Digital Pass-band Transmission (Digital modulation)

Pass-band modulation is the process by which an information signal is converted to a sinusoidal waveform, for digital modulation, such a sinusoidal of duration T is referred to as a digital symbol. The sinusoidal has just three features that can be used to distinguish it from other sinusoids: amplitude, frequency, and phase. Thus pass-band modulation can be defined as the process whereby the amplitude, frequency and phase of an RF carrier, or a combination of them, is varied in accordance with the information to be transmitted. The most common digital modulation formats: -

1- Amplitude shift keying (ASK).
2- Frequency shift keying (FSK).
3- Phase shift keying (PSK).
4- Amplitude phase keying (APK).

➢ Amplitude shift keying (ASK)

In amplitude shift keying, the amplitude of high frequency carrier signal is switched between two or more values. For the binary case, the usual choice is on-off keying (OOK). Assume a sequence of binary pulses, as shown in figure below the 1’s turn ON the carrier of amplitude A, the 0’s turn it OFF. The ASK waveform for all pulses (i.e. binary 1) can be written as: -
\[ \phi(t) = \begin{cases} A \sin \omega_c t & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases} \]

\textbf{Detection of ASK:-}

The impulse response of the match filter for optimum detection of this ASK waveform in the presence of white noise is:

\[ h(t) = \phi(T - t) \]

The matched filter output for the (noiseless) input \( \phi(t) \)

\[ y(t) = \phi(t) \otimes h(t) \]

\[ = \int_{-\infty}^{\infty} \phi(\tau)\phi(T - t + \tau) d\tau \]

\[ = r\phi(T - t) \]

where \( r\phi(t) \) is the time autocorrelation for \( \phi(t) \)

The optimum decision time is for \( t = T \), so that

\[ y(T) = r\phi(0) = E \]
A sketch of the match filter output is shown in figure below. The signal energy may be find as

$$E = \int_0^T A^2 \sin^2 \omega_c t \, dt = \frac{A^2 T}{2}$$

➢ **Frequency shift keying (FSK)**

Binary FSK is a form of constant amplitude, angle modulation. FSK waveform can be considered as composed of two ASK waveforms of differing carrier frequencies. Thus to convey either of the binary symbols, we have a choice of the two waveforms:

$$\phi_1(t) = \begin{cases} A \sin \omega_1 t & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases}$$

$$\phi_2(t) = \begin{cases} A \sin \omega_2 t & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases}$$

Thus a typical pair of sinusoidal waveform is described by:-
\[ s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_i t & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \]

where \( i = 1, 2 \) and \( E_b \) is the transmitted energy per bit. A simple binary FSK output waveform shown below

- **Generation and detection of binary FSK signals:**

  To generate a binary FSK signal, the scheme shown in figure below may be used:
The incoming binary data sequence is first applied to an on-off level encoder, at the output of which symbol 1 is represented by a constant amplitude of $\sqrt{Eb}$ volt and symbol 0 is represented zero volt. By using inverter in the lower channel, we in effect make sure that when we have symbol 1 at the input, the oscillator with frequency $f_1$ in the upper channel is switched on, while the oscillator with frequency $f_2$ in the lower channel is switched off, with the result that frequency $f_1$ is transmitted. Conversely, when we have symbol 0 at the input, the oscillator in the upper channel is switched off and the oscillator in the lower channel is switched on, with the result that frequency $f_2$ is transmitted. The two frequencies $f_1$ and $f_2$ are chosen to equal integer multiple of bit rate $\frac{1}{T_b}$.

To detect the original binary sequences given the noisy received signal $x(t)$, we may use the receiver shown in figure below (coherent detection).

For noncoherent detection of frequency modulated wave, the receiver consists of a pair of matched filters followed by envelope detectors, as shown in figure below:
The filter in the upper path of the receiver is matched to \( \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) \), and filter in the lower path is matched to \( \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \), and \( 0 \leq t \leq T_b \). The resulting outputs form envelope detectors are sampled at \( t = T_b \), and their values are compared. The envelope samples of the upper and lower paths in above figure are shown as \( L_1 \) and \( L_2 \), respectively. Then if \( L_1 > L_2 \), the receiver decided in a favor of symbol 1, and if \( L_1 < L_2 \), the receiver decided in a favor of symbol 0. If \( L_1 = L_2 \) the receiver simply makes a guess in favor of symbol 1 or 0.
Bandwidth consideration of FSK:

The frequency deviation given by

\[
\Delta f = \frac{f_2 - f_1}{2}
\]

where \( f_1 = f_c - \Delta f \), \( f_2 = f_c + \Delta f \).

The output spectrum for an FSK signal can be represented as shown in figure below.

The bandwidth for FSK can be approximated as:

\[
B, W = 2\Delta f = 2B
\]

where B is the original base-band binary signal bandwidth.

Phase shift keying (PSK)

PSK is another form of angle modulated, constant-amplitude digital modulation. PSK is similar to conventional phase modulation except that with PSK the input signal in binary signal and limited number of output phases are possible.
1) **Binary phase shift keying (BPSK):**

With BPSK two output phases are possible for a single carrier frequency. One output phase representing logic 1 and the other logic 0. At the input digital signal change state, the phase of the output carrier shifts between two angles that are 180° out of phase. Other name of BPSK is phase reversal keying (PRK).

![](image1.png)

**BPSK transmitter:**

Figure below shows a simplified block diagram of a BPSK modulator. The balanced modulator acts as a phase reversing switch depending on the logic condition of the digital input, the carrier is transferred to the output either in phase or 180° out of phase with the reference carrier oscillator.

![](image2.png)

- **Ring modulator:**

Figure below shows the schematic diagram of a balance ring modulator. If the binary input is a **logic 1** (positive voltage), $D_1$ and $D_2$ are on (forward biased). While $D_3$ and $D_4$ are off (reversed biased). Therefore, the output carrier is in phase with input carrier. If the binary input is a **logic 0** (negative voltage), $D_3$ and $D_4$ are on. While $D_1$ and $D_2$
are off. Therefore, the output carrier is 180° out of phase with input carrier.
Figure below shows output phase-versus-time relationship, truth table, phaser diagram, and constellation diagram (some time called state-space diagram).
\**BPSK receiver:**

Figure below shows the block diagram of BPSK receiver.

The input received signal may be $+\sin \omega_c t$ or $-\sin \omega_c t$. For a BPSK input signal of $+\sin \omega_c t$ (logic 1), the output of the balanced modulator is

$$\text{output} = \sin \omega_c t \times \sin \omega_c t = \sin^2 \omega_c t$$

$$\sin^2 \omega_c t = \frac{1}{2} (1 - \cos 2\omega_c t) = \frac{1}{2} - \frac{1}{2} \cos 2\omega_c t$$

$$\frac{1}{2} \cos 2\omega_c t \text{ Filtered out by LPF}$$

$$\therefore \text{output} = + \frac{1}{2} V = \text{logic 1}$$

For a BPSK input signal of $-\sin \omega_c t$ (logic 0), the output of the balanced modulator is

$$\text{output} = -\sin \omega_c t \times \sin \omega_c t = -\sin^2 \omega_c t$$

$$-\sin^2 \omega_c t = -\frac{1}{2} (1 - \cos 2\omega_c t) = -\frac{1}{2} + \frac{1}{2} \cos 2\omega_c t$$

$\diamondsuit$
\[ \frac{1}{2} \cos 2\omega_c t \] Filtered out by LPF
\[ \therefore \text{output} = -\frac{1}{2} V = \log ic0 \]

**Bandwidth consideration of BPSK:**

\[ \therefore \text{output} = \sin \omega_c t \times \sin \omega_t \]
\[ = \frac{1}{2} \cos(\omega_c - \omega_d)t - \frac{1}{2} \cos(\omega_c + \omega_d)t \]
\[ \therefore \text{The minimum duple side Nyquist bandwidth (f}_N\text{) is} \]
\[ (\omega_c - \omega_d) - (\omega_c + \omega_d) = 2\omega_a = 2f_a \]
\[ \therefore f_a = \frac{R_b}{2} \]
\[ \therefore f_N = 2\left(\frac{R_b}{2}\right) = R_b \]

**Example:-**
For BPSK modulator \( f_c=70\text{MHz} \), input bit rate \( R_b=10\text{Mbps} \). Determine
(1) maximum and minimum upper and lower side frequency.
(2) the minimum Nyquist bandwidth.
(3) baud rate.

**Solution:-**
\[ \therefore \text{output} = \sin \omega_a t \times \sin \omega_c t \]
\[ \frac{1}{2} \cos(\omega_c - \omega_d)t - \frac{1}{2} \cos(\omega_c + \omega_d)t \]

\[ = \frac{1}{2} \cos 2\pi (70 - 5) \times 10^6 t - \frac{1}{2} \cos 2\pi (70 + 5) \times 10^6 t \]

\[ \therefore \text{LSB}= 70\text{MHz}-5\text{MHz} = 65\text{MHz} \]

\[ \text{USB}= 70\text{MHz}+5\text{MHz} = 75\text{MHz} \]

\[ \text{min. Nyquist B.W (}f_N\text{)} = 75-65 = 10\text{MHz} \]

Baud rate (symbol rate) = bit rate = 10Mbaud

2) **M-ary encoding**

M-ary is term derived from the word “binary”. M is simply a digital that represents the number of conditions possible.

\[ m = \log_2 M \quad M = 2^m \]

Where \( m \) = number of bits, \( M \) = number of output conditions possible with \( m \) bits. For example, if \( m = 1 \) (binary signal), \( M = 2 \). If \( m = 2 \) (quaternary signal), \( M = 4 \) (four different output conditions are possible).

2.1 **Quaternary phase shift keying (QPSK)**

With QPSK four output phases are possible for a single carrier frequency therefore must be four input conditions. With 2-bit (\( M=4 \)), there are four possible conditions (00, 01, 10, 11). Therefore, with QPSK, the binary input data are combined into group of two bits called
dibits. Each dibits code generator one of four possible output phases. Therefore, for each 2-bit clocked into the modulator, a single output change occurs. Therefore, the rate of change at the output (baud rate) is one half of the input bit rate.

\textbf{QPSK Transmitter}

QPSK modulator is two BPSK modulators combined in parallel. When the linear summer combined the two quadrature (90° out of phase) signals, there are four possible resultant phases given by these expressions ($+ \sin \omega_c t + \cos \omega_c t$, $+ \sin \omega_c t - \cos \omega_c t$, $- \sin \omega_c t + \cos \omega_c t$, $- \sin \omega_c t - \cos \omega_c t$). A block diagram of QPSK transmitter is shown in figure below.
The output phase versus time relationship, truth table, phaser diagram, and the constellation diagram, for a QPSK modulator are shown below:

\begin{itemize}
\item \textbf{Bandwidth consideration of QPSK}
\end{itemize}

The bit rate in either I or Q channel is equal to one half of the input data rate \((R_b/2)\).

\[ \therefore \text{The highest fundamental frequency (}f_a\text{) present at the data input to the I or the Q balanced modulator=} R_b/4 \]
\[ \text{Minimum required bandwidth} = 2f_a = R_b/2, \quad \text{and baud rate} = \text{bit rate}/2 = R_b/2. \]

\[ \text{Bandwidth required for QPSK} = 1/2 \text{ bandwidth required for BPSK}. \]

**Example:-**
For QPSK modulator \( f_c = 70\text{MHz} \), input bit rate \( R_b = 10\text{Mbps} \). Determine
(1) the minimum Nyquist bandwidth.
(2) baud rate.
(3) compare the results with BPSK.

**Solution:-**
\[ R_{bQ} = R_{bl} = \frac{R_b}{2} = 5\text{Mbps} \]
\[ f_a = \frac{R_{bQ}}{2} = \frac{R_{bl}}{2} = 2.5\text{Mbps} \]
\[ \text{min. B.W} = 2f_a = 5\text{MHZ} \]

\[ \text{Output} = \sin \omega_at \times \sin \omega_ct \]
\[ = \frac{1}{2} \cos(\omega_c - \omega_a)t - \frac{1}{2} \cos(\omega_c + \omega_a)t \]
\[ = \cos 2\pi(70 - 2.5) \times 10^6 t - \frac{1}{2} \cos 2\pi(70 + 2.5) \times 10^6 t \]
\[ = \cos 2\pi(67.5) \times 10^6 t - \frac{1}{2} \cos 2\pi(72.5) \times 10^6 t \]

\[ \Delta \]
min. Nyquist B.W \( (f_N) = 72.5 - 67.5 = 5\text{MHz} \)

Baud rate (symbol rate) = bit rate/2 = 5Mbaud

\[ \therefore \text{B.W for QPSK} = \frac{1}{2} \text{B.W for BPSK} \]

**QPSK receiver**

The block diagram of QPSK receiver is shown below.

---

**Example:**

Prove how we can find binary out from QPSK demodulator?

**Solution:**

Let the incoming QPSK signal be \(- \sin \omega_c t + \cos \omega_c t\)

\[ \therefore I - \text{channel} = \sin \omega_c t(-\sin \omega_c t + \cos \omega_c t) \]

\[ = -\sin^2 \omega_c t + \cos \omega_c t \sin \omega_c t \]

\[ = -\frac{1}{2}(1 - \cos 2\omega_c t) - \frac{1}{2} \sin 2\omega_c t + \frac{1}{2} \sin 0 \]

\[ \therefore \]
= \frac{1}{2} - \frac{1}{2} \cos 2\omega_c t - \frac{1}{2} \sin 2\omega_c t

The terms \( \frac{1}{2} \cos 2\omega_c t - \frac{1}{2} \sin 2\omega_c t \) filtered by LPF.

\[ \therefore \text{output} = \frac{1}{2} \text{ (logic 0)} \therefore I = 1 \]

\[ Q \text{ - channel} = \cos \omega_c t (-\sin \omega_c t + \cos \omega_c t) \]

\[ = -\cos \omega_c t \sin \omega_c t + \cos^2 \omega_c t \]

\[ = \frac{1}{2} (1 + \cos 2\omega_c t) - \frac{1}{2} \sin 2\omega_c t - \frac{1}{2} \sin 0 \]

\[ = \frac{1}{2} + \frac{1}{2} \cos 2\omega_c t - \frac{1}{2} \sin 2\omega_c t \]

The terms \( \frac{1}{2} \cos 2\omega_c t - \frac{1}{2} \sin 2\omega_c t \) filtered by LPF.

\[ \therefore \text{output} = \frac{1}{2} \text{ (logic 1)} \therefore Q = 1 \]

2.2 Eight-phase shift keying (8-PSK)

With 8-PSK modulator, there are eight possible output phases (M=8). To encoded eight different phases, the incoming bits are considered in group of 3-bits, called tribits \( (2^3=8) \).
8-PSK transmitter

Figure below shows the 8-PSK transmitters. The incoming serial bits stream enters the bit splitter, where it is converted to parallel, three channel output (the I or inphase channel, the Q or quadrature phase channel and C or control channel). The bit rate in each three channel is $R_b/3$. The 2-to-4 level converter are parallel input digital to analog converter (ADC), with 2 input bits, four output voltages are possible.

Figure below shows the truth table and corresponding output conditions for the 2-to-4 level converters.
Example:-

For Q=0, I=0, and c=0 (000), determine the output phase for the 8-PSK modulator.

Solution:-

\[ I = 0 \& c = 0 \]

\[ \therefore the\ output\ from\ the\ 2\text{-to-}4\ level\ converter\ in\ I\text{-channel}= -0.541. \]

\[ Q = 0 \& c = 1 \]

\[ \therefore the\ output\ from\ the\ 2\text{-to-}4\ level\ converter\ in\ I\text{-channel}= -1.307. \]

\[ I = -0.541\sin \omega_c t \]

\[ Q = -1.307\cos \omega_c t \]

\[ \text{summer output} = (-0.541\cos \omega_c t)(-1.307\cos \omega_c t) \]

\[ = 1.41\sin(\omega_c t - 112.5^\circ) \]

Figure below shows the output phase versus time relationship, truth table, phaser diagram, and constellation diagram for 8PSK modulator:-
**Bandwidth consideration of 8-PSK**

- With 8-PSK the data divided into three channels.
- The bit rate in the I, Q and c = \( R_b/3 \).
- The highest fundamental frequency \( f_a = R_b/6 \).

and the minimum required bandwidth = \( 2f_a = R_b/3 \), and baud rate = bit rate/3 = \( R_b/3 \).

\[ \therefore \text{Bandwidth required for 8-PSK} = \frac{1}{3} \text{bandwidth required for BPSK}. \]

**Example:**

For 8-PSK modulator \( f_c = 70 \text{MHz} \), input bit rate \( R_b = 10 \text{Mbps} \). Determine

1. the minimum Nyquist bandwidth.
2. USB and LSB
3. baud rate.

**solution:**

\[
R_{bQ} = R_{bl} = R_{bc} = \frac{R_b}{3} = 3.33 \text{Mbps}
\]

\[
f_a = \frac{R_{bQ}}{2} = \frac{R_{bl}}{2} = \frac{R_{bc}}{2} = 1.667 \text{Mbps}
\]

\[ \therefore \text{min BW} = 2f_a = 3.33 \text{MHZ} \]

\[ \therefore \text{output} = \sin \omega_c t \times \sin \omega_a t \]

\[ = \frac{1}{2} \cos(\omega_c - \omega_a)t - \frac{1}{2} \cos(\omega_c + \omega_a)t \]

\[ \therefore \]
\[
\frac{1}{2} \cos 2\pi(70 - 1.667) \times 10^6 t - \frac{1}{2} \cos 2\pi(70 + 1.667) \times 10^6 t
\]

\[
= \frac{1}{2} \cos 2\pi(68.333) \times 10^6 t - \frac{1}{2} \cos 2\pi(71.667) \times 10^6 t
\]

min. Nyquist B.W (\(f_N\)) = 71.667 - 68.333 = 3.33MHz

Baud rate (symbol rate) = bit rate/3 = 3.33Mbaud

\[\because\text{ B.W for 8-PSK=}1/3 \text{ B.W for BPSK}\]

\[\textbf{8-PSK receiver}\]

The block diagram of 8PSK receiver is shown below. (Prove how we can find binary out?).
2.3 Sixteen-phase shift keying (16-PSK)

With 16-PSK there are 16 different output phases possible. Baud rate = $R_b/4$. With 16-PSK the angular separation between adjacent output phases is 22.5°. The truth table and constellation diagram of 16-PSK are shown below.

➢ Offset QPSK (OQPSK)

OQPSK is a modified form of QPSK where the bit waveform on the I and Q channels are offset or shifted in phase from each other by one half of a bit time. As shown in figure below.
Because change in the I channel occur at the mid point of the Q channel bit, and vice versa, there is never more than a single changes in the dibit code, and therefore, there is never more than a $90^\circ$ shift in the output phase. Therefore, an advantage of OQPSK is the limited phase shift that must be imparted during modulation. A disadvantages of OQPSK is that changes in the output phase occur at twice the data rate in either the I and Q channels. Consequently with OQPSK the baud and minimum bandwidth are twice that of conventional QPSK for a given transmission bit rate. OQPSK is some times called offset-keyed QPSK (OKQPSK).

- **Differential phase shift keying (DPSK)**

  DPSK is an alternative of digital modulation where the binary input information is contained in the difference between two successive signaling element rather than absolute phase. With DPSK it is not necessary to recover a phase coherent carrier. Instead, a received signaling element is delayed by one signaling element time slot and then compared to the next received signaling element. The difference in
the phase of the two signaling elements determines the logic condition of the data. DBPSK transmitter is shown in figure below.

Figure below shows the block diagram and timing sequence of DPSK receiver.

The primary advantage of DBPSK is the simplicity with which it can be implemented. With DBPSK no carrier recovery circuit is needed. A disadvantage of DBPSK is that it required between 1dB to 3dB more SNR to achieve the same bit error rate as that absolute value.
Ñ Quadrature amplitude modulation (QAM)

QAM is a form of digital modulation where the digital information is contained in both the amplitude and phase of the transmitted carrier.

1) Eight QAM (8-QAM)

8-QAM is an M-ary encoding techniques where M=8. Unlike 8-PSK, the output signal from an 8-QAM modulator is not a constant amplitude signal.

¢ 8-QAM transmitter

Figure below shows a block diagram of 8-QAM transmitter. We can see only the difference between 8-QAM and 8-PSK transmitters is the omission of the inverter between the C channel and Q channel.
Example

For a tribit of Q=0, I=0, and C=0. Determine the output amplitude and phase for the 8-QAM transmitter.

Solution:-

As in 8-PSK, I and Q bits determine the polarity of the PAM signal, and C bit determine the value of levels.

At I-channel I=0, C=0

\[ \Rightarrow \text{The output from 2-4 level converter} = -0.541 \]

and the output from modulator = \(-0.541\sin \omega_c t\)

At Q-channel Q=0, C=0

\[ \Rightarrow \text{The output from 2-4 level converter} = -0.541 \]

and the output from modulator = \(-0.541\cos \omega_c t\)

the output from I and Q channel product modulator are combined in a linear summer and produce a modulated output of

\[ \text{summer output} = -0.541\sin \omega_c t - 0.541\cos \omega_c t \]

\[ = 0.765\sin(\omega_c t - 135^\circ) \]

For the remaining codes (001, 010, 011, 100, 101, 111) the procedure is the same. The results are shown in figure below.
8-QAM receiver

8-QAM receiver is almost identical to the 8PSK receiver. The differences are the PAM levels at the output of the product detectors and the binary signals at the output of the A/D converter. Because that there two transmitted amplitudes possible with 8QAM that there are different from those achievable with 8PSK, the four demodulated PAM levels in 8-QAM are different from those in 8PSK. There is the conversion factor for A/D converter must also be different. Also, with QAM the binary output from I channel A/D converter are the I & C bits, and from Q channel are the Q & C bits.

Note The minimum bandwidth required for 8-QAM is the same as in 8-PSK, $R_b/3$. 
2) *Sixteen QAM (16-QAM)*

As with 16-PSK, 16-QAM is an M-ary system where M=16. The input data are acted on in group of four \(2^4=16\).

**16-QAM transmitter**

The block diagram of 16-QAM transmitter is shown in figure below.

---

**Example**

For a quadbit input I=0, I`=0, Q=0, and Q`=0 \((0000)\). Determine the output amplitude and phase for the 16-QAM transmitter.

**Solution:-**

The I-channel output = \(-0.22 \sin \omega_c t\)

The Q-channel output = \(-0.22 \cos \omega_c t\)

Summer output = \(-0.22 \sin \omega_c t - 0.22 \cos \omega_c t\)

\[= 0.311 \sin(\omega_c t - 135^\circ)\]
For the remaining codes the procedure is the same. The results are shown in figure below.

➢ **Note**

The minimum bandwidth required for 8-QAM is the same as in 8-PSK, $R_b/3$. 
Example:-

For 16-QAM modulator, \( f_c = 70\text{MHz} \), input bit rate \( R_b = 10\text{Mbps} \). Determine (1) USB and LSB. (2) the minimum Nyquist bandwidth. (3) baud rate.

solution:-

\[
\text{min. } B.W = \frac{R_b}{4} = 2.5 \text{MHz}
\]

(1) USB = 70 + 1.25 = 71.25 MHz.

LSB = 70 - 1.25 = 68.75 MHz.

(2) min. Nyquist B.W \( (f_N) = 71.25 - 68.75 = 2.5 \text{ MHz} \)

(3) Baud rate (symbol rate) = bit rate/4 = 2.5 Mbaud

4) Minimum shift keying (MSK)

MSK is modified form of OQPSK in that I and Q channel sinusoidal pulse shaping is employed prior to multiplication by the carrier, as shown in figure below, the transmitted MSK signal can be represented by:

\[
f(t) = a_n \cos\left(\frac{2\pi t}{4T_b}\right)\cos(2\pi f_c t) + b_n \sin\left(\frac{2\pi t}{4T_b}\right)\sin(2\pi f_c t)
\]

Where \( a_n \) and \( b_n \) are the \( n^{th} \) I and Q channel symbols. The MSK modulator is shown in figure below.
Various components of the MSK signal are shown in figure below for an in out binary sequence 1001001. The even index sample values shown in figure (a) are indicated by \(-1, +1\), held constant over two bit periods \((T_s=2T_b)\), and weighted by \(\cos(\frac{\pi t}{2T_b})\). While Q channel weighted by \(\sin(\frac{\pi t}{2T_b})\), shown in figure (c). The modulation in phase and quadrature carrier terms are shown in figures (b and d) respectively. Subtracting these two waveforms yields the MSK waveform shown in figure (e). MSK also called continuous phase frequency shift keying (CPFSK).
Bandwidth consideration and Bandwidth efficiency

Bandwidth efficiency (or information density as it is sometimes called) is often used to compare the performance of one digital modulation techniques to another.

\[ B.W = \frac{\text{transmission rate } R_b (\text{bps})}{\text{min. bandwidth (Hz)}} = \text{bps/Hz} \]

See table below:

<table>
<thead>
<tr>
<th>Modulation type</th>
<th>No. of bit per Symbol</th>
<th>Minimum Bandwidth</th>
<th>Bandwidth efficiency(bps/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>1</td>
<td>( R_b )</td>
<td>1</td>
</tr>
<tr>
<td>QPSK</td>
<td>2</td>
<td>( R_b/2 )</td>
<td>2</td>
</tr>
<tr>
<td>8-PSK &amp; 8-QAM</td>
<td>3</td>
<td>( R_b/3 )</td>
<td>3</td>
</tr>
<tr>
<td>16PSK&amp;16QAM</td>
<td>4</td>
<td>( R_b/4 )</td>
<td>4</td>
</tr>
</tbody>
</table>
> **Error performance for digital modulation systems**

1) **Probability of error for ASK**

For coherent ASK detection

\[ P_E = \text{Erfc} \left( \sqrt{\frac{E}{2 N_0}} \right) = \text{Erfc} \left( \sqrt{\frac{S}{2 N}} \right) \]

where \( S \) = signal power, \( N \) = noise power.

For noncoherent detection

\[ P_E = \frac{1}{2} \exp \left( - \frac{E}{4 N_0} \right) + \frac{1}{2} \text{Erfc} \left( \frac{2 E}{N_0} \right) \]

where \( N_0 \) = noise power spectral density = \( N \)/bandwidth, \( E \) = signal energy = \( S \)T.

2) **Probability of error for FSK**

For coherent FSK detection

\[ P_E = \text{Erfc} \left( \frac{E}{N_0} \right) \]

For coherent FSK if \( f_c \gg \Delta f \) and \( \omega_c T \gg 1 \)

\[ P_E = \text{Erfc} \left( \frac{1.21 E}{N_0} \right) \]

For noncoherent FSK detection

\[ P_E = \frac{1}{2} \exp \left( - \frac{E}{2 N_0} \right) \]

\( \Box \)
Example

NRZ binary system with bit rate=300 bit/sec., using FSK with transmitted frequencies of 2025, 2225 Hz. (a) if B.W=800 Hz centered at carrier, calculate minimum $P_E$, if $S/N=8$dB. (b) repeat for $S/N=7$dB.

Solution:-

(a) $f_c = \frac{f_1 + f_2}{2} = \frac{2025 + 2225}{2} = 2125$Hz

$2\Delta f = f_2 - f_1 = 2225 - 2025 = 200$Hz $\Rightarrow \Delta f = 100$Hz

$T_b = \frac{1}{300}$ sec.

$\therefore \omega_c T_b >> 1 \quad \therefore f_c >> \Delta f$

$P_E = Erfc\left(\frac{1.21E}{\sqrt{\frac{N_0}{N_0}}}\right) = Erfc\left(\frac{1.21ST_b}{\sqrt{N_0}}\right)$

$\frac{S}{N} = 8$dB $= 10^{0.8}$

$N = BN_0 = 800N_0$

$\frac{S}{800N_0} = 10^{0.8}$

$P_E = Erfc(4.51) = 3.26 \times 10^{-6}$

(b) $\frac{S}{N} = 7$dB $= 10^{0.7}$ The same procedure in (a)
3) **Probability of error for BPSK**

\[ P_E = \text{Erfc} \left( \frac{2E}{\sqrt{N_0}} \right) \]

Generally the modulation index for BPSK \( m = \cos(\Delta \theta) \) where \( 0 < m < 1 \), \( \Delta \theta \) = peak phase deviation.

\[ P_E = \text{Erfc} \left( \frac{2E(1 - m^2)}{\sqrt{N_0}} \right) \]

For DBPSK (noncoherent detection), the probability of error is:

\[ P_E = \frac{1}{2} \text{Exp} \left( - \frac{E}{N_0} \right) \]

4) **Probability of error for QPSK & QAM system**

The probability of error for QPSK & QAM systems are

\[ P_E = 2\text{Erfc} \left( \frac{E_s}{\sqrt{N_0}} \right) \]

where \( E_s \) = symbol energy.

5) **Probability of error for M-ary PSK systems (M>2)**

For M-ary PSK systems (M>2), the Probability of error given by

\[ P_E = 2\text{Erfc} \left( \frac{2E_s}{\sqrt{N_0}} \sin^2 \frac{\pi}{M} \right) \]
It should be noted that these results are for the symbol probability of error. The bit probability of error, $P_{Eb}$, can be found by

$$P_{Eb} \approx \frac{P_E}{\log_2 M}$$

A derivation of probability of error for M-ary DPSK is rather involved, an approximation for large SNR is

$$P_E = 2\text{Erfc} \sqrt{\frac{2E_s}{N_0}} \sin^2 \frac{\pi}{\sqrt{2M}}$$

6) **Probability of error for M-ary QAM systems**

The probability of error (Symbol error) for M-ary QAM is approximately given by

$$P_E = 4(1 - \frac{1}{\sqrt{M}}) \text{Erfc} \sqrt{\frac{2E_s}{N_0}}$$

7) **Probability of error for MSK system**

The probability of error for MSK system can be calculated by

$$P_E = \text{Erfc} \sqrt{\frac{2E}{N_0}}$$

where $E$ = bit Energy

• **Note:** $P_E$ for MSK = $P_E$ for BPSK.
**Example 1:**

Find $P_E$ for a 1 Mbit/sec. MSK transmission with a received carrier power of -130 dB and noise power spectral density = -200 dB/Hz.

Solution:-

$$T_b = \frac{1}{R_b} = 10^{-6} \mu \text{sec.}$$

$$N_0 = -200\, dB = 10^{-20} \frac{W}{Hz}.$$ 

$$S = -130\, dB = 10^{-13} W.$$ 

$$E_b = ST_b = 10^{-13} \times 10^{-6} = 10^{-19} J.$$ 

$$\frac{E_b}{N_0} = \frac{10^{-19}}{10^{-20}} = 10$$

For MSK

$$P_E = Erfc \sqrt{\frac{2E}{N_0}} = Erfc \sqrt{20} = 3.88 \times 10^{-6}$$

**Example 2:**

An MPSK, B.W.=120 KHz, $R_b$=900 kbit/sec., what minimum S/N is required to maintain reception with a $P_{Eb}$ no worse than $10^{-6}$. 
Solution:

Bandwidth efficiency $= \frac{900 \text{kbit}}{120 \text{KHz}} = 7.5 \frac{\text{bit/sec}}{\text{Hz}}$

$\therefore m = 8 \rightarrow M = 2^m = 2^8 = 256$

$P_E = P_{Eb} \log_2 M = 8 \times 10^{-6}$

For MPSK (M=256)

$P_E = 2Erfc \sqrt{\frac{2E_s}{N_0} \sin^2 \frac{\pi}{M}}$

$\therefore 8 \times 10^{-6} = 2Erfc \sqrt{\frac{2E_s}{N_0} \sin^2 \frac{\pi}{M}}$

$4 \times 10^{-6} = Erfc \sqrt{\frac{2E_s}{N_0} \sin^2 \frac{\pi}{M}}$

From table

$\sqrt{\frac{2E_s}{N_0} \sin^2 \frac{\pi}{M}} = 4.47$

$\frac{2E_s}{N_0} \sin^2 \frac{\pi}{M} = 19.98$

$\frac{E_s}{N_0} = 66338$
\[ E_s = E_b \log_2 M \rightarrow E_b = \frac{E_s}{\log_2 M} = \frac{E_s}{8} \]

\[ \frac{E_b}{N_0} = 8292.25 \]

\[ \frac{S}{N} = \frac{E_b}{N_0} \times \frac{R_b}{B} = 8292.25 \times 7.5 = 61988 = 47.9 dB \]