10-Frequency Response

Basic Concept

\[ X_C = \frac{1}{2\pi fC} \]

This formula shows that the capacitive reactance varies inversely with frequency. At lower frequencies, the reactance is greater, and it decreases as the frequency increases. At lower frequencies, capacitively coupled amplifiers such as those in Fig 10-1 have less voltage gain than they have at higher frequencies. The reason is that at lower frequencies, more signal voltage is dropped across \( C_1 \) and \( C_3 \) because their reactances are higher. This higher signal voltage drop at lower frequencies reduces the voltage gain.

![Capacitive coupled BJT amplifier](Fig10-1)

Effect of bypass capacitors

At lower frequencies, the reactance of the emitter bypasses capacitor, \( C_2 \) becomes significant, and the emitter is at ac ground. The capacitive reactance \( X_{C2} \) in parallel with \( R_E \) creates an impedance that reduces the gain.

At higher frequency, \( X_C \approx 0 \Omega \) and the voltage gain is \( A_v = \frac{R_C}{r_e} \).

At lower frequencies, \( X_C >> 0 \Omega \) and the voltage gain is \( A_v = \frac{R_C}{r_e + Z_e} \).

![Nonzero \( X_C \parallel R_E \), produce \( Z_e \), which reduces \( A_v \)](Fig10-2)

Effect of internal transistor capacitances

At high frequencies, the coupling and bypass capacitors become ac shorts and do not affect an amplifier's response. Internal transistor junction capacitances do come into play, reducing an amplifier's gain and introducing phase shift as the signal frequency increases.

In BJT, \( C_{be} \) (\( C_{ib} \) is the B-E junction capacitance, and \( C_{bc} \) (\( C_{ob} \) is the B-C junction capacitance. In JFET, \( C_{gs} \) (\( C_{iss} \) is the G-S junction capacitance, and \( C_{gd} \) (\( C_{rss} \) is the G-D junction capacitance.

![Internal transistor capacitances](Fig10-3)
At lower frequencies, the internal capacitances have a very high reactance because of their low capacitance value (usually only a few pf) and the low frequency value. Therefore, they look like opens and have no effect on the transistor’s performance. As the frequency goes up, the internal capacitive reactance’s go down, and at some point they begin to have a significant effect on the transistor’s gain.

When the reactance of $C_{be}$ (or $C_{gs}$) becomes small enough, a significant amount of the signal voltage is lost due to a voltage-divider effect of the source resistance and the reactance of $C_{be}$. Fig10-4(a).

When the reactance of $C_{bc}$ (or $C_{gd}$) becomes small enough, a significant amount of output signal voltage is fed back out of phase with the input (negative feedback), thus effectively reducing the voltage gain. Fig10-4(b).

Miller’s Theorem

Miller’s theorem will be used later to simplify the analysis of inverting amplifiers at high frequencies where the internal transistor capacitances are important. The capacitance $C_{bc}$ in BJTs between the input B and the output C is shown in Figure 10-5(a) in a generalized form. $A_v$ is the voltage gain of the amplifier at midrange frequencies, and $C$ represents either $C_{bc}$ or $C_{gd}$.

![Fig10-5 General case of Miller input and output capacitance. C represents $C_{bc}$ or $C_{gd}$.](image)

Miller theorems state that $C$ effectively appears as a capacitance from input to ground as shown in fig 10-5(b) that can be expressed as follows:

$$C_{in(Miller)} = C(A_v +1) \quad [10-1]$$

This formula shows that $C_{bc}$ has a much greater impact on input capacitance than its actual value.

Fig10-6 shows how this effective input capacitance appears in the actual equivalent circuit in parallel with $C_{be}$.
Miller's theorems also state that $C$ effectively appears as a capacitance from output to ground as shown in fig 10-5(b) that can be expressed as follows:

$$C_{\text{out(Miller)}} = C\left(\frac{A_v + 1}{A_v}\right)$$  \[10-2\]

This indicates that if the voltage gain is 10 or greater $C_{\text{out(Miller)}}$ is approximately equal to $C_{bc}$ because $(A_v + 1) / A_v$ is approximately equal to 1

Decibels

Decibel is a form of gain measurement and is commonly used to express amplifier response. The decibel unit important in amplifier measurements, the basis for the decibel unit stems from the logarithmic response of the human ear to the intensity of sound. The decibel is a logarithm measurement of the ratio of one power to another or one voltage to another.

$$A_p(dB) = 10 \log A_p$$  \[10-3\]

$$A_v(dB) = 20 \log A_v$$  \[10-4\]

If $A_v$ is greater than 1, the dB gain is positive. If $A_v$ is less than 1, the dB gain is negative and is usually called attenuation.

---

The bel (B) was defined by the following equation to relate power levels $P_1$ and $P_2$

$$G = \log \frac{P_2}{P_1}$$ bel

It was found, that the bel was too large a unit of measurement for practical purposes, so the decibel (dB) was defined such that 10 decibels = 1 bel. Therefore,

$$G_{\text{dB}} = 10 \log \frac{P_2}{P_1}$$ dB

Consider the following mathematical equations:

$a$, $b$, and $x$ are the same in each equation. If $a$ determined by taking the same base $b$ to the $x$ power, the same $x$ will result if the log of $a$ is taken to the base $b$.

If $b=10$ and $x=2$,

$$a = b^x = (10)^2 = 100$$

$$x = \log_{10} 100 = 2$$

For the electrical/electronics, the base in the logarithmic equation is limited to 10
For the neutral, the base in the logarithmic is limited to the number $e=2.71828…$

The two are related by

\[
\log_e a = 2.3 \log_{10} a
\]

0 dB reference

It is often convenient in amplifiers to assign a certain value of gain as the 0 dB reference. This does not mean that the actual voltage gain is 1 (which is 0 dB); it means that the reference gain is used as a reference with which to compare other values of gain and is therefore assigned a 0 dB value. The maximum gain is called the midrange gain and is assigned a 0 dB value. Any value of gain below midrange can be referenced to 0 dB and expressed as a negative dB value. For example, if the midrange voltage gain of a certain amplifier is 100 and the gain at a certain frequency below midrange is 50, then this reduced voltage gain can be expressed as:

\[
20 \log \left(\frac{50}{100}\right) = 20 \log (0.5) = -6 \text{ dB}.
\]

This indicates that it is 6 dB below the 0 dB reference. Halving the output voltage for a steady input voltage is always a 6 dB reduction in the gain. Correspondingly, a doubling of the output voltage is always a 6 dB increase in the gain. Fig10-7 illustrates a normalized gain-versus-frequency showing several dB points, the term normalized means that the midrange voltage gain is assigned a value of 1 or 0 dB.

![Normalized $A_v$ versus frequency curve](image)

The Critical frequency

The critical frequency (cutoff frequency or corner frequency) is the frequency at which the output power drops to one-half of its midrange value; this corresponds to a 3 dB reduction in the power gain, as expressed in dB by the following formula:

\[
A_p(\text{dB}) = 10 \log A_p = 10 \log(0.5) = -3 \text{ dB}
\]
Also, at the critical frequency the output voltage is 0.707 percent of its midrange value and is expressed in dB as:

\[ A_v(dB) = 20 \log A_v = 20 \log(0.707) = -3dB \]

At the critical frequency, the voltage gain is down 3 dB or is 70.7% of its midrange value. At this frequency, the power is one-half of its midrange value.

**dB power Measurement**

A unit that is often used in measuring power is the dBm, the term dBm means decibels referenced to 1 mW of power. When dBm is used, all power measurements are relative to a reference level of 1 mW, a 3dBm increase corresponds to doubling of the power, and a 3 dBm decrease corresponds to a halving of the power.

For example, +3 dBm corresponds to 2 mW (twice 1 mW), and -3dBm corresponds to 0.5 mW (half of 1 mW).

<table>
<thead>
<tr>
<th>POWER</th>
<th>dBm</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 mW</td>
<td>15 dBm</td>
</tr>
<tr>
<td>16 mW</td>
<td>17 dBm</td>
</tr>
<tr>
<td>8 mW</td>
<td>17 dBm</td>
</tr>
<tr>
<td>4 mW</td>
<td>16 dBm</td>
</tr>
<tr>
<td>2 mW</td>
<td>15 dBm</td>
</tr>
<tr>
<td>1 mW</td>
<td>14 dBm</td>
</tr>
<tr>
<td>0.5 mW</td>
<td>13 dBm</td>
</tr>
<tr>
<td>0.25 mW</td>
<td>12 dBm</td>
</tr>
<tr>
<td>0.125 mW</td>
<td>11 dBm</td>
</tr>
</tbody>
</table>

Table10-2 power in terms of dBm

**BJT Amplifier**

A typical capacitively coupled CE Amp is shown in Fig10-8. Assuming that the coupling and bypass capacitors are ideal shorts at the midrange signal frequency, you can determine the midrange voltage using Eq [10-5], where

\[ R_C = R_C \parallel R_L \]

\[ A_v(mid) = \frac{R_C}{r_e} \]  

[10-5]

Fig10-8 a capacitive coupled amp  
Fig10-9 the low-frequency ac equivalent cct of amp

The BJT amplifier in Fig10-8 has three high-pass RC circuits that affect its gain as the frequency is reduced below midrange. These are shown in the low-frequency ac equivalent circuit in Fig10-9. Which represented midrange response \( X_C \approx 0\Omega \), the low-frequency equivalent circuit retains the coupling and bypass capacitors because \( X_C \) is not small enough to neglect when the signal frequency is low.

One RC circuit is formed by the input coupling capacitor \( C_1 \) and the input resistance of the amplifier. The second RC circuit is formed by the output coupling capacitor \( C_3 \), the resistance looking in at the collector, and the load resistance. The third RC circuit that affects the low-frequency response is formed by the emitter-bypass capacitor \( C_2 \) and the resistance looking in at the emitter.
1-The input RC Circuit
The input RC circuit for the BJT amplifier in Fig10-8 is formed by $C_1$ and the amplifier’s input resistance is shown in Fig10-10, as the signal frequency decreases, $X_{C1}$ increase, This causes less voltage across the input resistance of the amplifier at the base because more voltage is dropped across $C_1$ and because of this, the overall voltage gain of the amplifier is reduced.

The base voltage for the input RC circuit in Fig10-10 is:

$$V_{\text{base}} = \left( \frac{R_{\text{in}}}{\sqrt{R_{\text{in}}^2 + X_{C1}^2}} \right) V_{\text{in}}$$

Fig10-10 input RC circuit formed by $C_1$ & $R_{\text{in}}$

A critical point in the amplifier’s response occurs when the output voltage is 70.7 percent of its midrange value. This condition occurs in the input RC circuit when $X_{C1} = R_{\text{in}}$

$$V_{\text{base}} = \left( \frac{R_{\text{in}}}{\sqrt{R_{\text{in}}^2 + R_{\text{in}}^2}} \right) V_{\text{in}} = \left( \frac{R_{\text{in}}}{\sqrt{2R_{\text{in}}^2}} \right) V_{\text{in}} = \left( \frac{R_{\text{in}}}{\sqrt{2R_{\text{in}}}} \right) V_{\text{in}} = \left( \frac{1}{\sqrt{2}} \right) V_{\text{in}} = 0.707V_{\text{in}}$$

In terms of measurement in decibels

$$20 \log \left( \frac{V_{\text{base}}}{V_{\text{in}}} \right) = 20 \log(0.707) = -3 \text{ dB}$$

Lower critical frequency
The condition where the gain is down 3 dB is logically called the -3dB point of the amplifier response; the overall gain is 3dB less than at midrange frequencies because of the attenuation of the input RC circuit. The frequency, $f_c$, at which this condition occurs is called the lower critical frequency (lower cutoff frequency, lower corner frequency, or lower break frequency)

$$X_{c1} = \frac{1}{2\pi f_c C_1} = R_{\text{in}}$$

$$f_c = \frac{1}{2\pi R_{\text{in}} C_1}$$

$$f_c = \frac{1}{2\pi (R_1 + R_{\text{in}}) C_1}$$

Example 1: for an input RC cct in a certain Amp, $R_{\text{in}}=1k\Omega$ & $C_1=1\mu F$. Neglect the source resistance
(a) Determine the lower critical frequency
(b) what is the attenuation of the RC circuit at the lower critical frequency
(c) if the midrange voltage gain of the Amp is 100, what is the gain at the lower critical frequency?

Solution:

(a) $f_c = \frac{1}{2\pi R_{\text{in}} C_1} = \frac{1}{2\pi (1.0k\Omega)(1\mu F)} = 159 \text{ Hz}$

(b) At $f_c$, $X_{c1} = R_{\text{in}}$. Therefore,

$$\text{Attenuation} = \frac{V_{\text{base}}}{V_{\text{in}}} = 0.707$$

(c) $A_v = 0.707A_{v(mid)} = 0.707(100) = 70.7$
Voltage gain roll-off at low frequencies
The input RC circuit reduces the overall voltage gain of an amplifier by 3 dB when the frequency is reduced to the critical value $f_C$. As the frequency continues to decrease below $f_C$, the overall voltage gain also continues to decrease. The decrease in voltage gain with frequency is called roll-off. For each ten times reduction in frequency below $f_C$, there is a 20 dB reduction in voltage gain. Let’s take a frequency that is one-tenth of the critical frequency ($f = 0.1f_C$). Since $X_{C1} = R_{in}$ at $f_C$, then $X_{C1} = 10 R_{in}$ at $0.1f_C$ because of the inverse relationship of $X_{C1}$ and $f$. The attenuation of the input RC circuit is, therefore,

$$\text{Attenuation} = \frac{V_{in}}{V_{out}} = \frac{R_{in}}{\sqrt{R_{in}^2 + X_{C1}^2}} = \frac{R_{in}}{\sqrt{R_{in}^2 + (10R_{in})^2}} = \frac{R_{in}}{\sqrt{R_{in}^2 + 100R_{in}^2}}$$

$$= \frac{R_{in}}{\sqrt{R_{in}^2(1+100)}} = \frac{R_{in}}{R_{in} \sqrt{101}} = \frac{1}{\sqrt{101}} \approx \frac{1}{10} = 0.1$$

The dB attenuation is

$$20 \log\left(\frac{V_{in}}{V_{out}}\right) = 20 \log(0.1) = -20 \text{ dB}$$

dB/decade
A ten-time change in frequency is called a decade. So, for the input RC circuit, the attenuation is reduced by 20 dB for each decade that the frequency decreases below the critical frequency. This causes the overall voltage gain to drop 20 dB per decade.

For example, if the frequency is reduced to one-hundredth of $f_C$ (a two-decade decrease), the amplifier voltage gain drops 20 dB for each decade, giving a total decrease in voltage gain of

$$-20 \text{ dB} + (-20 \text{ dB}) = -40 \text{ dB}.$$ 

---

**Example 2:** the midrange voltage gain of a certain Amp is 100. the input RC cct has a lower critical frequency of 1 kHz. Determine the actual voltage gain at $f = 1$ kHz, $f = 100$ Hz, and $f = 10$ Hz

**Solution:** when $f = 1$ kHz, $A_v$ is 3dB less than at midrange. At 0 dB, $A_v$ reduced by 0.707

$$A_v = (0.707)(100) = 70.7$$

When $f = 100$ Hz = 0.1$f_C$, $A_v$ is 20 dB less than at $f_C$. $A_v$ at -20dB is one tenth (1/10) of that at midrange frequencies

$$A_v = (0.1)(100) = 10$$

When $f = 10$ Hz = 0.01$f_C$, $A_v$ is 20 dB less than at $f = 0.1f_C$ or -40 dB. $A_v$ at -40 dB is one-tenth of that at -20 dB or one-hundredth that at the midrange frequencies

$$A_v = (0.01)(100) = 1$$

The midrange voltage gain of an amp is 300. The lower critical frequency of the input RC circuit is 400 Hz. Determine the actual voltage gain at 400 Hz, 40 Hz and 4 Hz.
Phase shift in the input RC circuit

In addition to reducing the voltage gain, the input RC circuit also cause an increasing phase shift through an amplifier as the frequency decreases. At midrange frequencies the phase shift through the input RC circuit is approximately zero because $X_{C1} \approx 0 \Omega$, at lower frequencies, higher values of $X_{C1}$ cause a phase shift to be introduced, and the output voltage of the RC circuit leads the input voltage, the phase angle in an input RC circuit is expressed as:

$$\theta = \tan^{-1}\left(\frac{X_{C1}}{R_{in}}\right)$$

At the critical frequency, $X_{C1} = R_{in}$, so

$$\theta = \tan^{-1}\left(0 \Omega / R_{in}\right) = \tan^{-1}(0) = 0^\circ$$

A decade below the critical frequency, $X_{C1} = 10 R_{in}$, so

$$\theta = \tan^{-1}\left(10 R_{in} / R_{in}\right) = \tan^{-1}(10) = 84.3^\circ$$

A continuation of this analysis will show that the phase shift through the input RC circuit approaches 90º as the frequency approaches zero. The result is that the voltage at the base of the transistor leads the input signal voltage in phase below midrange, as shown in fig10-13.

![Fig10-12 phase angle versus f for IP- RC cct](image)

![Fig10-13 base voltage lead ip voltage by θ](image)

2-The output RC circuit

The second high-pass RC circuit in the BJT amplifier is formed by the coupling capacitor $C_3$, the resistance looking in at the collector, and the load resistance $R_L$. In determining the output resistance, looking in at the collector, the transistor is treated as an ideal current source and the upper end of $R_C$ is effectively at ac ground, as shown in fig 10-14(b).

![Fig10-14 Development of the equivalent low-frequency output RC circuit](image)

Thevenizing the circuit to the left of capacitor $C_3$ produces an equivalent voltage source equal to the collector voltage and a series resistance equal to $R_C$, as shown in fig10-14(c).
At the signal frequency decreases, $X_{C3}$ increases. This causes less voltage across the load resistance because more voltage is dropped across $C_3$. The signal voltage is reduced by a factor of 0.707 when frequency is reduced to the lower critical value, $f_c$, for the circuit. This corresponds to a 3 dB reduction in voltage gain.

**Example 3:** an output RC cct in a certain amplifier, $RC=10k\Omega$, $C_3=0.1\mu F$, $RL=10k\Omega$

(a) Determine the critical frequency (b) What is the attenuation of the output RC circuit at the critical frequency (c) if the midrange voltage gain of the amplifier is 50, what is the gain at the critical frequency?

**Solution:**

(a) $f_c = \frac{1}{2\pi(R_C + R_L)C_3} = \frac{1}{2\pi(20k\Omega + 0.1\mu F)} = 79.6$ Hz

(b) For the midrange frequencies, $X_{C3} \approx 0\Omega$, thus the attenuation of the circuit

Or in dB, $V_{out}/V_{collector} = 20\log(0.5) = -6$ dB, this shows that, in this case, the midrange voltage gain is reduced by 6 dB because of the load resistor. At the critical frequency, $X_{C3} = R_C + R_L$ and the attenuation is

Or in dB, $V_{out}/V_{collector} = 20\log(0.354) = -9$ dB. As you can see, the gain at $f_c$ is 3dB less than the gain at midrange.

(c) $A_v = 0.707 A_{v(mid)} = 0.707(50) = 35.4$

**Phase shift in the output RC circuit**

The phase shift in the output RC circuit is

$$\theta = \tan^{-1} \left( \frac{X_{C3}}{R_C + R_L} \right)$$

$\theta \approx 0$ for the midrange frequency and approaches $90^\circ$ as the frequency approaches zero ($X_{C3}$ approaches infinity). At the critical frequency $f_c$, the phase shift is $45^\circ$.

**3-The bypass RC circuit**

The third RC circuit that affects the low-frequency gain of the BJT amplifier is the bypass capacitor $C_2$. For midrange frequencies, it is assumed that $X_{C2} \approx 0\Omega$, effectively shorting the emitter to ground so that the amplifier gain is $A_v = R_C / r_e$.

---

Fig 10-15 at low frequencies $X_{C2} \parallel R_E$ reduces $A_v$
As the frequency is reduced, $X_{C2}$ increases and provides a sufficiently low reactance to effectively place the emitter at ac ground. Because the impedance from emitter to ground increases, the gain decreases is: $A_v = \frac{R_C}{(r_e + R_e)}$

$R_e$ is replaced by an impedance formed by $R_E$ in parallel with $X_{C2}$. The bypass RC circuit is formed by $C_2$ and the resistance looking in at emitter $R_{in(\text{emitter})}$, Fig10-16(a). First, Thevenin's theorem is applied looking from the base of the transistor toward the input source $V_{in}$, Fig10-16(b). This results in an equivalent resistance ($R_{th}$) and an equivalent voltage source $V_{th(1)}$ in series with the base, Fig10-16(c). The resistance looking in at the emitter is determined with the equivalent input source shorted, Fig10-16(d),

$$R_{in(\text{emitter})} = \frac{V_v}{I_v} = \frac{V_b}{\beta_{ac} I_b} + r'_e = \frac{I_b R_{th}}{\beta_{ac} I_b} + r'_e$$

Looking from capacitor $C_2$, $\frac{R_{th}}{\beta_{ac} + r_e}$ is in parallel with $R_E$, fig10-16(e), thevenizing again, we get the equivalent RC circuit fig10-16(f), the $f_c$ for this equivalent bypass RC circuit is

$$f_c = \frac{1}{2\pi [(r'_e + R_{th}/\beta_{ac}) [R_E] C_2]}$$

Fig10-16 Development of the equivalent bypass circuit
Example 4: Determine $f_c$ of the bypass RC circuit for the amplifier in fig10-17 ($r_e = 12\Omega$)

Solution: Thevenize the base circuit (looking from the base toward the input source)

$R_{in} = R_1 \parallel R_2 \parallel R_e = 62\,k\Omega \parallel 22\,k\Omega \parallel 1.0\,k\Omega = 942\,\Omega$

$R_{in\,(emitter)} = r_e' + \frac{R_{th}}{\beta_{ae}} = 12\,\Omega + 9.42\,\Omega = 21.4\,\Omega$

$f_c = \frac{1}{2\pi R_{in\,(emitter)}C_2} = \frac{1}{2\pi(21\,\Omega)(100\,\mu\text{F})} = 75.8\text{ Hz}$

The Bode Plot
A plot of dB voltage gain versus frequency on semilog graph paper (logarithmic horizontal axis scale and a linear vertical axis scale) is called a bode plot. The source of the spacing between the lines of the log plot is shown on fig10-18, the log of 2 to the base 10 is approximately 0.3. The distance from 1 (log 10 1 = 0) to 2 is therefore 30% of the span.

Fig10-18 Semi log graph paper

It is important to note the resulting numerical value and the spacing since plots will typically only have the tic marks indicated in Fig10-19 due to a lack of space.
A generalize bode plot for an RC circuit like that shown in Fig 10-20(a) appears in part (b) of the figure. The ideal response curve is shown in upper. Notice that it is flat (0 dB) down to the critical frequency, at which point the gain drops at -20 dB/decade as shown. The actual response curve is shown in lower. Notice that it decreases gradually in midrange and is down to -3 dB at the critical frequency, the critical frequency at which the curve “breaks” into a -20 dB/decade drop is sometimes called the lower break frequency.

The critical frequencies of the three RC circuits are not necessarily all equal. If one of the RC circuits has a critical (break) frequency higher than the other two, then it is dominant RC circuit. The dominant circuit determines the frequency at which the overall voltage gain of the amplifier begins to drop at -20 dB/decade roll-off below their respective critical (break) frequencies. In fig 10-21 each RC circuit has a different critical frequency, the input RC circuit is dominant (highest \( f_c \)) in this case and the bypass RC has the lowest. As the frequency is reduced from midrange, the first “break point” occurs at the critical frequency of the input RC circuit, \( f_c(\text{input}) \), and the gain begins to drop at -20dB/decade. This constant roll-off rate continues until the critical frequency of the output RC circuit, \( f_c(\text{output}) \), is reached. At this break point, the output RC circuit adds another -20 dB/decade to make a total roll-off of -40 dB/decade. This constant -40 dB/decade roll-off continues until the critical frequency of the bypass RC circuit, \( f_c(\text{bypass}) \), is reached. At this break point, the bypass RC circuit adds still another -20dB/decade, making the gain roll-off at -60 dB/decade.
If all RC circuits have the same critical frequency, the response curve has one break point at that value of $f_c$, and the voltage gain rolls off at -60 dB/decade below that value, as shown by 10-22. Actually, the midrange voltage gain does not extend down to the dominant critical frequency but is really at -9 dB below the midrange voltage gain at that (-3 dB for each RC circuit).

**Example 5:** Determine the total low-frequency response of the BJT amplifier in Fig10-23, $\beta_{ac}=100$ and $r_e=16\,\Omega$
Solution: Each RC circuit is analyzed to determine its critical frequency. For the input RC circuit with the source resistance, $R_S$, taken into account

$$f_{c \text{(input)}} = \frac{1}{2 \pi (R_S + R_e) C_1} = \frac{1}{2 \pi (600 \Omega + 1.46 \Omega)(0.1 \mu F)} = 773 \text{ Hz}$$

$$R_S = R_1 \parallel R_2 \parallel R_e = 62 \text{ k}\Omega \parallel 22 \text{ k}\Omega \parallel 600 \Omega = 579 \text{ \Omega}$$

$$R_{in \text{(emitter)}} = \frac{R_{th}}{\beta_{ac}} + r_e' = \frac{579 \Omega}{100} + 16 \Omega = 21.8 \Omega$$

$$f_{c \text{(bypass)}} = \frac{1}{2 \pi (R_{in \text{(emitter)}}) C_2} = \frac{1}{2 \pi (21.8 \Omega)(10 \mu F)} = 747 \text{ Hz}$$

For the output RC circuit:

$$f_{c \text{(output)}} = \frac{1}{2 \pi (R_C + R_2) C_3} = \frac{1}{2 \pi (2.2 \text{ k}\Omega + 10 \text{ \Omega})(0.1 \mu F)} = 130.5 \text{ Hz}$$

$$A_v \text{(mid)} = \frac{R_C}{r_e'} = \frac{2.2 \text{ k}\Omega}{16 \Omega} = 113$$

The midrange attenuation of the input circuit is

$$\frac{R_1 \parallel R_2 \parallel \beta_{ac} r_e'}{R_s + R_1 \parallel R_2 \parallel \beta_{ac} r_e'} = \frac{62 \text{ k}\Omega \parallel 22 \text{ k}\Omega \parallel 600 \Omega}{600 \Omega + 62 \text{ k}\Omega \parallel 22 \text{ k}\Omega \parallel 600 \Omega} = \frac{1456}{2056} = 0.708$$

The overall voltage gain is

$$A_v \text{(mid)} = 0.708(113) = 80$$

and is expressed in dB as

$$A_v \text{(mid)(dB)} = 20 \log(80) = 38.1 \text{ dB}$$

The ideal Bode plot of the low frequency response is shown in fig10-24. As a practical matter, $f_{c \text{(input)}}$ and $f_{c \text{(bypass)}}$ are close in value that the difference would be difficult to measure.

![Fig10-24 Ideal Bode plot of f- response Ex5](image)

High-Frequency Amplifier Response

A high frequency ac equivalent circuit for the BJT amplifier in fig10-25(a) is shown in fig 10-25(b). The internal capacitances, $C_{be}$ and $C_{bc}$, which are significant only at high frequencies, do appear in the diagram.
Miller's Theorem in high-frequency analysis

By applying Miller's theorem to the circuit in fig10-25(b) and using the midrange voltage gain, you have a circuit that can be analyzed for high-frequency response.

\[ C_{in(miller)} = C_{be}(A_v + 1) \]

\[ C_{out(miller)} = C_{be}\left(\frac{A_v + 1}{A_v}\right) \]

These two capacitances create a high-frequency input RC circuit and a high-frequency output RC circuit. These two circuits differ from the low-frequency input and output circuit, which act as high-pass filters, because the capacitances go to ground and therefore act as low-pass filters.

1-The Input RC Circuit

At high frequencies, the input circuit is as shown in Fig10-27(a), where \( \beta_{ac} r_e \) is the input resistance at the base of the transistor because the bypass capacitor effectively shorts the emitter to ground. By combining \( C_{be} \) and \( C_{in(miller)} \) in parallel and repositioning, you get the simplified circuit shown in Fig10-27(b). Next, by thevenizing the circuit to the left of the capacitor, the input RC circuit is reduced to the equivalent form shown in Fig10-27(c).

As the frequency increases, the capacitive reactance becomes smaller. This causes the signal voltage at the base to decrease, so the amplifier's voltage gain decreases. The reason for this is that the capacitance and resistance act as a voltage divider and, as the frequency increases, more voltage is dropped across the resistance and less across the capacitance.

At the critical frequency, the gain is 3 dB less than its midrange value. Just as with the low-frequency response, the critical high frequency, \( f_c \), is the frequency at which the capacitive reactance is equal to the total resistance

\[ X_{C_{tot}} = R_s || R_1 || R_2 || \beta_{ac} r_e' \]

\[ \frac{1}{2\pi f_c C_{tot}} = R_s || R_1 || R_2 || \beta_{ac} r_e' \]
Where $R_s$ is the resistance of the signal source and $C_{tot} = C_{be} + C_{(in)(Miller)}$

As the frequency goes above in the input RC circuit causes the gain to roll off at a rate of -20 dB/decade just as with the low-frequency response.

**Example 6:** Derive the input RC circuit for the BJT amplifier in fig10-28, also determine the critical frequency. The transistor’s data sheet provides the following: $\beta_{as} = 125$, $C_{be} = 20 \text{ pF}$, and $C_{bc} = 2.4 \text{ pF}$

**Solution:** First, find $r_e$ as follows:

$$V_b = \left( \frac{R_2}{R_1 + R_2} \right) V_{cc} = \left( \frac{4.7 \text{ k}\Omega}{26.7 \text{ k}\Omega} \right) 10 \text{ V} = 1.76 \text{ V}$$

$$V_b = V_B = 0.7 \text{ V} - 1.06 \text{ V}$$

$$V_E = 1.06 \text{ V}$$

$$R_E = \frac{470 \Omega}{22 \text{ k}\Omega} = 2.26 \text{ mA}$$

$$r_e' = \frac{25 \text{ mV}}{1.06 \text{ V}} = 11.1 \Omega$$

$$R_{(a(ris))} = R_s \parallel R_1 \parallel R_2 \parallel \beta_{ac}r_e' = 600 \\Omega \parallel 22 \text{ k}\Omega \parallel 4.7 \text{ k}\Omega \parallel 125(11.1 \Omega) = 378 \Omega$$

Next, in order to determine the capacitance, you must calculate the midrange gain of the amplifier so that you can apply Miller’s theorem
The resulting high-frequency input RC circuit is shown in Fig10-29. The critical frequency is

\[ f_c = \frac{1}{2\pi R C} \]

The output RC circuit reduces the gain by 3 dB at the critical frequency. When the frequency goes above the critical value, the gain drops at a -20dB/decade rate. The phase shift introduced by the output RC circuit is:

\[ \frac{10-15}{10-15}\]
Example 7: Determine the critical frequency of the amplifier in Example 6 (Fig 10-28) due to its output RC circuit.

Solution: The Miller output capacitance is calculated as follows:

\[ C_{out(Miller)} = C_{be} \left( \frac{A_v + 1}{A_v} \right) = (2.4 \text{ pF}) \left( \frac{99 + 1}{99} \right) = 2.4 \text{ pF} \]

The equivalent resistance is:

\[ R_e = R_C \parallel R_L = 2.2 \text{ k}\Omega \parallel 2.2 \text{ k}\Omega = 1.1 \text{ k}\Omega \]

The equivalent output RC circuit is shown in Fig 10-31, and the \( f_c \) is determined as follows:

\[ f_c = \frac{1}{2\pi R_e C_{be}} = \frac{1}{2\pi (1.1 \text{ k}\Omega)(2.4 \text{ pF})} = 60.3 \text{ MHz} \]

Total High-Frequency Response of an Amplifier

The two RC circuits created by the internal transistor capacitances influence the high-frequency response of BJT amplifiers. As the frequency increases and reaches the high end of its midrange values, one of the RC will cause the amplifier's gain to begin dropping off. The frequency at which this occurs is the dominant critical frequency; it is the lower of the two critical high frequencies. An ideal high-frequency Bode plot is shown in Fig 10-32(a). It shows the first break point at \( f_{c(input)} \) where the voltage gain begins to roll off at -20 dB/decade. At \( f_{c(output)} \), the gain begins dropping at -40 dB/decade because each RC circuit is providing a -20 dB/decade roll-off. Fig 10-32(b) shows a non-ideal Bode plot where the voltage gain is actually -3dB below midrange at \( f_{c(input)} \).
Total Amplifier Frequency Response

Fig10-33(b) shows a generalized ideal response curve (Bode plot) for the BJT Amplifier shown in Fig10-33(a). The three break points at the lower frequencies ($f_{c1}$, $f_{c2}$, and $f_{c3}$) are produced by the three low-frequency RC circuit formed by the coupling and bypass capacitors. The break points at the upper critical frequencies, $f_{c4}$ and $f_{c5}$, are produced by the two high-frequency RC circuits formed by the transistor's internal capacitances. The two dominant critical frequencies, $f_{c3}$ and $f_{c4}$ (fig10-33(b)), these two frequencies are where the voltage gain of the amplifier is 3 dB below its midrange value. These dominant frequencies are referred to as the lower critical frequency, $f_{cL}$, and the upper critical frequency $f_{cu}$.

Band width

An amplifier normally operates with signal frequencies between $f_{cL}$ and $f_{cu}$. If the signal frequency drops below $f_{cL}$, the gain and thus the output signal level drops at 20 dB/decade until the next critical frequency is reached. The same occurs when the signal frequency goes above $f_{cu}$. The range (band) of frequencies lying between $f_{cL}$ and $f_{cu}$ is defined as the bandwidth of the amplifier. Only the dominant critical frequencies appear in the response curve because they determine the bandwidth. The amplifier’s bandwidth is expressed in units of hertz as

$$BW = f_{cu} - f_{cL}$$

Ideally, all signal frequencies lying in an amplifier's bandwidth are amplified equally. For example: if a 10 mV rms signal is applied to an amplifier with a voltage gain of 20, it is amplified to 200 mV rms for all frequencies in the bandwidth. Actually, the gain is down 3 dB at $f_{cL}$ and $f_{cu}$.
Example 8: What is the bandwidth of an amplifier having an $f_{cl}$ of 200Hz and $f_{cu}$ of 2 KHz?

Solution

If $f_{cl}$ is increased, does the bandwidth increase or decrease? If $f_{cu}$ is increased, does the bandwidth increase or decrease?

Gain-Bandwidth Product
One characteristic of amplifiers is that the product of the voltage gain and the bandwidth is always constant when the roll-off is -20dB/decade. This characteristic is called the gain-bandwidth product. Let’s assume that the lower critical frequency of a particular amplifier is much less than the upper critical frequency.

Unity-Gain Frequency
The simplified Bode plot for this condition is shown in Fig10-35. Notice that $f_{cl}$ is neglected because it is so much smaller than $f_{cu}$ and the band-width approximately equals $f_{cu}$. Beginning at $f_{cu}$, the gain rolls off until unity gain (0 dB) is reached. The frequency at which the amplifier's gain is 1 is called the unity-gain frequency, $f_T$.

$$f_T = \frac{A_{v(mid)}BW}{f_{cl}}$$

Example 9: A certain transistor has an $f_T$ of 175MHz. When this transistor is used in an amplifier with a midrange voltage gain of 50, what bandwidth can be achieved ideally?

Solution:

$$f_T = \frac{A_{v(mid)}BW}{f_{cl}}$$

$$BW = \frac{f_T}{A_{v(mid)}} = \frac{175 MHz}{50} = 3.5 MHz$$

Half-Power Point
The upper and lower critical frequencies are sometimes called the half-power frequencies, this term is derived from the fact that the output power of an amplifier at its critical frequencies is
one-half of its midrange power, as previously mentioned. This can be shown of follows, starting with the fact that the output voltage is 0.707 of its midrange value at the critical frequencies

\[ V_{\text{out}(f_c)} = 0.707 V_{\text{out(mid)}} \]

\[ P_{\text{out}(f_c)} = \frac{V_{\text{out}(f_c)}^2}{R_{\text{out}}} = \frac{(0.707 V_{\text{out(mid)}})^2}{R_{\text{out}}} = 0.5 \frac{V_{\text{out(mid)}}^2}{R_{\text{out}}} = 0.5 P_{\text{out(mid)}} \]

**Multistage Amplifier**

One of the advantages of the logarithmic relationship is the manner in which it can be applied to cascaded stages, for example, the magnitude of the overall voltage gain of a cascaded system is given by:

\[ |A_{v_i}| = |A_{v_1}| |A_{v_2}| |A_{v_3}| \cdots |A_{v_n}| \]

Applying the proper logarithmic relationship result in:

\[ G_c = 20 \log_{10} |A_{v_i}| = 20 \log_{10} |A_{v_1}| + 20 \log_{10} |A_{v_2}| + \cdots + 20 \log_{10} |A_{v_n}| \quad \text{(dB)} \]

i.e. the gain of a cascaded system is simply the sum of the decibel gains of each stage

\[ G_{\text{dB}} = G_{\text{dB}_1} + G_{\text{dB}_2} + G_{\text{dB}_3} + \cdots + G_{\text{dB}_n} \quad \text{dB} \]

**Methods of Coupling Transistor**

The circuitry used to connect the output of one stage of a multi stage amplifier to the input of the next stage is called the coupling method, one such method:

**1-Cascade Amplifier**

A cascade connection is a series connection it has one on top of anther, for a cascade connection, amplification is the product of the stage gains, a cascade connection provides a high input impedance with low voltage gain to ensure that the miller capacitance is at a minimum with the CB stage providing good high-frequency operation and a low output impedance

![Fig 10-36 Cascade configuration](image)

When amplifier stages are cascaded to form a multistage amplifier, the dominant frequency response is determined by the responses of the individual stages. There are two cases to consider:

1- Each stage has a different lower critical frequency and a different upper critical frequency
2- Each stage has the same lower critical frequency and the same upper critical frequency

**1-Different Critical Frequencies**

When the lower critical frequency, \( f_{cl} \), of each amplifier stage is different, the dominant lower critical frequency, \( f'_{cl} \), equals the critical frequency of the stage with the highest \( f_{cl} \). When the upper critical frequency \( f_{cu} \), of each amplifier stage is different, the dominant upper critical frequency \( f'_{cu} \), equals the critical frequency of the stage with the lowest \( f_{cu} \)

**Overall Bandwidth**
The bandwidth of a multistage amplifier is the difference between the dominant lower critical frequency and the dominant upper critical frequency.

\[ BW = f'_{cu} - f'_{cl} \]

**Example 10:** In a certain 2-stage amplifier, one stage has a lower critical frequency of 850 Hz and an upper critical frequency of 100 kHz. The other has a lower critical frequency of 1 kHz and an upper critical frequency of 230 kHz. Determine the overall bandwidth of the 2-stage amplifier.

**Solution:**

\[ f'_{cl} = 1 \text{ kHz} \]
\[ f'_{cu} = 100 \text{ kHz} \]
\[ BW = f'_{cu} - f'_{cl} = 100 \text{ kHz} - 1 \text{ kHz} = 99 \text{ kHz} \]

### 2-Equal Critical Frequencies

When each amplifier stage in a multi stage arrangement has equal critical frequencies, the dominant lower critical frequency is increased by a factor of \( \frac{1}{\sqrt{2^{1/n}} - 1} \) as shown by the following formula (\( n \) is the number of stages in the multi stage amplifier):

\[ f'_{cl} = \frac{f_{cl}}{\sqrt{2^{1/n}} - 1} \]  \[10-18\]

When the upper critical frequencies of each stage are all the same, the dominant upper critical frequency is reduced by a factor of \( \sqrt{2^{1/n}} - 1 \), as shown by the following formula:

\[ f'_{cu} = f_{cu} \sqrt{2^{1/n} - 1} \]  \[10-19\]

**Example 11:** Both stages in a certain 2-stage amplifier have a lower critical frequency of 500 Hz and an upper critical frequency of 80 kHz. Determine the overall bandwidth

\[ f'_{cl} = \frac{500 \text{ Hz}}{\sqrt{2^{0.5}} - 1} = \frac{500 \text{ Hz}}{0.644} = 776 \text{ Hz} \]
\[ f'_{cu} = f_{cu} \sqrt{2^{1/2} - 1} = (80 \text{ kHz})(0.644) = 51.5 \text{ kHz} \]
\[ BW = f'_{cu} - f'_{cl} = 51.5 \text{ kHz} - 776 \text{ Hz} = 50.7 \text{ kHz} \]

**Example 12:** Calculate the voltage gain for the cascade amplifier of fig10-37

**Solution:**

\[ V_{vi} = 4.9 \text{ V} \quad V_{bv} = 10.8 \text{ V} \quad I_{C1} \approx I_{C1} = 3.8 \text{ mA} \]
\[ r_v = \frac{26 \Omega}{3.8 \text{ mA}} = 6.8 \Omega \]
Capacitor coupling, also called RC Coupling because the inter stage circuitry is equivalent to a high-pass RC network. RC coupling is used to prevent dc current from flowing between the output of one amplifier stage and the input of the next stage. The capacitor connected in the path between amplifier stages makes it possible to have a dc bias voltage at the output of one stage that is different from the dc bias voltage at the input to the next stage. This idea is illustrated in Fig10-38

\[
A_v = \frac{R_C}{r_e} = \frac{1.8 \text{ k}\Omega}{6.8 \Omega} = 265
\]

\[
A_v = A_v A_{v_o} = (-1)(265) = -265
\]

2-Capacitor coupling (RC coupling)

Example 13: Fig10-39 shows an amplifier consisting of a CE stage driving an E-follower stage; the transistors have the following parameter values:

Q1: \( r_e = 15 \Omega, \beta_1 = 180, r_e = \infty \)
Q2: \( r_e = 25 \Omega, \beta_2 = 100 \)

Solution:

\[
f_1(C_1) = \frac{1}{2\pi r_m(\text{stage } 1) + r_e} \frac{1}{C_1} = \frac{1}{2\pi (2.48 \times 10^3 + 100)6 \times 10^{-6}} = 10.3 \text{ Hz}
\]

\[
f_1(C_2) = \frac{1}{2\pi R_e C_E}
\]

where \( C_E = C_2 = 40 \mu\text{F} \) and

\[
R_e = R_E \left( \frac{r_e \| R_E}{\beta_1} + r_e \right)
\]

\[
= (1.5 \times 10^3) \left[ \frac{100 \| (150 \times 10^3) \| (39 \times 10^3)}{180} + 15 \right] \approx 15 \Omega
\]
Fig 10-41 Bode plot for the gain

\[ f_1(C_2) = \frac{1}{2\pi(15)(40 \times 10^{-6})} = 265.3 \text{ Hz} \]

\[ f_1(C_3) = \frac{1}{2\pi[r_o(\text{stage 1}) + r_o(\text{stage 2})] C_3} = \frac{1}{2\pi(4.7 \times 10^3 + 5.9 \times 10^3)(0.4 \times 10^{-6})} = 37.5 \text{ Hz} \]

\[ f_1(C_4) = \frac{1}{2\pi[r_o(\text{stage 2}) + r_o] C_4} = \frac{1}{2\pi(64.8 + 50)(20 \times 10^{-6})} = 69.3 \text{ Hz} \]

3-Direct-Coupled Amplifier

Direct coupling is the coupling method in which the output of one stage is electrically connected directly to the input of the next stage. In other words, both the dc and ac voltages at the output of one stage are identical to those at the input of the next stage. Clearly, any change in the dc voltage at the output of one stage produces an identical change in dc voltage at the input to the next stage, so a direct-coupled amplifier behaves like a direct-current amplifier. Direct coupling is used in differential and operational amplifiers.

Fig 10-42 direct-coupled CE amplifier

The output of the first stage (collector of Q1) is connected directly to the input of the second stage (base of Q2) we will first analyze the dc bias of the circuit and then consider its ac performance.

The current in \( R_{C1} = I_{C1} + I_{B2} \)

\( I_{B2} \) is negligibly small in comparison to \( I_{C1} \), \( V_{B1} \) is determined essentially by the \( R_1-R_2 \) voltage divider

\[ V_{B1} = \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} \]
The important point to note is that $V_{C1} = V_{B2}$. The voltage gain of the first stage is

$$A_{v1} = \frac{-r_d(\text{stage 1}) || r_m(\text{stage 2})}{r_n + R_{E1}}$$

$R_C$ and $r_m$ (stage 2) = $\beta_2(r_c + R_{E2})$.

$$A_{v2} = \frac{-r_d(\text{stage 2})}{r_{e2} + R_{E2}} = \frac{-R_{C2}}{r_{e2} + R_{E2}}$$

Finally, the overall gain is the product of the stage gains:

$$A_{v(overall)} = A_{v1}A_{v2}$$

If a load resistance $R_L$ is direct-coupled between the output (collector of Q2) and ground, then the ac load resistance on the second stage is $r_L = R_{C2} || R_L$.

$$A_{v2} \approx \frac{R_{C2} || R_L}{r_{e2} + R_{E2}}$$

It is important to realize that direct-coupling an output load resistance changes the dc value of $V_{C2}$ and $V_{CE2}$; to demonstrate this fact, let us regard the transistor as a constant-current source, as shown in fig10-43(b). We can then apply $V_L (= V_C)$ due to each source in the circuit, as shown in fig10-43(c) and (d).

Combining the contributions of each source leads to:

$$V_L = V_{CC} - I_C \left( \frac{R_C}{R_L + R_C} \right) R_L$$

Example 14: Si transistor in fig10-44 has the following parameters:
Q1: β₁ = 100, rₑ₁ = 6 Ω, rₑ₁ = ∞  
Q2: β₂ = 60, rₑ₂ = 10 Ω, rₑ₂ = ∞  

Fig10-44 network for Ex14:  
(1) Find the quiescent values of Vₑ₁, Iₑ₂, Vₑ₂, Vₑ₂  
(2) find Aᵥ  
(3) Repeat (1) & (2) is a 10-kΩ load is direct-coupled between the collector of Q₂ & ground  

Solution:  

\[ R_{m1} = \beta_1 R_{e1} = 100 \times 75 = 7.5 \text{ kΩ} \]

\[ V_{e1} = \left( \frac{R_2 \parallel R_{m1}}{R_1 + R_2 \parallel R_{m1}} \right) V_{cc} = \left[ \frac{(11 \text{ kΩ}) \parallel (7.5 \text{ kΩ})}{(100 \text{ kΩ}) + (11 \text{ kΩ}) \parallel (7.5 \text{ kΩ})} \right] (24 \text{ V}) = 1.2 \text{ V} \]

\[ V_{e1} \approx V_{m1} = 0.7 = 1.02 - 0.7 = 0.32 \text{ V} \]

\[ I_{c1} = I_{e1} = V_{e1}/R_{e1} = (0.32 \text{ V})/(75 \Omega) = 4.26 \text{ mA} \]

\[ V_{c1} = V_{cc} - I_{c1} R_{e1} = 24 - (4.26 \text{ mA}) (4.7 \text{ kΩ}) = 3.95 \text{ V} \]

\[ V_{c2} = V_{cc} - I_{c2} R_{e2} = 24 - (3.25 \text{ mA}) (3.3 \text{ kΩ}) = 13.3 \text{ V} \]

\[ V_{c2} = V_{c2} - V_{e2} = 13.3 - 3.25 = 10.05 \text{ V} \]

\[ A_{v1} = \frac{-R_{c1} \parallel \beta_1 (R_{e2} + r_{e2})}{R_{e1} + r_{e1}} = \frac{(-4.7 \text{ kΩ}) \parallel 60(1 \text{ kΩ}) + (10 \Omega)}{(75 \Omega) + 6 \Omega} = -53.8 \]

\[ A_{v2} = \frac{-R_{e2} \parallel r_{e2}}{R_{e2} + r_{e2}} = \frac{-3.3 \text{ kΩ}}{(1 \text{ kΩ}) + (10 \Omega)} = -3.3 \]

\[ A_{v(overall)} = A_{v1}/A_{v2} = (-53.8)(-3.3) = 177.5 \]

\[ V_{L} = V_{e2} = \left[ \frac{10 \text{ kΩ}}{(10 \text{ kΩ}) + (3.3 \text{ kΩ})} \right] (24 - (3.25 \text{ mA})(3.3 \text{ kΩ})) = 10 \text{ V} \]

Therefore, \( V_{c2} = V_{e2} - 10 - 3.25 = 6.75 \text{ V} \). Now,

\[ I_{L} = V_{L}/R_{L} = \frac{10 \text{ V}}{10 \text{ kΩ}} = 1 \text{ mA} \]

\[ I_{e2} = (3.25 \text{ mA}) + (1 \text{ mA}) = 4.25 \text{ mA} \]

\[ A_{v2} = \frac{-R_{e2} \parallel R_{L}}{R_{e2} + r_{e2}} = \frac{-3.3 \text{ kΩ} \parallel (10 \text{ kΩ})}{(1 \text{ kΩ}) + (10 \Omega)} = -2.48 \]

\[ A_{v(overall)} = (-53.8)(-2.48) = 133.4 \]
The voltage drop across the emitter resistor is:

\[ V_{CC} - V_{E2} = 24 - 20.1 = 3.9 \text{ V} \]

and so

\[ I_{E2} = \frac{(3.9 \text{ V})}{(1 \text{ k}\Omega)} = 3.9 \text{ mA} \approx I_{C2} \]

Clearly,

\[ V_{C2} = I_{C2}R_{C2} = (3.9 \text{ mA})(2.7 \text{ k}\Omega) = 10.5 \text{ V} \]

**Example 15:** The Si transistor in fig 10-46 has the following parameters: Q1: \( \beta_1 = 100, r_{e1} \approx \infty \), \( C_{bc} = 4 \text{ pF} \), \( C_{be} = 10 \text{ pF} \); Q2: \( \alpha_2 \approx 1 \), \( r_{e2} \approx \infty \). (1) find dc current & voltages \( I_{C1}, I_{C2}, V_{C1}, V_{C2} \) (2) the small signal voltage gain \( \frac{v_L}{v_s} \) (3) the break frequency \( f_{2(CA)} \) due to shunt capacitance at the input of Q1

**Solution:**

\[ V_{\beta 1} = \frac{10 \text{ k}\Omega}{(10 \text{ k}\Omega) + (33 \text{ k}\Omega)} \times (12 \text{ V}) = 2.8 \text{ V} \]

\[ V_{E1} = V_{\beta 1} - 0.7 = 2.8 - 0.7 = 2.1 \text{ V} \]

\[ I_{C1} = I_{E1} = \frac{V_{E1}}{R_{E1}} = \frac{2.1 \text{ V}}{1 \text{ k}\Omega} = 2.1 \text{ mA} = I_{E2} \approx I_{C2} \]

\[ V_{E2} = \frac{10 \text{ k}\Omega}{(10 \text{ k}\Omega) + (10 \text{ k}\Omega)} \times (12 \text{ V}) = 6 \text{ V} \]

\[ V_{C1} = V_{E2} - 0.7 = 6 - 0.7 = 5.3 \text{ V} \]

\[ V_{C2} = V_{CC} - I_{C2}R_{C2} = 12 - (2.1 \text{ mA})(2 \text{ k}\Omega) = 7.8 \text{ V} \]

2. Since \( I_{E1} = I_{E2} \),

\[ r_{e2} = r_{e1} = \frac{0.026}{0.026} = \frac{0.026}{2.1 \text{ mA}} = 12.4 \Omega \]

\[ r_{(\text{stage 1})} = \frac{33 \text{ k}\Omega}{(10 \text{ k}\Omega) || 100(12.4 \Omega)} = 1.07 \text{ k}\Omega \]

\[ A_{pe1} = \frac{-r_{e2}}{r_{e1}} = -1 \]
Example 16: (a) Determine the lower cutoff frequency for the network of fig10-47 using the following parameters: $C_s = 10 \mu F$, $C_E = 20 \mu F$, $C_c = 1 \mu F$, $R_s = 1 \Omega$, $R_1 = 40 \Omega$, $R_2 = 10 \Omega$, $R_E = 2 \Omega$, $R_c = 4 \Omega$, $R_L = 2.2 \Omega$, $V_{cc} = 100$, $r_o = \infty \Omega$, $V_{cc} = 20V$

(b) Sketch the frequency response using a log plot

Solution:  
(a) Determining $r_e$ for dc condition

\[
\beta R_E = (100)(2 \Omega) = 200 \Omega, \quad > > 10R_2 = 100 \Omega
\]

The result is:

\[
V_R = \frac{R_2 V_{cc}}{R_2 + R_1} = \frac{10 \Omega(20 V)}{10 \Omega + 40 \Omega} = \frac{200 V}{50} = 4 V
\]

with

\[
I_E = \frac{V_E}{R_E} = \frac{4 V - 0.7 V}{2 \Omega} = \frac{3.3 V}{2 \Omega} = 1.65 mA
\]

so that

\[
r_e = \frac{26 mV}{1.65 mA} = 15.76 \Omega
\]

and

\[
\beta r_e = 100(15.76 \Omega) = 1576 \Omega = 1.576 k\Omega
\]

Midband Gain:

\[
A_v = \frac{V_o}{V_i} = \frac{-R_c R_L}{r_e} = \frac{(4 k\Omega)(2.2 k\Omega)}{15.76 \Omega} \approx -90
\]

The input impedance

\[
Z_i = R_i = R_1 || R_2 || \beta r_e
\]

\[
= 40 k\Omega || 10 k\Omega || 1.576 k\Omega
\]

\[
= 1.32 k\Omega
\]

\[
V_i = \frac{-R_c V_o}{R_1 + R_2}
\]
Fig 10-48 the effect of $R_s$ on the gain $A_v$

$C_s$

$$R_s = R_1 || R_2 || \beta r_e = 40 \, \Omega || 10 \, \Omega || 1.576 \, k\Omega \approx 1.32 \, k\Omega$$

$$f_{Lc} = \frac{1}{2\pi(R_s + R_c)C_s}$$

$$= \frac{1}{(6.28)(1 \, k\Omega + 1.32 \, k\Omega)(10 \mu F)}$$

$$f_{Lc} \approx 6.86 \, \text{Hz}$$

Recall that the 3-dB point corresponds with a gain equal to 0.707 of the midband value, $C_c$

$$f_{Lc} = \frac{1}{2\pi(R_c + R_e)C_c}$$

$$= \frac{1}{(6.28)(4 \, k\Omega + 2.2 \, k\Omega)(1 \times 10^{-6})}$$

$$\approx 25.68 \, \text{Hz}$$

$C_e$

$$R_s = R_2 || R_{B1} || R_{B2} = 1 \, k\Omega || 40 \, k\Omega || 10 \, k\Omega \approx 1 \, k\Omega$$

$$R_e = R_e \left( \frac{R_e}{\beta} + r_v \right) = 2 \, k\Omega \left( \frac{1 \, k\Omega}{100} + 15.76 \right) = 2 \, k\Omega \left( 10 + 15.76 \right)$$

$$= 2 \, k\Omega || 25.76 \, \Omega \approx 25.76 \, \Omega$$

$$f_{Le} = \frac{1}{2\pi R_e C_e} = \frac{1}{(6.28)(25.76)(20 \times 10^{-6})} = \frac{10^6}{3235.46} \approx 309.1 \, \text{Hz}$$

$$P_L = \frac{V_L^2}{R_L} = \frac{(0.707V_{max})^2}{R_L} = 0.5V_{max}^2/R_L$$

= 0.5$P_{max}$

Fig 10-49 dB plot of the low-frequency response of the BJT amplifier
SUMMARY
1- In an ac amplifier, which capacitors affect the low frequency gain?
2- How is the high frequency gain of an amplifier limited?
3- When can coupling and bypass capacitors be neglected?
4- Determine $C_{\text{in(Miller)}}$ if $A_v = 50$ and $C_{bc} = 5 \text{pF}$?
5- Determine $C_{\text{out(Miller)}}$ if $A_v = 25$ and $C_{bc} = 3 \text{pF}$?
6- How much increase in actual voltage gain corresponds to $+12 \text{ dB}$?
7- Convert a power gain of 25 to decibels
8- What power corresponds to $0 \text{dBm}$?
9- A certain BJT amplifier exhibits three critical frequencies in its low-frequency response: $f_{C1} = 130 \text{Hz}$, $f_{C2} = 167 \text{ Hz}$, $f_{C3} = 75 \text{ Hz}$. Which is the dominant critical frequency?
10- If the midrange voltage gain of the amplifier in Q:9 is $50 \text{ dB}$, what is the gain at the dominant $f_c$?
11- A certain RC circuit has an $f_c = 235 \text{ Hz}$, above which the attenuation is $0 \text{ dB}$ . What is the dB attenuation at $23.5 \text{ Hz}$?
12- What is the amount of phase shift contributed by an input circuit when $X_C = 0.5 R_{\text{in}}$ at a certain frequency below $f_{C1}$?
13- What determines the high-frequency response of an amplifier?
14- If an amplifier has a midrange voltage gain of 80, the transistor’s $C_{bc}$ is $4 \text{ pF}$, and $C_{be} = 8 \text{ pF}$, what is the total input capacitance?
15- A certain amplifier has $f_{c(\text{input})} = 3.5 \text{ MHz}$ and $f_{c(\text{output})} = 8.2 \text{ MHz}$. Which circuit dominates the high-frequency response?
16- What is the voltage gain of an amplifier at $f_T$?
17- What is the bandwidth of an amplifier when $f_{cu} = 25 \text{ kHz}$ and $f_{cL} = 100\text{Hz}$?
18- The $f_T$ of a certain transistor is $130 \text{ MHz}$. What voltage gain can be achieved with a bandwidth of $50 \text{ MHz}$?
19- One stage in an amplifier has $f_{cL} = 1 \text{ kHz}$ and the other stage has $f_{cL} = 325 \text{ Hz}$. What is the dominant lower critical frequency?
20- In a certain 3-stage amplifier $f_{cu(1)} = 50 \text{ kHz}$, $f_{cu(2)} = 55 \text{ kHz}$, and $f_{cu(3)} = 49 \text{ kHz}$. What is the dominant upper frequency?
21- When more identical stages are added to a multistage amplifier with each stage having the same critical frequency, does the bandwidth increase or decrease?
22- The logarithm of a number will give you the power to which the base must be brought to obtain the same number. If the base is 10, it is referred to as the common logarithm; if the base is $e = 2.71828...$, it is called the natural logarithm.
23- Since the decibel rating of any piece of equipment is a comparison between levels, a reference level must be selected for each area of application. For audio system, terns the reference level is generally accepted as $1 \text{ mW}$.
24- The dB gain of cascaded systems is simply the sum of the dB gains of each stage.
25- It is the capacitive elements of a network that determine the bandwidth of a system. The larger capacitive elements of the basic design will determine the low cutoff frequency, whereas the smaller parasitic capacitors will determine the high cutoff frequencies.
26- The narrower the bandwidth, the smaller the range of frequencies that will permit a transfer of power to the load that is at least 50% of the mid-band level.
27- A change in frequency by a factor of 2, equivalent to 1 decade, results in a 6-dB change in gain. For a 10:1 change in frequency, there is a 20-dB change in gain.
28- For any inverting amplifier, the input capacitance will be increased by a miller effect capacitance determined by the gain of the amplifier and the interelectrode (parasitic) capacitance between the input and output terminals of the active device.
11-Feedback Amplifiers

Feedback Theory

A typical feedback connection is shown in Fig11-1, the input signal, $V_s$ is applied to a mixer network, where it is combined with a feedback signal, $V_f$. The difference of these signals is then the input voltage to the amplifier. $V_o$, the amplifier output is connected to the feedback network ($\beta$), which provides a reduced portion of the output as feedback signal to the input mixer network.

![Fig11-1 Simple block diagram of feedback Amplifier](image)

If the feedback signal is of opposite polarity to the input signal as shown in fig11-1, negative feedback results, negative feedback results in reduced over-all voltage gain.

A number of improvements are obtained:
1. Higher input impedance.
2. Better stabilized voltage gain.
3. Improved frequency response.

Feedback connection Types

There are four basic ways of connecting the feedback signal. Both voltage and current can be fed back to the input either in series or in parallel, there can be:

1. Voltage- Series Amplifiers

![Voltage-Series feedback, $A_f = \frac{V_o}{V_s}$](image)

2. Voltage- Shunt Amplifiers

![Voltage-Shunt feedback, $A_f = \frac{V_o}{I_s}$](image)
3. Current-Series Amplifiers

(c) Current-Series feedback, $A_f = I_o/V_s$

4. Current-Shunt Amplifiers

(d) Current-Shunt feedback, $A_f = I_o/I_s$

Fig 11-2 Feedback amplifier types

Voltage refers to connecting the output voltage as input to the feedback network.
Current refers to tapping off some output current through the feedback network.
Series refers to connecting the feedback signal in series with the input signal voltage.
Shunt refers to connecting the feedback signal in shunt (parallel) with an input current source.
Series feedback connection tends to increase the input resistance, while shunt feedback connections tend to decrease the input resistance. Voltage feedback tends to decrease the output impedance, while current feedback tends to increase the output impedance. Typically, higher input and lower output impedances are desired for the most cascade amplifiers. Both of these are provided using the voltage-series feedback connection.

Gain with Feedback

Below a summary of the gain without feedback $A$, feedback factor $\beta$, and gain with feedback $A_f$ of Fig 11-2, the overall gain of the circuit is reduced by a factor $(1 + \beta A)$ as shown in Table 11-1.

<table>
<thead>
<tr>
<th>Voltage-Series</th>
<th>Voltage-Shunt</th>
<th>Current-Series</th>
<th>Current-Shunt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_o$</td>
<td>$V_o$</td>
<td>$I_o$</td>
<td>$I_o$</td>
</tr>
<tr>
<td>$V_i$</td>
<td>$I_i$</td>
<td>$V_i$</td>
<td>$I_i$</td>
</tr>
</tbody>
</table>

Gain without feedback $A$

Feedback $\beta$

Gain with feedback $A_f$
1. Voltage-Series Amplifiers
If $V_f = 0$ (there is no feedback), Fig11-2(a)

$$A = \frac{V_o}{V_i} = \frac{V_o}{V_i}$$

If feedback signal is connected, $V_f$ is connected in series with the input, then

$$V_i = V_s - V_f$$

$$V_o = AV_i = A(V_s - V_f) = AV_s - AV_f = AV_s - A(\beta V_o)$$

$$(1 + \beta A)V_o = AV_s$$

So that the overall voltage gain with feedback is

$$A_f = \frac{V_o}{V_f} = \frac{A}{1 + \beta A}$$

[11-2]

The overall gain of the circuit is reduced by a factor $(1 + \beta A)$

Input Impedance with Voltage-Series Feedback
In fig 11-3 the input impedance can be determined as follows

$$I_i = \frac{V_i}{Z_i} = \frac{V_s - V_f}{Z_i} = \frac{V_s - \beta V_i}{Z_i}$$

$$I_i Z_i = V_s - \beta V_i$$

$$V_s = I_i Z_i + \beta A V_i$$

$$V_i = I_i Z_i + \beta A I_i Z_i$$

$$Z_o = \frac{V_o}{I_r} = Z_i + (\beta A) Z_i = Z_i (1 + \beta A)$$

[11-3]

Output Impedance with Voltage-Series feedback
The output impedance for the connection of Fig11-2 is dependent on whether voltage or current feedback is used. For voltage feedback, the output impedance is decreased. While current feedback increases the output impedance. Fig11-3, the output impedance is
determined by applying a voltage, $V$, resulting in a current, $I$, with $V_s$ shorted out ($V_s = 0$) the voltage $V$ is then:

$$V = IZ_o + AV_i$$

$$V_i = -V_f$$

$$V = IZ_o - AV_f = IZ_o - A(\beta V)$$

$$V + \beta AV = IZ_i$$

$$Z_{df} = \frac{V}{I} = \frac{Z_o}{1 + \beta A}$$

With voltage series feedback the output impedance is reduced from that without feedback by the factor $(1 + \beta A)$

2. Voltage-Shunt Amplifiers

The gain with feedback for the network of Fig11-2b is:

$$A_f = \frac{V_o}{I_s} = \frac{A I_i}{I_i + I_f} = \frac{A I_i}{I_i + \beta V_o} = \frac{A I_i}{I_i + \beta A I_i}$$

$$A_f = \frac{A}{1 + \beta A}$$

[11-5]

Input Impedance with Voltage-Shunt Feedback

A more detailed voltage-shunt feedback connection is shown in Fig11-4. The input impedance can be determined to be:

$$Z_{df} = \frac{V_i}{I_s} = \frac{V_i}{I_i + I_f} = \frac{V_i}{I_i + \beta V_o}$$

$$= \frac{V_i/I_i}{I_f/I_i + \beta V_o/I_i}$$

$$Z_{df} = \frac{Z_i}{1 + \beta A}$$

[11-6]

Fig11-4 Voltage-Shunt Feedback connection

This reduced input impedance applies to the voltage-series connection of Fig11-2a and the voltage-shunt connection of Fig11-2b

3. Current-Series Amplifiers

The output impedance with current-series feedback can be determined by applying a signal $V$ to the output with $V_s$ shorted out, resulting in a current $I$, the ratio of $V$ to $I$ being the output
impedance (Fig11-5). For the output part of a current-series feedback connection (Fig11-5), the resulting output impedance is determined as follows with $V_s = 0$

$$V_i = V_f$$

$$I = \frac{V}{Z_o} - AV_i = \frac{V}{Z_o} - A \cdot \frac{V}{Z_o} = \frac{V}{Z_o} - \frac{A \beta I}{Z_o}$$

$$Z_o (1 + \beta A) I = V$$

$$Z_{of} = \frac{V}{I} = Z_o (1 + \beta A)$$

[11-7]

A summary of effect of feedback on input and output impedance is provided in table 11-2.

<table>
<thead>
<tr>
<th>Voltage-Series</th>
<th>Current-Series</th>
<th>Voltage-Shunt</th>
<th>Current-Shunt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{if}$</td>
<td>$Z_o (1 + \beta A)$</td>
<td>$\frac{Z_o}{1 + \beta A}$</td>
<td>$\frac{Z_v}{1 + \beta A}$</td>
</tr>
<tr>
<td>(increased)</td>
<td>(increased)</td>
<td>(decreased)</td>
<td>(decreased)</td>
</tr>
<tr>
<td>$Z_{of}$</td>
<td>$\frac{Z_o}{1 + \beta A}$</td>
<td>$Z_v (1 + \beta A)$</td>
<td>$Z_v (1 + \beta A)$</td>
</tr>
<tr>
<td>(decreased)</td>
<td>(increased)</td>
<td>(decreased)</td>
<td>(increased)</td>
</tr>
</tbody>
</table>

**Example 1:** Determine the voltage gain, input, and output impedance with feedback for voltage series feedback having $A$ = -100, $R_1$ = 20kΩ for feedback of (a) $\beta = -0.1$ and (b) $\beta = -0.5$

**Solution:**

(a) $A_f = \frac{A}{1 + \beta A} = \frac{-100}{1 + (-0.1)(-100)} = \frac{-100}{11} = -9.09$

$Z_{if} = Z_s (1 + \beta A) = 10 \text{kΩ} \ (11) = 110 \text{kΩ}$

$Z_{of} = \frac{Z_o}{1 + \beta A} = \frac{20 \times 10^3}{11} = 1.82 \text{kΩ}$

(b) $A_f = \frac{A}{1 + \beta A} = \frac{-100}{1 + (-0.5)(-100)} = \frac{-100}{51} = -1.96$

$Z_{if} = Z_s (1 + \beta A) = 10 \text{kΩ} \ (51) = 510 \text{kΩ}$

$Z_{of} = \frac{Z_o}{1 + \beta A} = \frac{20 \times 10^3}{51} = 392.16 \Omega$

**Effect of negative feedback on gain and bandwidth**

The overall gain with negative feedback is Eq11-2

$$A_f = \frac{A}{1 + \beta A} \approx \frac{A}{\beta A} = \frac{1}{\beta} \quad \text{for } \beta A \gg 1$$
For a practical amplifier the open-loop gain drops off at high frequency due to the active device and circuit capacitance. Gain may also drop off at low frequencies for capacitively coupled amplifier stages. Once the open-loop gain $A$ drops low enough and the factor $\beta A$ is much larger than 1, the Eq. above $A_f \approx 1/\beta$ be true. Fig 11-6 shows that the amplifier with negative feedback has more bandwidth ($\beta_f$) than the amplifier without feedback ($\beta$), the feedback amplifier has a higher upper 3-dB frequency and smaller lower 3-dB frequency.

![Graph of amplifier with and without feedback](image)

**Fig11-6 Effect of negative feedback on gain and bandwidth**

**Gain Stability with Feedback**

By differentiating Eq11-2 we can see how stable the feedback amplifier is compared to an amplifier without feedback.

\[
\left| \frac{dA_f}{A_f} \right| = \frac{1}{1 + \beta A} \left| \frac{dA}{A} \right| \]

\[
\left| \frac{dA_f}{A_f} \right| \approx \frac{1}{\beta A} \left| \frac{dA}{A} \right| \quad \text{for } \beta A \gg 1
\]

This shows that magnitude of the relative change in gain $\left| \frac{dA_f}{A_f} \right|$ is reduced by the factor $|\beta A|$ compared to that without feedback $\left| \frac{dA}{A} \right|$.

**Example 2:** If an amplifier with gain of -1000 and feedback of $\beta = -0.1$ has a gain change of 20% due to temperature, calculate the change in gain of the feedback amplifier.

**Solution:** Using Eq11-9

\[
\left| \frac{dA_f}{A_f} \right| \approx \frac{1}{\beta A} \left| \frac{dA}{A} \right| = \frac{1}{-0.1(-1000)}(20\%) = 0.2\%
\]

The improvement is 100 times. Thus, while the amplifier gain change from $|A| = 1000$ by 20%, the gain with feedback change from $|A_f| = 100$ by only 0.2%.

**Practical feedback circuits**

**Voltage- series Feedback:** Fig 11-7 shows an FET amplifier stage with voltage-series feedback.

**Without feedback the amplifier gain is**

\[
A = \frac{V_o}{V_i} = -g_{in} R_L 
\]

Where $R_L$ is the parallel combination of resistors.
The feedback network provides a feedback factor of
\[ \beta = \frac{V_f}{V_o} = \frac{-R_2}{R_1 + R_2} \]  

Using the value of \( A \) & \( \beta \) above in Eq11-2, the gain with negative feedback to be
\[ A_f = \frac{A}{1 + \beta A} = \frac{-g_m R_L}{1 + [R_2 R_f / (R_1 + R_2)] g_m} \]  

If \( \beta A \gg 1 \), we have
\[ A_f = \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} \]  

**Example 3:** Calculate the gain without and with feedback for the FET Amp circuit of fig11-7 and the following circuit values: \( R_1 = 80K\Omega, R_2 = 20K\Omega, R_O = 10K\Omega, R_D = 10K\Omega, \quad g_m = 4000\mu S \)

**Solution:**

\[ R_L = \frac{R_D R_O}{R_1 + R_2} = \frac{10 k\Omega \times (10 k\Omega)}{10 k\Omega + 10 k\Omega} = 5 k\Omega \]

Neglecting 100 k\Omega resistance of \( R_1 \) and \( R_2 \) in series
\[ A = -g_m R_L = -(4000 \times 10^{-6} \mu S)(5 k\Omega) = -20 \]

The feedback factor is
\[ \beta = \frac{-R_2}{R_1 + R_2} = \frac{-20 k\Omega}{80 k\Omega + 20 k\Omega} = -0.2 \]

The gain with feedback is
\[ A_f = \frac{A}{1 + \beta A} = \frac{-20}{1 + (-0.2)(-20)} = \frac{-20}{5} = -4 \]

The emitter-follower circuit of Fig11-8 provides voltage-series feedback. The signal voltage, \( V_s \), is the input voltage, \( V_i \) the output voltage, \( V_o \) is also the feedback voltage in series with the input voltage.
Without feedback $V_f = 0$ so that

$$A = \frac{V_o}{V_s} = \frac{h_{fe}I_fR_E}{V_s} = \frac{h_{fe}R_E(V_s/h_{ie})}{V_s} = \frac{h_{fe}R_E}{h_{ie}}$$

$$\beta = \frac{V_f}{V_o} = 1$$

With feedback

$$A_f = \frac{V_o}{V_s} = \frac{A}{1 + \beta A} = \frac{h_{fe}R_E/h_{ie}}{1 + (1)(h_{fe}R_E/h_{ie})}$$

$$= \frac{h_{fe}R_E}{h_{ie} + h_{fe}R_E}$$

For $h_{fe}R_E \gg h_{ie}$,

$$A_f \approx 1$$

**Current-series feedback**

Fig11-9, the current through resistance $R_E$ results in a feedback voltage that opposes the source signal applied so that the output voltage $V_o$ is reduced. To remove the current-series feedback, the emitter resistor must be either removed or bypassed by a capacitor.

Without feedback

$$A = \frac{I_o}{I_i} = \frac{-I_e h_{fe}}{I_e h_{ie} + R_E} = \frac{-h_{fe}}{h_{ie} + R_E}$$
The input & output impedance are
\[ Z_i = R_B \| (h_{ie} + R_F) \equiv h_{ic} + R_E \]  
\[ Z_o = R_C \]  

With feedback
\[ A_f = \frac{I_o}{I_i} = \frac{A}{1 + \beta A} = \frac{-h_{ie}/h_{ie}}{1 + (-R_F)\left(\frac{-h_{fe}}{h_{ie} + h_{fe}R_E}\right)} = \frac{-h_{fe}}{h_{ic} + h_{fe}R_E} \]

The input & output impedance
\[ Z_{if} = Z_i (1 + \beta A) = h_{ie}\left(1 + \frac{h_{fe}R_E}{h_{ie}}\right) = h_{ic} + h_{fe}R_E \]
\[ Z_{of} = Z_o (1 + \beta A) = R_C\left(1 + \frac{h_{fe}R_E}{h_{ie}}\right) \]

The voltage gain \((A_f)\) with feedback is
\[ A_{vf} = \frac{V_o}{V_i} = \frac{I_o R_C}{V_o} = \left(\frac{I_o}{V_o}\right)R_C = A_f R_C = \frac{-h_{fe}R_C}{h_{ic} + h_{fe}R_E} \]

Example 4: Calculate the voltage gain of Fig11-10

Solution: Without feedback
\[ A = \frac{I_o}{V_i} = \frac{-h_{fe}}{h_{ic} + R_E} = \frac{-120}{900 + 510} = -0.085 \]
\[ \beta = \frac{V_f}{I_o} = -R_E = -510 \]

The factor \(1 + \beta A\) is then
\[ 1 + \beta A = 1 + (-0.085)(-510) = 44.35 \]

The gain with feedback is then
\[ A_f = \frac{I_o}{V_i} = \frac{A}{1 + \beta A} = \frac{-0.085}{44.35} = -1.92 \times 10^{-3} \]

and the voltage gain with feedback \(A_{vf}\) is
\[ A_{vf} = \frac{V_o}{V_i} = A_f R_C = (-1.92 \times 10^{-3})(2.2 \times 10^3) = -4.2 \]
Voltage-shunt feedback

The circuit of fig 11-11 is a voltage-shunt feedback amplifier using an FET with feedback, $V_f = 0$

\[
\begin{align*}
A_v &= \frac{-R_c}{r_c} = \frac{-2.2 \times 10^3}{7.5} = -293.3 \\
\end{align*}
\]

**Voltage-shunt feedback**

The circuit of fig 11-11 is a voltage-shunt feedback amplifier using an FET with feedback, $V_f = 0$

\[A = \frac{V_o}{I_i} = -g_mR_D R_S\]

\[\beta = \frac{I_f}{V_o} = -\frac{1}{R_F}\]

With feedback, the gain of the circuit is

\[A_f = \frac{V_o}{I_s} = \frac{A}{1 + \beta A} = \frac{-g_mR_D R_S}{1 + \left(-\frac{1}{R_F}\right)(-g_mR_D R_S)}\]

\[= \frac{-g_mR_D R_S R_F}{R_F + g_mR_D R_S}\]

The voltage gain of the circuit with feedback is then

\[A_{vf} = \frac{V_o}{I_f} = \frac{V_o}{I_s} \frac{I_s}{I_f} = \frac{-g_mR_D R_S R_F}{R_F + g_mR_D R_S} \left(\frac{1}{R_F}\right)\]

\[= \frac{-g_mR_D R_S}{R_F + g_mR_D R_S} \frac{R_F}{R_F + g_mR_D R_S}\]

**Example 5**: Calculate the voltage gain with and without feedback for the circuit of fig11-12 with values of $g_m = 5mS$, $R_D = 5.1k\Omega$, $R_S = 1k\Omega$, and $R_F = 20k\Omega$

**Solution: Without feedback**, the voltage gain is

\[A_v = -g_m R_D = -(5 \times 10^{-3})(5.1 \times 10^3) = -25.5\]

**With feedback the gain is reduced to**

\[A_{vf} = \frac{-g_mR_D}{R_F + g_mR_D R_S} \frac{R_F}{R_F + g_mR_D R_S}\]

\[= (-25.5) \frac{20 \times 10^3}{(20 \times 10^3) + (5 \times 10^{-3})(5.1 \times 10^3)(1 \times 10^3)}\]

\[= -25.5(0.44) = -11.2\]