**

Assume a constant heat flux at wall, \( \frac{dq}{dx} = 0 \)

The heat flow conducted into and out of the annular element are

\[ q_r = -k 2\pi r dx \frac{\partial T}{\partial r} \]

\[ q_{r-o} = q_r + \frac{\partial T}{\partial r} dr \]

The net heat convected out of the element is

\[ 2\pi rdr c_p u \frac{\partial T}{\partial x} dx \]

The energy balance is
Net energy convected out = net heat conducted in

Neglecting second-order differentials, the energy balance gives

\[
\frac{1}{ur} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial x}
\]

Assume heat flux \( q_w = \text{constant} \), then the temperature increases linearly with \( x \), i.e.

\[
\frac{\partial T}{\partial x} = C
\]

Boundary conditions:

\[
\left. \frac{\partial T}{\partial r} \right|_{r = 0} = 0 \quad \text{at} \quad r = 0
\]

\[
E \left. \frac{\partial T}{\partial r} \right|_{r = r_o} = \text{const.} \quad \text{at} \quad r = r_o
\]

The solution is

\[
T = \frac{1}{\alpha} \frac{\partial T}{\partial x} \left[ \frac{u_o}{4} \frac{r^3}{16r_o} + C_1 \ln r + C_2 \right]
\]

\[
T - T_o = \frac{1}{\alpha} \frac{\partial T}{\partial x} \left[ \frac{r^3}{4 \frac{r_o}{r}} \left( \frac{1}{r^3} - \frac{1}{4 \frac{r_o}{r}} \right) \right]
\]

\( T_o \): centerline temperature

---

**The bulk temperature**

**Local heat transfer**

\[
q'' = h(T_u - T_o)
\]

\[
T_o = \frac{\int r^2 \rho 2\pi r u_o CT \, dr}{\int \rho 2\pi r u_o C \, dr}
\]

is so-called bulk temperature or energy average fluid temperature across the tube.

\[
T_o = T_e + \frac{7}{96} \frac{u_o r_o^2}{\alpha} \frac{\partial T}{\partial x}
\]

\[
T_u = T_e + \frac{3}{10} \frac{u_o r_o^2}{\alpha} \frac{\partial T}{\partial x}
\]

\[
q = hA(T_u - T_o) = kA \left. \frac{\partial T}{\partial r} \right|_{r = 0}
\]

\[
h = \frac{k}{T_u - T_o} = \frac{24 k}{11 r_o} = \frac{48 k}{11 d_o}
\]

\[
Nu = \frac{h d_o}{k} = 4.364
\]
5-13 SUMMARY

Our presentation of convection heat transfer is incomplete at this time and will be developed further in Chapters 6 and 7. Even so, we begin to see the structure of a procedure for solution of convection problems:

1. Establish the geometry of the situation; for now we are mainly restricted to flow over flat plates.
2. Determine the fluid involved and evaluate the fluid properties. This will usually be at the film temperature.
3. Establish the boundary conditions (i.e., constant temperature or constant heat flux).
4. Establish the flow regime as determined by the Reynolds number.
5. Select the appropriate equation, taking into account the flow regime and any fluid property restrictions which may apply.
6. Calculate the value(s) of the convection heat-transfer coefficient and/or heat transfer.

Table 5-2: Summary of equations for flow over flat plates. Properties evaluated at $T_f = (T_w + T_\infty)/2$ unless otherwise noted.

<table>
<thead>
<tr>
<th>Flow regime</th>
<th>Restrictions</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar, local</td>
<td>$T_w = \text{const}$, $Re_x &lt; 5 \times 10^5$, $0.6 &lt; Pr &lt; 50$</td>
<td>$Nu_x = 0.332 \Pr^{1/3}Re_x^{1/2}$</td>
</tr>
<tr>
<td>Laminar, local</td>
<td>$T_w = \text{const}$, $Re_x &lt; 5 \times 10^5$, $Re_x, Pr &gt; 100$</td>
<td>$Nu_x = \frac{0.4387 Re_x^{1/3} Pr^{1/3}}{1 + \left( \frac{0.0468}{Pr} \right)^{2/3}}$</td>
</tr>
<tr>
<td>Laminar, local</td>
<td>$q_w = \text{const}$, $Re_x &lt; 5 \times 10^5$, $0.6 &lt; Pr &lt; 50$</td>
<td>$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}$</td>
</tr>
<tr>
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<td>$Nu_x = \frac{0.4637 Re_x^{1/3} Pr^{1/3}}{1 + \left( \frac{0.0207}{Pr} \right)^{2/3}}$</td>
</tr>
<tr>
<td>Laminar, average</td>
<td>$Re_x &lt; 5 \times 10^5$, $T_w = \text{const}$</td>
<td>$Nu_x = 2 Nu_x = 0.664 Re_x^{1/2} Pr^{1/3}$</td>
</tr>
<tr>
<td>Laminar, local</td>
<td>$T_w = \text{const}$, $Re_x &lt; 5 \times 10^5$, $Pr &lt; 1$ (liquid metals)</td>
<td>$Nu_x = 0.564 Re_x^{1/2} Pr^{1/3}$</td>
</tr>
<tr>
<td>Laminar, local</td>
<td>$T_w = \text{const}$, starting at $x = x_0$, $Re_x &lt; 5 \times 10^5$, $0.6 &lt; Pr &lt; 50$</td>
<td>$Nu_x = 0.332 Pr^{1/3} Re_x^{1/2} \left[ 1 - \left( \frac{0.7}{Pr} \right)^{3/4} \right]^{-1/3}$</td>
</tr>
<tr>
<td>Turbulent, local</td>
<td>$T_w = \text{const}$, $Re_x &lt; 5 \times 10^5$, $0.6 &lt; Pr &lt; 50$</td>
<td>$St = Pr^{2/3} = 0.0296 Re_x^{0.2}$</td>
</tr>
<tr>
<td>Turbulent, local</td>
<td>$T_w = \text{const}$, $Re_x &lt; 10^6$</td>
<td>$St = Pr^{2/3} = 0.185 (\log Re_x)^{-2.584}$</td>
</tr>
<tr>
<td>Turbulent, local</td>
<td>$q_w = \text{const}$, $Re_x &lt; 5 \times 10^5$</td>
<td>$Re_x &lt; 10^7$</td>
</tr>
<tr>
<td>Laminar turbulent, average</td>
<td>$T_w = \text{const}$, $Re_x &lt; 10^7$, $Re_x &lt; 10^5$</td>
<td>$St = Pr^{2/3} = 0.037 Re_x^{0.2} - 871 Re_x^{-1}$</td>
</tr>
<tr>
<td>Laminar turbulent, average</td>
<td>$Nu_x = Pr^{1/3} (0.037 Re_x^{0.2} - 871)$</td>
<td>$Nu_x = 0.036 Pr^{0.49} (Re_x^{0.8} - 9200) \left( \frac{\mu_{\infty}}{\mu} \right)^{1/4}$</td>
</tr>
</tbody>
</table>
**Boundary-layer thickness**

<table>
<thead>
<tr>
<th>Laminar</th>
<th>$\text{Re}_x &lt; 5 \times 10^5$</th>
<th>$\frac{\delta}{x} = 5.0 \text{Re}_x^{-1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>$\text{Re}_x &lt; 10^7$, $\delta = 0$ at $x = 0$</td>
<td>$\frac{\delta}{x} = 0.381 \text{Re}_x^{-1/5}$</td>
</tr>
<tr>
<td>Turbulent</td>
<td>$5 \times 10^5 &lt; \text{Re}<em>x &lt; 10^7$, $\text{Re}</em>{cm} = 5 \times 10^5$, $\delta = \delta_{lam}$ at $\text{Re}_{crit}$</td>
<td>$\frac{\delta}{x} = 0.381 \text{Re}_x^{-1/5} - 10,256 \text{Re}_x^{-1}$</td>
</tr>
</tbody>
</table>

---

**Mass Flow and Boundary-Layer Thickness**

**EXAMPLE 5-3**

Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. Calculate the mass flow that enters the boundary layer between $x = 20$ cm and $x = 40$ cm. The viscosity of air at 27°C is $1.85 \times 10^{-5}$ kg/m·s. Assume unit depth in the $z$ direction.

**Solution**

The density of air is calculated from

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(300)} = 1.177 \text{ kg/m}^3 \quad [0.073 \text{ lb}_m/\text{ft}^3]$$

The Reynolds number is calculated as

At $x = 20$ cm: $\text{Re} = \frac{(1.177)(2.0)(0.2)}{1.85 \times 10^{-5}} = 25,448$

At $x = 40$ cm: $\text{Re} = \frac{(1.177)(2.0)(0.4)}{1.85 \times 10^{-5}} = 50,897$

The boundary-layer thickness is calculated from Equation (5-21):

At $x = 20$ cm: $\delta = \frac{(4.64)(0.2)}{(25,448)^{1/2}} = 0.00582$ m [0.24 in]

At $x = 40$ cm: $\delta = \frac{(4.64)(0.4)}{(50,897)^{1/2}} = 0.00823$ m [0.4 in]
To calculate the mass flow that enters the boundary layer from the free stream between \( x = 20 \text{ cm} \) and \( x = 40 \text{ cm} \), we simply take the difference between the mass flow in the boundary layer at these two \( x \) positions. At any \( x \) position the mass flow in the boundary layer is given by the integral

\[
\int_0^\delta \rho u dy
\]

where the velocity is given by Equation (5-19),

\[
u = u_\infty \left[ \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right]
\]

Evaluating the integral with this velocity distribution, we have

\[
\int_0^\delta \rho u_\infty \left[ \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right] dy = \frac{5}{8} \rho u_\infty \delta
\]

Thus the mass flow entering the boundary layer is

\[
\Delta m = \frac{5}{8} \rho u_\infty (\delta_{40} - \delta_{20})
\]

\[
= \left( \frac{5}{8} \right) (1.177)(2.0)(0.0082 - 0.0058)
\]

\[
= 3.53 \times 10^{-3} \text{ kg/s} \quad \text{[7.78 \times 10^{-3} lbm/s]}
\]

---

**Isothermal Flat Plate Heated Over Entire Length**

**EXAMPLE 5-4**

For the flow system in Example 5-3 assume that the plate is heated over its entire length to a temperature of 60°C. Calculate the heat transferred in (a) the first 20 cm of the plate and (b) the first 40 cm of the plate.

**Solution**

The total heat transfer over a certain length of the plate is desired; so we wish to calculate average heat-transfer coefficients. For this purpose we use Equations (5-44) and (5-45), evaluating the properties at the film temperature:

\[
T_f = \frac{27 + 60}{2} = 43.5°C = 316.5 \text{ K} \quad [110.3°F]
\]

From Appendix A the properties are

\[
v = 17.36 \times 10^{-6} \text{ m}^2/\text{s} \quad [1.87 \times 10^{-4} \text{ ft}^2/\text{s}]
\]

\[
k = 0.02749 \text{ W/m} \cdot \text{°C} \quad [0.0159 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}]
\]

\[
Pr = 0.7
\]

\[
c_p = 1.006 \text{ kJ/kg} \cdot \text{°C} \quad [0.24 \text{ Btu/lbm} \cdot \text{°F}]
\]
At \( x = 20 \text{ cm} \)

\[
\begin{align*}
\text{Re}_x &= \frac{u_\infty x}{\nu} = \frac{(2)(0.2)}{17.36 \times 10^{-6}} = 23,041 \\
\text{Nu}_x &= \frac{h_x x}{k} = 0.332\text{Re}_x^{1/2}\text{Pr}^{1/3} \\
&= (0.332)(23,041)^{1/2}(0.7)^{1/3} = 44.74 \\
\bar{h} &= \text{Nu}_x \left( \frac{k}{x} \right) = \frac{(44.74)(0.02749)}{0.2} \\
&= 6.15 \text{ W/m}^2 \cdot ^\circ\text{C} \quad [1.083 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}] 
\end{align*}
\]

The average value of the heat-transfer coefficient is twice this value, or

\[
\bar{h} = (2)(6.15) = 12.3 \text{ W/m}^2 \cdot ^\circ\text{C} \quad [2.17 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}]
\]

The heat flow is

\[
q = \bar{h}A(T_w - T_\infty)
\]

If we assume unit depth in the \( z \) direction,

\[
q = (12.3)(0.2)(60 - 27) = 81.18 \text{ W} \quad [277 \text{ Btu/h}]
\]

At \( x = 40 \text{ cm} \)

\[
\begin{align*}
\text{Re}_x &= \frac{u_\infty x}{\nu} = \frac{(2)(0.4)}{17.36 \times 10^{-6}} = 46,082 \\
\text{Nu}_x &= (0.332)(46,082)^{1/2}(0.7)^{1/3} = 63.28 \\
\bar{h} &= \frac{(63.28)(0.02749)}{0.4} = 4.349 \text{ W/m}^2 \cdot ^\circ\text{C} \\
\bar{h} &= (2)(4.349) = 8.698 \text{ W/m}^2 \cdot ^\circ\text{C} \quad [1.53 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}] \\
q &= (8.698)(0.4)(60 - 27) = 114.8 \text{ W} \quad [392 \text{ Btu/h}]
\end{align*}
\]

---

**EXAMPLE 5-5  Flat Plate with Constant Heat Flux**

A 1.0-kW heater is constructed of a glass plate with an electrically conducting film that produces a constant heat flux. The plate is 60 cm by 60 cm and placed in an airstream at 27°C, 1 atm with \( u_\infty = 5 \text{ m/s} \). Calculate the average temperature difference along the plate and the temperature difference at the trailing edge.

**Solution**

Properties should be evaluated at the film temperature, but we do not know the plate temperature. So for an initial calculation, we take the properties at the free-stream conditions of

\[
\begin{align*}
T_\infty &= 27^\circ\text{C} = 300 \text{ K} \\
v &= 15.69 \times 10^{-6} \text{ m}^2/\text{s} \\
\text{Pr} &= 0.708 \\
k &= 0.02624 \text{ W/m} \cdot ^\circ\text{C} \\
\text{Re}_L &= \frac{(0.6)(5)}{15.69 \times 10^{-6}} = 1.91 \times 10^5
\end{align*}
\]

From Equation (5-50) the average temperature difference is

\[
\frac{T_w - T_\infty}{[1000/(0.6)^2}(0.6)/0.02624 \frac{0.6795(1.91 \times 10^5)^{1/2}(0.708)^{1/3}}{240^\circ\text{C}}
\]
Now, we go back and evaluate properties at
\[ T_f = \frac{240 + 27 + 27}{2} = 147^\circ C = 420 \text{ K} \]
and obtain
\[ \nu = 28.22 \times 10^{-6} \text{ m}^2/\text{s} \quad \text{Pr} = 0.687 \quad k = 0.035 \text{ W/m} \cdot \text{°C} \]
\[ \text{Re}_L = \frac{0.6(5)}{28.22 \times 10^{-6}} = 1.06 \times 10^5 \]
\[ \frac{T_w - T_\infty}{0.6795(1.06 \times 10^5)^{1/2}(0.687)^{1/3}} = 243^\circ C \]
At the end of the plate \((x = L = 0.6 \text{ m})\) the temperature difference is obtained from Equations (5-48) and (5-50) with the constant 0.453 to give
\[ (T_w - T_\infty)_{x=L} = \frac{(243.6)(0.6795)}{0.453} = 365.4^\circ C \]
An alternate solution would be to base the Nusselt number on Equation (5-51).

---

**Plate with Unheated Starting Length**

**EXAMPLE 5-6**

Air at 1 atm and 300 K flows across a 20-cm-square plate at a free-stream velocity of 20 m/s. The last half of the plate is heated to a constant temperature of 350 K. Calculate the heat lost by the plate.

**Solution**

First we evaluate the air properties at the film temperature
\[ T_f = (T_w + T_\infty)/2 = 325 \text{ K} \]
and obtain
\[ \nu = 18.23 \times 10^{-6} \text{ m}^2/\text{s} \quad k = 0.02814 \text{ W/m} \cdot \text{°C} \quad \text{Pr} = 0.7 \]
At the trailing edge of the plate the Reynolds number is
\[ \text{Re}_L = u_\infty L/\nu = (20)(0.2)/18.23 \times 10^{-6} = 2.194 \times 10^5 \]
or, laminar flow over the length of the plate.

Heating does not start until the last half of the plate, or at a position \(x_0 = 0.1 \text{ m}\). The local heat-transfer coefficient for this condition is given by Equation (5-41):
\[
h_x = 0.332k \text{Pr}^{1/3}(u_\infty/\nu x)^{1/2}[1 - (x_0/x)^{0.75}]^{-1/3} \tag{a} \]
Inserting the property values along with \(x_0 = 0.1 \text{ gives}\)
\[ h_x = 8.683x^{-1/2}(1 - 0.17783x^{-0.75})^{-1/3} \tag{b} \]
The plate is 0.2 m wide so the heat transfer is obtained by integrating over the heated length \(x_0 < x < L\)
\[ q = (0.2)(T_w - T_\infty) \int_{x_0 = 0.1}^{L = 0.2} h_x dx \tag{c} \]
Inserting Equation (b) in Equation (c) and performing the numerical integration gives

\[ q = (0.2)(8.6883)(0.4845)(350 - 300) = 421 \text{ W} \]  
\[ \text{[d]} \]

The average value of the heat-transfer coefficient over the heated length is given by

\[ h = q/(T_u - T_\infty)(L - x_0)W = 421/(350 - 300)(0.2 - 0.1)(0.2) = 421 \text{ W/m}^2 \cdot ^\circ \text{C} \]

where \( W \) is the width of the plate.

An easier calculation can be made by applying Equation (5-45b) to determine the average heat transfer coefficient over the heated portion of the plate. The result is

\[ h = 425.66 \text{ W/m}^2 \cdot ^\circ \text{C} \quad \text{and} \quad q = 425.66 \text{ W} \]

which indicates, of course, only a small error in the numerical integration.

---

**EXAMPLE 5-7**

**Oil Flow Over Heated Flat Plate**

Engine oil at 20°C is forced over a 20-cm-square plate at a velocity of 1.2 m/s. The plate is heated to a uniform temperature of 60°C. Calculate the heat lost by the plate.

**Solution**

We first evaluate the film temperature:

\[ T_f = \frac{20 + 60}{2} = 40^\circ \text{C} \]

The properties of engine oil are

\( \rho = 876 \text{ kg/m}^3 \quad \quad \nu = 0.00024 \text{ m}^2/\text{s} \)

\( k = 0.144 \text{ W/m} \cdot ^\circ \text{C} \quad \quad \text{Pr} = 2870 \)

The Reynolds number is

\[ \text{Re} = \frac{u_\infty L}{\nu} = \frac{(1.2)(0.2)}{0.00024} = 1000 \]

Because the Prandtl number is so large we will employ Equation (5-51) for the solution. We see that \( h_x \) varies with \( x \) in the same fashion as in Equation (5-44), that is, \( h_x \propto x^{-1/2} \), so that we get the same solution as in Equation (5-45) for the average heat-transfer coefficient. Evaluating Equation (5-51) at \( x = 0.2 \) gives

\[ \text{Nu}_x = \frac{(0.3387)(1000)^{1/2}(2870)^{1/3}}{1 + \left(\frac{0.0468}{2870}\right)^{2/3}}^{1/4} = 152.2 \]

and

\[ h_x = \frac{(152.2)(0.144)}{0.2} = 109.6 \text{ W/m}^2 \cdot ^\circ \text{C} \]

The average value of the convection coefficient is

\[ h = (2)(109.6) = 219.2 \text{ W/m}^2 \cdot ^\circ \text{C} \]

so that the total heat transfer is

\[ q = hA(T_u - T_\infty) = (219.2)(0.2)^2(60 - 20) = 350.6 \text{ W} \]
Turbulent Heat Transfer from Isothermal Flat Plate

Air at 20°C and 1 atm flows over a flat plate at 35 m/s. The plate is 75 cm long and is maintained at 60°C. Assuming unit depth in the z direction, calculate the heat transfer from the plate.

Solution

We evaluate properties at the film temperature:

\[ T_f = \frac{20 + 60}{2} = 40°C = 313 K \]

\[ \rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(313)} = 1.128 \text{ kg/m}^3 \]

\[ \mu = 1.906 \times 10^{-5} \text{ kg/m} \cdot \text{s} \]

\[ Pr = 0.7 \quad k = 0.02723 \text{ W/m} \cdot \text{°C} \quad c_p = 1.007 \text{ kJ/kg} \cdot \text{°C} \]

The Reynolds number is

\[ Re_L = \frac{\rho u \infty L}{\mu} = \frac{(1.128)(35)(0.75)}{1.906 \times 10^{-5}} = 1.553 \times 10^6 \]

and the boundary layer is turbulent because the Reynolds number is greater than \(5 \times 10^5\). Therefore, we use Equation (5-85) to calculate the average heat transfer over the plate:

\[ \bar{h} = \frac{k}{Nu} = \frac{k}{(0.037)(1.553 \times 10^6)^{0.8}} = 2180 \]

\[ \bar{h} = \frac{2180(0.02723)}{0.75} = 79.1 \text{ W/m}^2 \cdot \text{°C} \quad [13.9 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}] \]

\[ q = \bar{h} A (T_w - T_\infty) = (79.1)(0.75)(60 - 20) = 2373 \text{ W} \quad [8159 \text{ Btu/h}] \]

Example

Experimental results for the local heat transfer coefficient \(h_x\) for flow over a flat plate with an extremely rough surface were found to fit the relation

\[ h_x(x) = ax^{-0.1} \]

where \(a\) is a coefficient \((\text{W/m}^{1.9} \cdot \text{K})\) and \(x\) (m) is the distance from the leading edge of the plate.

1. Develop an expression for the ratio of the average heat transfer coefficient \(\bar{h}_x\) for a plate of length \(x\) to the local heat transfer coefficient \(h_x\) at \(x\).
2. Plot the variation of \(h_x\) and \(\bar{h}_x\) as a function of \(x\).
**Solution**

**Known:** Variation of the local heat transfer coefficient, \( h_x(x) \).

**Find:**
1. The ratio of the average heat transfer coefficient \( \bar{h}(x) \) to the local value \( h_x(x) \).
2. Plot of the variation of \( h_x \) and \( \bar{h}_x \) with \( x \).

**Schematic:**

![Schematic Diagram]

**Analysis:**

1. From Equation 6.14 the average value of the convection heat transfer coefficient over the region from 0 to \( x \) is

\[
\bar{h}_x = \bar{h}_x(x) = \frac{1}{x} \int_0^x h_x(x) \, dx
\]

Substituting the expression for the local heat transfer coefficient

\[
h_x(x) = ax^{-0.1}
\]

and integrating, we obtain

\[
\bar{h}_x = \frac{1}{x} \int_0^x ax^{-0.1} \, dx = \frac{a}{x} \int_0^x x^{-0.1} \, dx = \frac{a}{x} \left( \frac{x^{0.9}}{0.9} \right) = 1.11ax^{-0.1}
\]

or

\[
\bar{h}_x = 1.11h_x
\]

2. The variation of \( h_x \) and \( \bar{h}_x \) with \( x \) is as follows:

![Graph of \( h_x \) and \( \bar{h}_x \) vs. \( x \)]

**Comments:** Boundary layer development causes both the local and average coefficients to decrease with increasing distance from the leading edge. The average coefficient up to \( x \) must therefore exceed the local value at \( x \).
Example

Water flows at a velocity $u_\infty = 1 \text{ m/s}$ over a flat plate of length $L = 0.6 \text{ m}$. Consider two cases, one for which the water temperature is approximately 300 K and the other for an approximate water temperature of 350 K. In the laminar and turbulent regions, experimental measurements show that the local convection coefficients are well described by

$$h_{\text{lam}}(x) = C_{\text{lam}} x^{-0.5} \quad h_{\text{urb}}(x) = C_{\text{urb}} x^{-0.2}$$

where $x$ has units of m. At 300 K,

$$C_{\text{lam},300} = 395 \text{ W/m}^{1.5} \cdot \text{K} \quad C_{\text{urb},300} = 2330 \text{ W/m}^{1.8} \cdot \text{K}$$

while at 350 K,

$$C_{\text{lam},350} = 477 \text{ W/m}^{1.5} \cdot \text{K} \quad C_{\text{urb},350} = 3600 \text{ W/m}^{1.8} \cdot \text{K}$$

As is evident, the constant $C$ depends on the nature of the flow as well as the water temperature because of the thermal dependence of various properties of the fluid.

Determine the average convection coefficient, $\bar{h}$, over the entire plate for the two water temperatures.

**SOLUTION**

**Known:** Water flow over a flat plate, expressions for the dependence of the local convection coefficient with distance from the plate’s leading edge $x$, and approximate temperature of the water.

**Find:** Average convection coefficient, $\bar{h}$.

**Schematic:**

![Schematic of water flow over a flat plate](image)

**Assumptions:**
1. Steady-state conditions.
2. Transition occurs at a critical Reynolds number of $Re_{x,c} = 5 \times 10^5$. 
**Properties:** Table A.6, water ($\bar{T} \approx 300$ K): $\rho = \nu_f^{-1} = 997$ kg/m$^3$, $\mu = 855 \times 10^{-6}$ N·s/m$^2$. Table A.6 ($\bar{T} \approx 350$ K): $\rho = \nu_f^{-1} = 974$ kg/m$^3$, $\mu = 365 \times 10^{-6}$ N·s/m$^2$.

**Analysis:** The local convection coefficient is highly dependent on whether laminar or turbulent conditions exist. Therefore, we first determine the extent to which these conditions exist by finding the location where transition occurs, $x_c$. From Equation 6.24, we know that at 300 K,

$$x_c = \frac{Re_x \mu}{\rho \mu_w} = \frac{5 \times 10^5 \times 855 \times 10^{-6} \text{N} \cdot \text{s/m}^2}{997 \text{kg/m}^3 \times 1 \text{m/s}} = 0.43 \text{ m}$$

while at 350 K,

$$x_c = \frac{Re_x \mu}{\rho \mu_w} = \frac{5 \times 10^5 \times 365 \times 10^{-6} \text{N} \cdot \text{s/m}^2}{974 \text{kg/m}^3 \times 1 \text{m/s}} = 0.19 \text{ m}$$

From Equation 6.14 we know that

$$\overline{h} = \frac{1}{L} \int_0^L h \, dx = \frac{1}{L} \left[ \int_0^{x_c} h_{\text{lam}} \, dx + \int_{x_c}^L h_{\text{turb}} \, dx \right]$$

or

$$\overline{h} = \frac{1}{L} \left[ \frac{C_{\text{lam}}}{0.5} x \right]_0^{x_c} + \frac{C_{\text{turb}}}{0.8} x \right]_0^{x_c}$$

At 300 K,

$$\overline{h} = \frac{1}{0.6 \text{ m}} \left[ \frac{395 \text{ W/m}^{1.5} \cdot \text{K}}{0.5} \times (0.43^{0.5}) \text{ m}^{0.5} + \frac{2330 \text{ W/m}^{1.8} \cdot \text{K}}{0.8} \times (0.6^{0.8} - 0.43^{0.8}) \text{ m}^{0.8} \right] = 1620 \text{ W/m}^2 \cdot \text{K}$$

while at 350 K,

$$\overline{h} = \frac{1}{0.6 \text{ m}} \left[ \frac{477 \text{ W/m}^{1.5} \cdot \text{K}}{0.5} \times (0.19^{0.5}) \text{ m}^{0.5} + \frac{3600 \text{ W/m}^{1.8} \cdot \text{K}}{0.8} \times (0.6^{0.8} - 0.19^{0.8}) \text{ m}^{0.8} \right] = 3710 \text{ W/m}^2 \cdot \text{K}$$

The local and average convection coefficient distributions for the plate are shown in the following figure.
Comments:

1. The average convection coefficient at $T \approx 350$ K is over twice as large as the value at $T \approx 300$ K. This strong temperature dependence is due primarily to the shift of $x_*$ that is associated with the smaller viscosity of the water at the higher temperature. Careful consideration of the temperature dependence of fluid properties is crucial when performing a convection heat transfer analysis.

2. Spatial variations in the local convection coefficient are significant. The largest local convection coefficients occur at the leading edge of the flat plate, where the laminar thermal boundary layer is extremely thin, and just downstream of $x_*$, where the turbulent boundary layer is thinnest.

Example

Experimental tests using air as the working fluid are conducted on a portion of the turbine blade shown in the sketch. The heat flux to the blade at a particular point ($x^*$) on the surface is measured to be $q'' = 95,000$ W/m². To maintain a steady-state surface temperature of $800^\circ$C, heat transferred to the blade is removed by circulating a coolant inside the blade.
1. Determine the heat flux to the blade at $x^*$ if its temperature is reduced to $T_{z,1} = 700^\circ C$ by increasing the coolant flow.

2. Determine the heat flux at the same dimensionless location $x^*$ for a similar turbine blade having a chord length of $L = 80$ mm, when the blade operates in an airflow at $T_{in} = 1150^\circ C$ and $V = 80$ m/s, with $T_z = 800^\circ C$.

**Solution**

**Known:** Operating conditions of an internally cooled turbine blade.

**Find:**
1. Heat flux to the blade at a point $x^*$ when the surface temperature is reduced.
2. Heat flux at the same dimensionless location to a larger turbine blade of the same shape with reduced air velocity.

**Schematic:**

**Assumptions:**
1. Steady-state, incompressible flow.
2. Constant air properties.
Analysis:

1. When the surface temperature is 800°C, the local convection heat transfer coefficient between the surface and the air at \( x^* \) can be obtained from Newton’s law of cooling:

\[
q'' = h(T_\infty - T_2)
\]

Thus,

\[
h = \frac{q''}{(T_\infty - T_2)}
\]

We proceed without calculating the value for now. From Equation 6.49, it follows that, for the prescribed geometry,

\[
Nu = \frac{hL}{k} = f(x^*, Re_L, Pr)
\]

Hence, since there is no change in \( x^* \), \( Re_L \), or \( Pr \) associated with a change in \( T \), for constant properties, the local Nusselt number is unchanged. Moreover, since \( L \) and \( k \) are unchanged, the local convection coefficient remains the same. Thus, when the surface temperature is reduced to 700°C, the heat flux may be obtained from Newton’s law of cooling, using the same local convection coefficient:

\[
q'' = h(T_\infty - T_{2,1}) = \frac{q''}{(T_\infty - T_2)}(T_\infty - T_{2,1}) = \frac{95,000 \text{ W/m}^2}{(1150 - 800)\text{°C}}(1150 - 700)\text{°C}
\]

\[= 122,000 \text{ W/m}^2\]

2. To determine the heat flux at \( x^* \) associated with the larger blade and the reduced airflow (case 2), we first note that, although \( L \) has increased by a factor of 2, the velocity has decreased by the same factor and the Reynolds number has not changed. That is,

\[
Re_{L,2} = \frac{V_2 L_2}{\nu} = \frac{V_1 L}{\nu} = Re_L
\]

Accordingly, since \( x^* \) and \( Pr \) are also unchanged, the local Nusselt number remains the same.

\[
Nu_2 = Nu
\]

Because the characteristic length is different, however, the local convection coefficient changes, where

\[
\frac{h_2 L_2}{k} = \frac{hL}{k} \quad \text{or} \quad h_2 = h \frac{L}{L_2} = \frac{q''}{(T_\infty - T_2)} \frac{L}{L_2}
\]

The heat flux at \( x^* \) is then

\[
q'' = h_2(T_\infty - T_2) = \frac{q''}{(T_\infty - T_2)} \frac{L}{L_2}
\]

\[
q'' = 95,000 \text{ W/m}^2 \times \frac{0.04 \text{ m}}{0.08 \text{ m}} = 47,500 \text{ W/m}^2
\]
Empirical and Practical Relations for Forced Convection Heat Transfer

Regrettably, it is not always possible to obtain analytical solutions to convection problems, and the individual is forced to resort to experimental methods to obtain design information, as well as to secure the more elusive data that increase the physical understanding of the heat-transfer processes.

6-2 EMPIRICAL RELATIONS FOR PIPE AND TUBE FLOW

In this section we present some of the more important and useful empirical relations and point out their limitations.

In Chapter 5 we noted that the bulk temperature represents energy average or “mixing cup” conditions. Thus, for the tube flow depicted in Figure 6-1 the total energy added can be expressed in terms of a bulk-temperature difference by
\[ q = \dot{m} c_p (T_b_2 - T_b_1) \]  \[6-1\]

provided \( c_p \) is reasonably constant over the length. In some differential length \( \delta x \) the heat added \( \delta q \) can be expressed either in terms of a bulk-temperature difference or in terms of the heat-transfer coefficient

\[ dq = \dot{m} c_p \delta T_b = h(2\pi r)dx(T_w - T_b) \]  \[6-2\]

where \( T_w \) and \( T_b \) are the wall and bulk temperatures at the particular \( x \) location. The total heat transfer can also be expressed as

\[ q = h A (T_w - T_b) \text{av} \]  \[6-3\]

where \( A \) is the total surface area for heat transfer. Because both \( T_w \) and \( T_b \) can vary along the length of the tube, a suitable averaging process must be adopted for use with Equation (6-3).

In this chapter most of our attention will be focused on methods for determining the convection heat transfer coefficient. A traditional expression for calculation of heat transfer in fully developed turbulent flow in smooth tubes is that recommended by Dittus and Boelter [1]:*

\[ Nu_d = 0.023 \, Re_d^{0.8} \, Pr^n \]  \[6-4a\]

The properties in this equation are evaluated at the average fluid bulk temperature, and the exponent \( n \) has the following values:

\[ n = 0.4 \] for heating of the fluid
\[ n = 0.3 \] for cooling of the fluid

Equation (6-4) is valid for fully developed turbulent flow in smooth tubes for fluids with Prandtl numbers ranging from about 0.6 to 100 and with moderate temperature differences between wall and fluid conditions. Better results for turbulent flow in smooth tubes may be obtained from the following:

\[ Nu = 0.0214(Re^{0.8} - 100) \, Pr^{0.4} \]  \[6-4b\]
As described above, we may anticipate that the heat-transfer data will be dependent on the Reynolds and Prandtl numbers. A power function for each of these parameters is a simple type of relation to use, so we assume

\[ \text{Nu}_d = C \text{Re}_d^m \text{Pr}_d^n \]

where \( C, m, \) and \( n \) are constants to be determined from the experimental data.

To take into account the property variations, Sieder and Tate [2] recommend the following relation:

\[ \text{Nu}_d = 0.027 \text{Re}_d^{0.8} \text{Pr}^{1/3} \left( \frac{\mu}{\mu_w} \right)^{0.14} \]

Equations (6-4) and (6-5) apply to fully developed turbulent flow in tubes. In the entrance region the flow is not developed, and Nusselt [3] recommended the following equation:

\[ \text{Nu}_d = 0.036 \text{Re}_d^{0.8} \text{Pr}^{1/3} \left( \frac{d}{L} \right)^{0.055} \quad \text{for} \quad 10 < \frac{L}{d} < 400 \]

where \( L \) is the length of the tube and \( d \) is the tube diameter. The properties in Equation (6-6) are evaluated at the mean bulk temperature.

If the channel through which the fluid flows is not of circular cross section, it is recommended that the heat-transfer correlations be based on the hydraulic diameter \( D_H \), defined by

\[ D_H = \frac{4A}{P} \]

where \( A \) is the cross-sectional area of the flow and \( P \) is the wetted perimeter.

The hydraulic diameter should be used in calculating the Nusselt and Reynolds numbers.
Turbulent Heat Transfer in a Tube

Air at 2 atm and 200°C is heated as it flows through a tube with a diameter of 1 m (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3-m length of the tube?

**Solution**

We first calculate the Reynolds number to determine if the flow is laminar or turbulent, and then select the appropriate empirical correlation to calculate the heat transfer. The properties of air at a bulk temperature of 200°C are:

\[
\rho = \frac{p}{RT} = \frac{(2)(1.0132 \times 10^5)}{(287)(473)} = 1.493 \text{ kg/m}^3 \quad [0.0932 \text{ lbm/ft}^3]
\]

\[
Pr = \frac{\mu}{\rho v} = \frac{2.57 \times 10^{-5} \text{ kg/m} \cdot \text{s}}{1.493 \times 10^3 \text{ kg/m} \cdot \text{s}} = 0.00017 \quad [0.0622 \text{ lbm/h} \cdot \text{ft}]
\]

\[
k = 0.0386 \text{ W/m} \cdot \text{°C} \quad [0.0223 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}]
\]

\[
c_p = 1.025 \text{ kJ/kg} \cdot \text{°C}
\]

\[
Re_d = \frac{\rho u_d d}{\mu} = \frac{(493)(10)(0.0254)}{2.57 \times 10^{-5}} = 14,756
\]

so that the flow is turbulent. We therefore use Equation (6-4a) to calculate the heat-transfer coefficient.

\[
Nu_d = \frac{h d}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = (0.023)(14,756)^{0.8}(0.681)^{0.4} = 0.023
\]

\[
h = \frac{k}{\text{Nu}_d} = \frac{0.0386(42.67)}{0.0254} = 64.85 \text{ W/m}^2 \cdot \text{°C} \quad [11.42 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}]
\]

The heat flow per unit length is then

\[
\frac{q}{L} = h\pi d(T_w - T_b) = (64.85)\pi(0.0254)(20) = 103.5 \text{ W/m} \quad [107.7 \text{ Btu/ft}]
\]

We can now make an energy balance to calculate the increase in bulk temperature in a 3.0-m length of tube:

\[
q = \dot{m}c_p \Delta T_b = L \left( \frac{q}{L} \right)
\]

We also have

\[
\dot{m} = \rho u_m \pi d \frac{2}{4} = (1.493)(10)\pi \frac{(0.0254)^2}{4}
\]

\[
= 7.565 \times 10^{-3} \text{ kg/s} \quad [0.0167 \text{ lbm/s}]
\]

so that we insert the numerical values in the energy balance to obtain

\[
(7.565 \times 10^{-3})(1025)\Delta T_b = (3.0)(103.5)
\]

and

\[
\Delta T_b = 40.04 \text{°C} \quad [104.07 \text{°F}]
\]


**EXAMPLE 6-2** Heating of Water in Laminar Tube Flow

Water at 60°C enters a tube of 1-in (2.54-cm) diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3.0 m long and the wall temperature is constant at 80°C.

**Solution**

We first evaluate the Reynolds number at the inlet bulk temperature to determine the flow regime. The properties of water at 60°C are:

\[
\begin{align*}
\rho &= 985 \text{ kg/m}^3 \\
\mu &= 4.71 \times 10^{-4} \text{ kg/m} \cdot \text{s} \\
k &= 0.651 \text{ W/m} \cdot \text{°C} \\
Pr &= 3.02
\end{align*}
\]

\[
\begin{align*}
Re_d &= \frac{\rho u_m d}{\mu} = \frac{(985)(0.02)(0.0254)}{4.71 \times 10^{-4}} = 1062
\end{align*}
\]

so the flow is laminar. Calculating the additional parameter, we have

\[
Re_d Pr = \frac{1062(3.02)(0.0254)}{3} = 27.15 > 10
\]

so Equation (6-10) is applicable. We do not yet know the mean bulk temperature to evaluate properties so we first make the calculation on the basis of 60°C, determine an exit bulk temperature, and then make a second iteration to obtain a more precise value. When inlet and outlet conditions are designated with the subscripts 1 and 2, respectively, the energy balance becomes

\[
q = h \pi d L \left( T_w - \frac{T_{b1} + T_{b2}}{2} \right) = \dot{m} c_p (T_{b2} - T_{b1}) \tag{a}
\]

At the wall temperature of 80°C we have

\[
\mu_w = 3.55 \times 10^{-4} \text{ kg/m} \cdot \text{s}
\]

From Equation (6-10)

\[
Nu_d = (1.86) \left[ \frac{(1062)(3.02)(0.0254)}{3} \right]^{1/3} \left( \frac{4.71}{3.55} \right)^{0.14} = 5.816
\]

\[
h = \frac{k Nu_d}{d} = \frac{(0.651)(5.816)}{0.0254} = 149.1 \text{ W/m}^2 \cdot \text{°C} \tag{26.26 Btu/h} \cdot \text{ft}^2 \cdot \text{°F}
\]

The mass flow rate is

\[
\dot{m} = \rho \frac{\pi d^2}{4} u_m = \frac{(985)(\pi)(0.0254)^2(0.02)}{4} = 9.982 \times 10^{-3} \text{ kg/s}
\]

Inserting the value for \( h \) into Equation (a) along with \( \dot{m} \) and \( T_{b1} = 60°C \) and \( T_w = 80°C \) gives

\[
(149.1)(\pi)(0.0254)(3.0) \left( 80 - \frac{T_{b2} + 60}{2} \right) = (9.982 \times 10^{-3})(4180)(T_{b2} - 60) \tag{b}
\]

This equation can be solved to give

\[
T_{b2} = 71.98°C
\]

Thus, we should go back and evaluate properties at

\[
T_{b, mean} = \frac{71.98 + 60}{2} = 66°C
\]
We obtain

\[ \rho = 982 \text{ kg/m}^3 \quad c_p = 4185 \text{ J/kg} \cdot \text{°C} \quad \mu = 4.36 \times 10^{-4} \text{ kg/m} \cdot \text{s} \]

\[ k = 0.656 \text{ W/m} \cdot \text{°C} \quad \text{Pr} = 2.78 \]

\[ \text{Re}_d = \frac{(1062)(4.71)}{4.36} = 1147 \]

\[ \text{Re}_d \frac{d}{L} = \frac{(1147)(2.78)(0.0254)}{3} = 27.00 \]

\[ \text{Nu}_d = (1.86)(27.00)^{1/3} \left( \frac{4.36}{3.55} \right)^{0.14} = 5.743 \]

\[ h = \frac{(0.656)(5.743)}{0.0254} = 148.3 \text{ W/m}^2 \cdot \text{°C} \]

We insert this value of \( h \) back into Equation (a) to obtain

\[ T_{b2} = 71.88 \text{°C} \quad [161.4 \text{°F}] \]

The iteration makes very little difference in this problem. If a large bulk-temperature difference had been encountered, the change in properties could have had a larger effect.

**Heating of Air in Laminar Tube**

**Flow for Constant Heat Flux**

**EXAMPLE 6-3**

Air at 1 atm and 27°C enters a 5.0-mm-diameter smooth tube with a velocity of 3.0 m/s. The length of the tube is 10 cm. A constant heat flux is imposed on the tube wall. Calculate the heat transfer if the exit bulk temperature is 77°C. Also calculate the exit wall temperature and the value of \( h \) at exit.

**Solution**

We first must evaluate the flow regime and do so by taking properties at the average bulk temperature

\[ \frac{T_b}{2} = \frac{27 + 77}{2} = 52 \text{°C} = 325 \text{ K} \]

\[ v = 18.22 \times 10^{-6} \text{ m}^2/\text{s} \quad \text{Pr} = 0.703 \quad k = 0.02814 \text{ W/m} \cdot \text{°C} \]

\[ \text{Re}_d = \frac{\mu d}{\nu} = \frac{(3)(0.005)}{18.22 \times 10^{-6}} = 823 \quad [a] \]

so that the flow is laminar. The tube length is rather short, so we expect a thermal entrance effect and shall consult Figure 6-5. The inverse Graetz number is computed as

\[ \text{Gz}^{-1} = \frac{1}{\text{Re}_d \text{Pr} d} = \frac{0.1}{(823)(0.703)(0.005)} = 0.0346 \]

Therefore, for \( q_w = \text{constant} \), we obtain the Nusselt number at exit from Figure 6-5 as

\[ \text{Nu} = \frac{h d}{k} = 4.7 = \frac{q_w d}{(T_w - T_b) k} \quad [b] \]

The total heat transfer is obtained in terms of the overall energy balance:

\[ q = \dot{m} c_p (T_{b2} - T_{b1}) \]

At entrance \( \rho = 1.1774 \text{ kg/m}^3 \), so the mass flow is

\[ \dot{m} = (1.1774)(\pi)(0.0025)^2(3.0) = 6.94 \times 10^{-5} \text{ kg/s} \]

and

\[ q = (6.94 \times 10^{-5})(1006)(77 - 27) = 3.49 \text{ W} \]
Thus we may find the heat transfer without actually determining wall temperatures or values of $h$. However, to determine $T_w$ we must compute $q_w$ for insertion in Equation (b). We have

$$q = q_w \pi dL = 3.49 \text{ W}$$

and

$$q_w = 2222 \text{ W/m}^2$$

Now, from Equation (b)

$$(T_w - T_h)_{x=L} = \frac{(2222)(0.005)}{(4.7)(0.02814)} = 84^\circ \text{C}$$

The wall temperature at exit is thus

$$T_w|_{x=L} = 84 + 77 = 161^\circ \text{C}$$

and the heat-transfer coefficient is

$$h_{x=L} = \frac{q_w}{(T_w - T_h)_{x=L}} = \frac{2222}{84} = 26.45 \text{ W/m}^2 \cdot ^\circ \text{C}$$

---

**EXAMPLE 6.4**

**Heating of Air with Isothermal Tube Wall**

Repeat Example 6-3 for the case of constant wall temperature.

**Solution**

We evaluate properties as before and now enter Figure 6-5 to determine $\overline{Nu_d}$ for $T_w = \text{constant}$. For $Gz^{-1} = 0.0346$ we read

$$\overline{Nu_d} = 5.15$$

We thus calculate the average heat-transfer coefficient as

$$h = (5.15) \left(\frac{k}{d}\right) = \frac{(5.15)(0.02814)}{0.005} = 29.98 \text{ W/m}^2 \cdot ^\circ \text{C}$$

We base the heat transfer on a mean bulk temperature of $52^\circ \text{C}$, so that

$$q = \overline{h} \pi dL (T_w - T_h) = 3.49 \text{ W}$$

and

$$T_w = 76.67 + 52 = 128.67^\circ \text{C}$$
Heat Transfer in a Rough Tube

EXAMPLE 6-5

A 2.0-cm-diameter tube having a relative roughness of 0.001 is maintained at a constant wall temperature of 90°C. Water enters the tube at 40°C and leaves at 60°C. If the entering velocity is 3 m/s, calculate the length of tube necessary to accomplish the heating.

Solution

We first calculate the heat transfer from

\[ q = \dot{m}c_p \Delta T_b = (989)(3.0)\pi(0.01)^2(4174)(60 - 40) = 77,812 \text{ W} \]

For the rough-tube condition, we may employ the Petukhov relation, Equation (6-7). The mean film temperature is

\[ T_f = \frac{90 + 50}{2} = 70\text{°C} \]

and the fluid properties are

\[ \rho = 978 \text{ kg/m}^3 \quad \mu = 4.0 \times 10^{-4} \text{ kg/m} \cdot \text{s} \]

\[ k = 0.664 \text{ W/m} \cdot \text{°C} \quad \text{Pr} = 2.54 \]

Also,

\[ \mu_b = 5.55 \times 10^{-4} \text{ kg/m} \cdot \text{s} \]

\[ \mu_w = 2.81 \times 10^{-4} \text{ kg/m} \cdot \text{s} \]

The Reynolds number is thus

\[ \text{Re}_d = \frac{(978)(3)(0.02)}{4 \times 10^{-4}} = 146,700 \]

Consulting Figure 6-4, we find the friction factor as

\[ f = 0.0218 \quad f/8 = 0.002725 \]

Because \( T_w > T_b \), we take \( n = 0.11 \) and obtain

\[ \text{Nu}_d = \frac{(0.002725)(146,700)(2.54)}{1.07 + (12.7)(0.002725)^{1/2}(2.54^{2/3} - 1)} \left( \frac{5.55}{2.81} \right)^{0.11} = 666.8 \]

\[ h = \frac{(666.8)(0.664)}{0.02} = 22138 \text{ W/m}^2 \cdot \text{°C} \]

The tube length is then obtained from the energy balance

\[ q = h\pi d L (T_w - T_b) = 77,812 \text{ W} \]

\[ L = 1.40 \text{ m} \]
EXAMPLE 6-6 Turbulent Heat Transfer in a Short Tube

Air at 300 K and 1 atm enters a smooth tube having a diameter of 2 cm and length of 10 cm. The air velocity is 40 m/s. What constant heat flux must be applied at the tube surface to result in an air temperature rise of 5°C? What average wall temperature would be necessary for this case?

Solution

Because of the relatively small value of \( \frac{L}{d} = \frac{10}{2} = 5 \) we may anticipate that thermal entrance effects will be present in the flow. First, we determine the air properties at 300 K as

\[
\begin{align*}
\nu &= 15.69 \times 10^{-6} \text{m}^2/\text{s} \\
k &= 0.02624 \text{W/m} \cdot \degree\text{C} \\
c_p &= 1006 \text{J/kg} \cdot \degree\text{C} \\
p &= 1.18 \text{kg/m}^3
\end{align*}
\]

We calculate the Reynolds number as

\[
\text{Re}_d = \frac{ud}{\nu} = \frac{(40)(0.02)}{15.69 \times 10^{-6}} = 50.988
\]

so the flow is turbulent. Consulting Figure 6-6 for this value of \( \text{Re}_d \), \( \text{Pr} = 0.7 \), and \( \frac{L}{d} = 5 \) we find

\[
\frac{\text{Nu}_x}{\text{Nu}_\infty} \cong 1.15
\]

or the heat-transfer coefficient is about 15 percent higher than it would be for thermally developed flow. We calculate the heat-transfer coefficient for developed flow using

\[
\text{Nu}_d = 0.023 \text{Re}_d^{0.8} \text{Pr}^{0.4}
\]

\[
= 0.023(50988)^{0.8}(0.7)^{0.4} = 116.3
\]

and

\[
h = k\text{Nu}_d/d = (0.02624)(116.3)/0.02 = 152.6 \text{W/m}^2 \cdot \degree\text{C}
\]

Increasing this value by 15 percent,

\[
h = (1.15)(152.6) = 175.5 \text{W/m}^2 \cdot \degree\text{C}
\]

Increasing this value by 15 percent,

\[
h = (1.15)(152.6) = 175.5 \text{W/m}^2 \cdot \degree\text{C}
\]

The mass flow is

\[
\dot{m} = \rho u A_c = (1.18)(40)(0.02)^2/4 = 0.0148 \text{kg/s}
\]

so the total heat transfer is

\[
q = \dot{m}c_p \Delta T_b = (0.0148)(1006)(5) = 74.4 \text{W}
\]

This heat flow is convected from a tube surface area of

\[
A = \pi dL = \pi (0.02)(0.1) = 0.0628 \text{m}^2
\]

so the heat flux is

\[
q/A = 74.4/0.0628 = 11841 \text{W/m}^2 = h(T_w - T_b)
\]

We have

\[
\overline{T}_b = (300 + 305)/2 = 302.5 \text{K}
\]

so that

\[
\overline{T}_w = \overline{T}_b + 11841/175.5 = 302.5 + 67.5 = 370 \text{K}
\]
EXAMPLE 6-7  
Airflow Across Isothermal Cylinder

Air at 1 atm and 35°C flows across a 5.0-cm-diameter cylinder at a velocity of 50 m/s. The cylinder surface is maintained at a temperature of 150°C. Calculate the heat loss per unit length of the cylinder.

Solution

We first determine the Reynolds number and then find the applicable constants from Table 6-2 for use with Equation (6-17). The properties of air are evaluated at the film temperature:

\[ T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°C = 365.5 \text{ K} \]

\[ \rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(365.5)} = 0.966 \text{ kg/m}^3 \quad [0.0603 \text{ lbm/ft}^3] \]

\[ \mu_f = 2.14 \times 10^{-5} \text{ kg/m} \cdot \text{s} \quad [0.0486 \text{ lbm/ft} \cdot \text{s}] \]

\[ k_f = 0.0312 \text{ W/m} \cdot \text{°C} \quad [0.018 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}] \]

\[ \text{Pr}_f = 0.695 \]

\[ \text{Re}_f = \frac{\rho_\infty u_d}{\mu} = \left( \frac{0.966}{2.14 \times 10^{-5}} \right) = 1.129 \times 10^5 \]

From Table 6-2

\[ C = 0.0266 \quad n = 0.805 \]

so from Equation (6-17)

\[ \frac{h d}{k_f} = (0.0266)(1.129 \times 10^5)^{0.805}(0.695)^{1/3} = 275.1 \]

\[ h = \frac{(275.1)(0.0312)}{0.05} = 171.7 \text{ W/m}^2 \cdot \text{°C} \quad [30.2 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}] \]

The heat transfer per unit length is therefore

\[ \frac{q}{L} = h \pi d (T_w - T_\infty) \]

\[ = (171.7)(\pi)(0.05)(150 - 35) \]

\[ = 3100 \text{ W/m} \quad [3226 \text{ Btu/ft}] \]

EXAMPLE 6-8  
Heat Transfer from Electrically Heated Wire

A fine wire having a diameter of 3.94 \times 10^{-5} \text{ m} is placed in a 1-atm airstream at 25°C having a flow velocity of 50 m/s perpendicular to the wire. An electric current is passed through the wire, raising its surface temperature to 50°C. Calculate the heat loss per unit length.

Solution

We first obtain the properties at the film temperature:

\[ T_f = \frac{(25 + 50)}{2} = 37.5°C = 310 \text{ K} \]

\[ v_f = 16.7 \times 10^{-6} \text{ m}^2/\text{s} \quad k = 0.02704 \text{ W/m} \cdot \text{°C} \]

\[ \text{Pr}_f = 0.706 \]
The Reynolds number is
\[
Re_d = \frac{u_\infty d}{v_f} = \frac{(50)(3.94 \times 10^{-5})}{16.7 \times 10^{-6}} = 118
\]

The Peclet number is \( Pe = Re Pr = 83.3 \), and we find that Equations (6-17), (6-21), or (6-19) apply. Let us make the calculation with both the simplest expression, (6-17), and the most complex, (6-21), and compare results.

Using Equation (6-17) with \( C = 0.683 \) and \( n = 0.466 \), we have
\[
Nu_d = (0.683)(118)^{0.466}(0.705)^{1/3} = 5.615
\]

and the value of the heat-transfer coefficient is
\[
h = Nu_d \left( \frac{k}{d} \right) = 5.615 \frac{0.02704}{3.94 \times 10^{-5}} = 3854 \text{ W/m}^2 \cdot \text{°C}
\]

The heat transfer per unit length is then
\[
\frac{q}{L} = \pi dh(T_w - T_\infty) = \pi(3.94 \times 10^{-5})(3854)(50 - 25)
\]
\[
= 11.93 \text{ W/m}
\]

Using Equation (6-21), we calculate the Nusselt number as
\[
Nu_d = 0.3 + \frac{(0.62)(118)^{1/2}(0.705)^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}} \left[1 + (118/282,000)^{5/8}\right]^{4/5}
\]
\[
= 5.593
\]

and
\[
h = \frac{(5.593)(0.02704)}{3.94 \times 10^{-5}} = 3838 \text{ W/m}^2 \cdot \text{°C}
\]

and
\[
\frac{q}{L} = (3838)\pi(3.94 \times 10^{-5})(50 - 25) = 11.88 \text{ W/m}
\]

Here, we find the two correlations differing by 0.4 percent if the value from Equation (6-21) is taken as correct, or 0.2 percent from the mean value. Data scatter of ±15 percent is not unusual for the original experiments.

\[\text{EXAMPLE 6.9} \quad \text{Heat Transfer from Sphere}\]

Air at 1 atm and 27°F blows across a 12-mm-diameter sphere at a free-stream velocity of 4 m/s. A small heater inside the sphere maintains the surface temperature at 77°C. Calculate the heat lost by the sphere.

\[\text{Solution}\]
Consulting Equation (6-30) we find that the Reynolds number is evaluated at the free-stream temperature. We therefore need the following properties: at \( T_\infty = 27°C = 300 \text{ K}, \)
\[
v = 15.69 \times 10^{-6} \text{ m}^2/\text{s} \quad k = 0.02624 \text{ W/m} \cdot \text{°C},
\]
\[
Pr = 0.708 \quad \mu_\infty = 1.8462 \times 10^{-5} \text{ kg/m} \cdot \text{s}
\]
At \( T_w = 77°C = 350 \text{ K}, \)
\[
\mu_w = 2.075 \times 10^{-5}
\]
In contrast to Chapter 5, which was mainly analytical in character, this chapter has dealt almost entirely with empirical correlations that may be used to calculate convection heat transfer. The general calculation procedure is as follows:

1. Establish the geometry of the situation.
2. Make a preliminary determination of appropriate fluid properties.
3. Establish the flow regime by calculating the Reynolds or Peclet number
4. Select an equation that fits the geometry and flow regime and reevaluate properties, if necessary, in accordance with stipulations and the equation.
5. Proceed to calculate the value of $h$ and/or the heat-transfer rate.

The Reynolds number is thus

$$Re_d = \frac{(4)(0.012)}{15.69 \times 10^{-6}} = 3059$$

From Equation (6-30),

$$\overline{Nu} = 2 + [(0.4)(3059)^{1/2} + (0.06)(3059)^{2/3}](0.708)^{0.4} \left(\frac{1.8462}{2.075}\right)^{1/4}$$

$$= 31.40$$

and

$$\overline{h} = \overline{Nu} \left(\frac{k}{d}\right) = \frac{(31.4)(0.02624)}{0.012} = 68.66 \text{ W/m}^2 \cdot \text{°C}$$

The heat transfer is then

$$q = \overline{h} A (T_w - T_\infty) = (68.66)(4\pi)(0.006)^2(77 - 27) = 1.553 \text{ W}$$

For comparison purposes let us also calculate the heat-transfer coefficient using Equation (6-25). The film temperature is

$$T_f = \frac{350 + 300}{2} = 325 \text{ K}$$

so that

$$v_f = 18.23 \times 10^{-6} \text{ m}^2/\text{s} \quad k_f = 0.02814 \text{ W/m} \cdot \text{°C}$$

and the Reynolds number is

$$Re_d = \frac{(4)(0.012)}{18.23 \times 10^{-6}} = 2633$$

From Equation (6-25)

$$Nu_f = (0.37)(2633)^{0.6} = 41.73$$

and $\overline{h}$ is calculated as

$$\overline{h} = Nu \left(\frac{k_f}{d}\right) = \frac{(41.73)(0.02814)}{0.012} = 97.9 \text{ W/m}^2 \cdot \text{°C}$$

or about 42 percent higher than the value calculated before.
### Table 6-8 | Summary of forced-convection relations. (See text for property evaluation.)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Equation</th>
<th>Restrictions</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube flow</td>
<td>$N_u_d = 0.023 \text{Re}_d^{0.8} \text{Pr}^n$</td>
<td>Fully developed turbulent flow, $n = 0.4$ for heating, $n = 0.3$ for cooling, $0.6 &lt; \text{Pr} &lt; 100$, $2500 &lt; \text{Re}_d &lt; 1.23 \times 10^6$</td>
<td>(6-4a)</td>
</tr>
<tr>
<td>Tube flow</td>
<td>$N_u_d = 0.012 \text{Re}_d^{0.8} \text{Pr}^{0.4}$</td>
<td>$0.5 &lt; \text{Pr} &lt; 1.5$, $10^4 &lt; \text{Re}_d &lt; 5 \times 10^6$ $1.5 &lt; \text{Pr} &lt; 500$, $3000 &lt; \text{Re}_d &lt; 10^6$</td>
<td>(6-4c)</td>
</tr>
<tr>
<td>Tube flow</td>
<td>$N_u_d = 0.027 \text{Re}_d^{0.4} \text{Pr}^{1.3} \left( \frac{H}{\mu w} \right)^{0.14}$</td>
<td>Fully developed turbulent flow</td>
<td>(6-3)</td>
</tr>
<tr>
<td>Tube flow, entrance region</td>
<td>$N_u_d = 0.036 \text{Re}_d^{0.4} \text{Pr}^{1.3} \left( \frac{d}{L} \right)^{0.055}$</td>
<td>Turbulent flow</td>
<td>(6-6)</td>
</tr>
<tr>
<td>Tube flow</td>
<td>See also Figures 6-5 and 6-6</td>
<td>Turbulent flow</td>
<td>(6-6)</td>
</tr>
<tr>
<td>Tube flow</td>
<td>Petukov relation</td>
<td>Fully developed turbulent flow, $0.5 &lt; \text{Pr} &lt; 1000$, $10^4 &lt; \text{Re}_d &lt; 5 \times 10^6$, $0 &lt; \frac{\rho w}{\mu} &lt; 40$</td>
<td>(6-7)</td>
</tr>
<tr>
<td>Tube flow</td>
<td>$N_u_d = 3.66 + \frac{0.0668(d/L) \text{Re}_d \text{Pr}}{1 + 0.04((d/L) \text{Re}_d \text{Pr})^{2/3}}$</td>
<td>Laminar, $T_w$ = const.</td>
<td>(6-9)</td>
</tr>
<tr>
<td>Tube flow</td>
<td>$N_u_d = 1.86(\text{Re}_d \text{Pr})^{1/3} \left( \frac{d}{L} \right)^{1/3} \left( \frac{\mu}{\mu w} \right)^{0.14}$</td>
<td>Fully developed laminar flow, $T_w$ = const, $\text{Re}_d \text{Pr} \frac{d}{L} &gt; 10$</td>
<td>(6-10)</td>
</tr>
<tr>
<td>Rough tubes</td>
<td>$St_f \frac{Pr}{f} = \frac{f}{8}$ or Equation (6-7)</td>
<td>Fully developed turbulent flow</td>
<td>(6-12)</td>
</tr>
</tbody>
</table>

| Rough tubes | Reynolds number evaluated on basis of hydraulic diameter: $\text{Re}_d = \frac{4A}{L}$, $A$ = flow cross-section area, $P$ = wetted perimeter |

| Noncircular ducts | Same as particular equation for tube flow |

| Flow across cylinders | $N_u = C \text{Re}_d^{1/2} \text{Pr}^{1/3}$ and $n$ from Table 6-2 | $0.4 < \text{Re}_d < 400,000$, $0.4 < \text{Pr} < 10^7$, $P_c > 0.2$ | (6-17) |
| Flow across cylinders | $N_u = a + b \text{Re}_d^{1/2} \text{Pr}^{1/3}$ | See text | (5-18) to (5-20) |
| Flow across cylinders | $N_u = a + b \text{Re}_d^{1/2} \text{Pr}^{1/3}$ | See Table 6-3 for values of $C$ and $n$. | (5-17) |
Subscripts. \( b \) = bulk temperature, \( f \) = film temperature, \( \infty \) = free stream temperature, 
\( w \) = wall temperature

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Equation</th>
<th>Restrictions</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow across spheres</td>
<td>( \text{Nu}_f = 0.37 \text{Re}_f^{0.6} )</td>
<td>( \text{Pr} \sim 0.7 ) (gases), ( 17 &lt; \text{Re} &lt; 70,000 )</td>
<td>(6-25)</td>
</tr>
<tr>
<td></td>
<td>( \text{Nu}_f \text{Pr}^{-0.3} \left( \frac{\mu_f}{\mu} \right)^{0.25} = 1.2 + 0.53 \text{Re}_f^{0.54} )</td>
<td>Water and oils ( 1 &lt; \text{Re} &lt; 200,000 ) Properties at ( T_{\infty} )</td>
<td>(6-29)</td>
</tr>
<tr>
<td></td>
<td>( \text{Nu}_f = 2 + \left( 0.4 \text{Re}_f^{1/2} + 0.06 \text{Re}_f^{2/3} \right) \text{Pr}^{-0.4} \left( \frac{\mu_f}{\mu} \right)^{1/2} )</td>
<td>( 0.7 &lt; \text{Pr} &lt; 380, 3.5 &lt; \text{Re}<em>f &lt; 80,000 ) Properties at ( T</em>{\infty} )</td>
<td>(6-30)</td>
</tr>
<tr>
<td>Flow across tube banks</td>
<td>( \text{Nu}_f = C \text{Re}_f^{1/2} \text{Pr}_f^{1/3} ) ( C ) and ( n ) from Table 6-4</td>
<td>See text</td>
<td>(6-17)</td>
</tr>
<tr>
<td>Flow across tube banks</td>
<td>( \text{Nu}_f = C \text{Re}_f^{1/2} \text{Pr}_f^{1/3} \left( \frac{\text{Pr}}{\text{Pr}_f} \right)^{1/4} )</td>
<td>( 0.7 &lt; \text{Pr} &lt; 500, 10 &lt; \text{Re}_f &lt; 10^6 )</td>
<td>(6-34)</td>
</tr>
<tr>
<td>Liquid metals</td>
<td>( \Delta \mu = f(L, \delta) \rho_f u_m^2 / 2 \mu_f ) ( u_m = Re / \mu A_c )</td>
<td>See text</td>
<td>(6-13)</td>
</tr>
<tr>
<td>Friction factor</td>
<td>( \Delta \mu = f(L, \delta) \rho_f u_m^2 / 2 \mu_f ) ( u_m = Re / \mu A_c )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6-6** | Constants for Zukauskas correlation [Equation (6-34)] for heat transfer in tube banks of 20 rows or more.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>( \text{Re}_{f, \max} )</th>
<th>( C )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-line</td>
<td>10–100</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>100–10^2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 10^3 )–2 ( \times ) 10^5</td>
<td>0.27</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>( &gt; 2 \times 10^5 )</td>
<td>0.21</td>
<td>0.84</td>
</tr>
<tr>
<td>Staggered</td>
<td>10–100</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>100–10^3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 10^3 )–2 ( \times ) 10^5</td>
<td>0.35 \left( \frac{S_n}{S_L} \right)^{0.2}</td>
<td>0.60 ( S_n &lt; S_L )</td>
</tr>
<tr>
<td></td>
<td>( &gt; 2 \times 10^5 )</td>
<td>0.40</td>
<td>0.60</td>
</tr>
</tbody>
</table>

From Reference 39.
1) Consider the following fluids at a film temperature of 300 K in parallel flow over a flat plate with velocity of 1 m/s: atmospheric air, water, and engine oil. For each fluid, determine the local convection coefficient at $x = 40$ mm, and the average value over the distance from $x = 0$ to $x = 40$ mm.

2) Air at atmospheric pressure and a temperature of 300°C flows steadily with a velocity of 10 m/s over a flat plate of length 0.5 m. Estimate the cooling rate per unit width of the plate needed to maintain a surface temperature of 27°C.

3) A flat plate of width $w = 1$ m is maintained at a uniform surface temperature, $T_s = 230^\circ$C, by using independently controlled, electrical strip heaters, each of which is 50 mm long. If atmospheric air at 25°C flows over the plates at a velocity of 50 m/s, what is the electrical power requirement for the fifth heater?

4) Engine oil at 100°C and a velocity of 0.1 m/s flows over both surfaces of a 1-m-long flat plate maintained at 20°C. Determine the following:
   (a) The local heat flux at the trailing edge.
   (b) The total heat transfer per unit width of the plate.

5) Air at a pressure and a temperature of 1 atm and 50°C, respectively, is in parallel flow over the top surface of a flat plate that is heated to a uniform temperature of 100°C. The plate has a length of 0.20 m (in the flow direction) and a width of 0.10 m. The Reynolds number based on the plate length is 40,000. What is the rate of heat transfer from the plate to the air?

6) Consider the wing of an aircraft as a flat plate of 2.5-m length in the flow direction. The plane is moving at 100 m/s in air that is at a pressure of 0.7 bar and a temperature of -10°C. The top surface of the wing absorbs solar radiation at a rate of 800 W/m². Assume the wing to be of solid construction and to have a single, uniform temperature. Estimate the steady-state temperature of the wing.

7) Consider the following fluids, each with a velocity of $u_\infty = 5$ m/s and a temperature of $T_\infty = 20^\circ$C, in cross flow over a 10-mm-diameter cylinder maintained at 50°C: atmospheric air, saturated water, and engine oil. Calculate the rate of heat transfer per unit length, $q^*$.

8) Assume that a person can be approximated as a cylinder of 0.3-m diameter and 1.8-m height with a surface temperature of 24°C. Calculate the body energy loss while this person is subjected to a 15-m/s wind whose temperature is 5°C.

9) Water at 20°C flows over a 20-mm-diameter sphere with a velocity of 5 m/s. The surface of the sphere is at 60°C. What is the rate of heat transfer from the sphere?

10) Air at 25°C flows over a 10-mm-diameter sphere with a velocity of 25 m/s, while the surface of the sphere is maintained at 75°C. What is the rate of heat transfer from the sphere?

11) Atmospheric air at 25°C and a velocity of 0.5 m/s flows over a 50-W incandescent bulb whose surface temperature is at 140°C. The bulb may be approximated as a sphere of 50-mm diameter. What is the rate of heat transfer by convection to the air?
Example 17.2  Laminar Flow over a Flat Plate

Air at atmospheric pressure and a temperature of 300°C flows steadily with a velocity of 10 m/s over a flat plate of length 0.5 m. Estimate the cooling rate per unit width of the plate needed to maintain a surface temperature of 27°C.

Solution

**Known:** Airflow over an isothermal flat plate.

**Find:** Cooling rate per unit width of the plate, \( q' \) (W/m).

**Schematic and Given Data:**

\[
\begin{align*}
T_i &= 300°C \\
T_s &= 27°C \\
\alpha &= 10 \text{ m/s} \\
L &= 0.5 \text{ m}
\end{align*}
\]

**Assumptions:**
1. Steady-state conditions.
2. Negligible radiation exchange with surroundings.

**Properties:** Table HT-3, air \((T_f = 437 \text{ K}, p = 1 \text{ atm})\): \(\nu = 30.84 \times 10^{-6} \text{ m}^2/\text{s}, k = 36.4 \times 10^{-3} \text{ W/m} \cdot \text{K}, Pr = 0.687\).

**Analysis:** For a plate of unit width, it follows from Newton’s law of cooling that the rate of convection heat transfer to the plate is

\[
q' = \dot{h}L(T_m - T_s)
\]

To select the appropriate convection correlation for estimating \( \dot{h} \), the Reynolds number must be determined to characterize the flow.

\[
Re = \frac{\alpha L}{\nu} = \frac{10 \text{ m/s} \times 0.5 \text{ m}}{30.84 \times 10^{-6} \text{ m}^2/\text{s}} = 1.62 \times 10^9
\]

Since \( Re < Re_c = 5 \times 10^6 \), the flow is laminar over the entire plate, and the appropriate correlation is given by Eq. 17.26 (see also Table 17.5, page 273).

\[
\bar{Nu} = 0.664 Re^{1/2} Pr^{1/3} = 0.664(1.62 \times 10^9)^{1/2}(0.687)^{1/3} = 236
\]

The average convection coefficient is then

\[
\bar{h} = \frac{\bar{Nu} \cdot k}{L} = \frac{236 \times 0.0336 \text{ W/m} \cdot \text{K}}{0.5 \text{ m}} = 17.2 \text{ W/m}^2 \cdot \text{K}
\]

and the required cooling rate per unit width of plate is

\[
q' = 17.2 \text{ W/m}^2 \cdot \text{K} \times 0.5 \text{ m}(300 - 27)^{\circ} \text{C} = 2348 \text{ W/m} < 1
\]

**Comments:**

1. Note that the thermophysical properties are evaluated at the film temperature, \( T_f = (T_m + T_s)/2 \), Eq. 17.20.
2. Using Eq. 7.21, the hydrodynamic boundary layer thickness at the trailing edge of the plate \((x = L = 0.5 \text{ m})\) is

\[
\delta = 5 \lambda Re^{1/2} = 5 \times 0.5 \text{ m}(1.62 \times 10^9)^{-1/2} \approx 0.0069 \text{ m} = 7.0 \text{ mm}
\]

The thermal boundary layer at the same location from Eq. 17.24 is

\[
\delta_t = 0.01 \text{ Pr}^{1/3} = 0.2 \text{ mm}(0.687)^{1/3} = 7.0 \text{ mm}
\]

Since \( Pr = 0.7 < 1 \), we find that \( \delta < \delta_t \). Still, note that the magnitudes of the boundary layer thicknesses, \( \delta \) and \( \delta_t \), are quite similar as expected for gases.
3. If upstream turbulence is induced by a fan or grill, or a strip wire were placed at the leading edge, a turbulent boundary condition could exist over the entire plate. For such a condition, Eq. 17.36 is the appropriate correlation to estimate the convection coefficient

\[
\bar{Nu} = 0.037 Re^{1/2} Pr^{1/3} = 0.037(1.62 \times 10^9)^{1/2} (0.687)^{1/3} = 480
\]

\[
\bar{h} = 480(36.4 \times 10^{-3} \text{ W/m} \cdot \text{K})/0.5 \text{ m} = 35.0 \text{ W/m}^2 \cdot \text{K}
\]

The cooling rate per unit plate width is

\[
q' = 35 \text{ W/m}^2 \cdot \text{K} \times 0.5 \text{ m}(300 - 27)^{\circ} \text{C} = 1778 \text{ W/m}
\]

The effect of inducing turbulence over the entire plate is to double the convection coefficient, and hence, double the required cooling rate.