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Fluid:-( Liquids or Gases): It is a substance which *deforms* continuously under the application of a *shear stress* in specific situations such as the simple shear situation shown in Fig.1:

There are *normal stresses* and *tangential stresses*.

- *Normal stress*:
- *Shear stress*:

  - *Pressure* is an example of a *normal* stress, and acts inward, toward the surface, and perpendicular to the surface.
  - A *shear stress* is an example of a *tangential stress*, i.e., it acts along the surface, parallel to the surface. Friction due to fluid viscosity is the primary source of shear stresses in a fluid.  
  - *Fluids at rest cannot resist a shear stress*; in other words, when a shear stress is applied to a fluid at rest, the fluid will not remain at rest, but will move because of the shear stress, as shown in Figs.2,3.

Therefore conclude:
- Fluid continues to deform (or move) under the application of a shear force.
- Fluid at rest cannot sustain a shear stress.

Fluid Mechanics:- It is the basic for all engineering science, it deals with TWO type of fluid and application of the laws force and motion:

- Static, The main equation required for this is Newton's second law for non-accelerating bodies,
- Dynamic, The main equation required for this is Newton's second law for accelerating bodies, i.e.

Fluid application in Ch. Eng. in Unit operation such as filtration, mixing, fixed & fluidization beds, sedimentation, distillation...etc. Therefore student learn Fundamental concept of fluid static and motions, including governing equations that describe the basic principles and Pressure-drop calculations for laminar, tubular, incompressible, compressible and flow in packed-beds of solid particles.
Dimensions & Units:- There are Four fundamental dimensions as:

- \([M]\)
- \([L]\)
- \([T]\)
- \([\theta]\)

and all others dimensions can be derived for ex, acceleration \([LT^{-2}]\), area \([L^2]\), density \([ML^{-3}]\), etc. and 2nd law Newton's related to \([MLT]\) as \(F=mass \cdot (acc.,a)\), therefore \([F]=\)

Systems: SI (International systems of units), and BG (British gravitational units).

Physical Properties of Fluids:-

A. Density, Specific Weight, Relative Density

Density \((\rho)\) = mass per unit volume of substance = \(\delta m/\delta v\); \([\rho] = [ML^{-3}]\).

Specific weight \((\gamma)\) = force exerted by the earth's gravity upon a unit volume of the substance = \(pg\); \([\gamma]\) = The common units used is \((N/m^3)\), (dyne/cm³).

sp. vol=1/\(\rho\)

Relative density (specific gravity) = ratio of mass density of the substance to that of water at a standard temperature and pressure = \(\rho/\rho_w\) (non-dimensional). i.e

\[sp.gr. = \text{mass of density/mass density of water}\]

Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

B. Viscosity

Viscosity, \(\mu\) (nu) is a measure of the importance of friction in fluid flow. Consider, for example, a fluid in two-dimensional steady shear between two parallel plates, as shown below. The bottom plate is fixed, while the upper plate is moving at a steady speed of \(U\), as shown in Fig3.

All fluids are viscous, "Newtonian Fluids" obey the linear relationship given by Newton's law of viscosity.

\[\tau = \frac{du}{dy} \text{ is the velocity gradient or rate of shear strain, (s⁻¹)}\]

\(\mu\) is the "coefficient of dynamic viscosity=

\[\text{viscosity} = \frac{\tau}{\frac{du}{dy}} = \frac{\text{force/velocity}}{\text{area}} = \frac{\text{force x time}}{\text{area}} = \frac{mass}{length \times area}\]

Units: Newton seconds per square metre, \(N m^{-2}\) or Kilograms per meter per second, \(kg m^{-1} s^{-1}\). or in Poise, P, where 10 P = 1 \(kg m^{-1} s^{-1}\).

Kinematic Viscosity \(\nu\) (nu)- It is defined as the ratio of dynamic viscosity to mass density. \(\nu = \cdots\), units: \(m^2/s\) or in stoke \((10^4 St = 1 m^2 s^{-1})\).
Newtonian, Non-Newtonian Fluids: Flows obey Newton's law if they are Newtonian fluids, i.e., "applied shear stress varies linearly with the rate of deformation" $\text{slope} = \mu$, and fluids that do not obey this law are called Non-Newtonian fluids, i.e., $\text{slope} \neq \mu$, as shown in Figs. 5, 6.

Typically, as temperature increases, the viscosity will decrease for a liquid, but will increase for a gas. The fluid is non-Newtonian if the relation between $\tau$ & $du/dy$ is non-linear.

C. Surface Tension and Capillarity

- Surface tension ($\sigma$, sigma) is a property of liquids which is felt at the interface between the liquid and another fluid (typically a gas). Surface tension has dimensions of force per unit length, and always acts parallel to the interface.
- Surface molecules are subject to an attractive force from nearby surface molecules so that the surface is in a state of tension, as shown in Fig. 7.
- A soap bubble is a good example to illustrate the effects of surface tension, as shown in half bubble Fig. 7.

![Half a bubble diagram]

$$2(2\pi R)\sigma = (\pi R^2)\Delta P_{\text{bubble}}$$

Fig. 7 a

Consider a soap bubble of radius $R$ with internal pressure $P_{\text{in}}$ and external (atmospheric) pressure $P_{\text{out}}$. Hence, balancing the forces due to surface tension and pressure difference:

$$\Delta P_{\text{bubble}} =$$

- 3 --Lect.1
Surface tension is also important at the interface between a liquid, a gas, and a solid. For example, a meniscus occurs when the surface of a liquid touches a solid wall, as most readily noticed when a capillary tube is placed in a liquid, as shown in Figs.:

\[ \rho g \pi R h = 2 \pi R \sigma_s \cos \theta \]

\[ h = \]

D. Vapor Pressure
Vapor pressure is defined as the pressure at which a liquid will boil (vaporize). Vapor pressure rises as temperature rises. Vapor pressure is important to fluid flows because, in general, pressure in a flow decreases as velocity increases. This can lead to cavitation, which is generally destructive and undesirable.

E. Ideal & Real Fluid
An ideal fluid is one that is incompressible, and having \( \mu = 0 \), this is imaginary fluid, and a fluid which possesses viscosity, is known as real fluid, all the fluids actually having viscosity.

The Common Symbol Flow Rate:
Volumetric flowrate, \( Q = u A \) (u-velocity, A-cross sectional area of flow).
Mass flowrate, \( m = Q \rho = u A \rho \)

Mass flux or mass velocity, \( G = \frac{m}{A} = u \rho \) has unit in SI \( \text{(Kg/m}^2\text{.s)} \)
F. Compressibility (E): All fluids are compressible under the application of external forces. The compressibility of a fluid is expressed by its bulk modulus of elasticity $E$, which is the ratio of the change in unit pressure to the corresponding volume change per unit volume.

$$E = \frac{\Delta V}{\Delta P}$$

G. Incompressible Fluid: If the difference in pressure ($\Delta p < 10\%$) i.e temp. is nearly constant then the physical properties can be considered constant with fluid flow, such as ($\rho, \mu$ ..). This fluid is called incompressible, for ex. all liquids, water.

H. Compressible Fluid: If the difference in pressure ($\Delta p > 10\%$) i.e temp. is change through flow then physical properties must be considered in designing data, for ex. all gases.

I. Pressure: It is the force /cross sectional area, common unit $(N/m^2)$=Pa. The pressure between two points refers ($\Delta p$), $P=\frac{h \rho g}{\Delta p}$ and $\Delta P=\Delta h \rho g$

Classification of Fluid:-

It can be classified as following:

```
FLUID
    ↓
DYNAMIC
    ↓ according to effect pressure
    ↓
COMPRESSIBLE (Temp. CHANGE)
    ↓ according to effect shear stress
    ↓
NEWTONIAN
    ↓
Time-Independent
    ↓ Bingham fluid
Pseudoplastic fluid
Dilatant fluid
```

```
INCOMPRESSIBLE (physical properties constant)
    ↓
NON-NEWTONIAN
    ↓
Time-Dependent
    ↓ Viscoelastic
    ↓
    ↓ Thixotropic liquid
```
EXAMPLES:-
EX.(1)- A body requires a force of 100 N to accelerate it at rate of 0.20 m/s². Determine the mass of the body in kg and in slugs.
sol.:

EX.(2)- A reservoir of CCl₄ has m=500 Kg & V=0.315 m³. Find the CCL₄: weight, mass density, sp.wt. & sp.gr..
sol.:

EX.(3)- Under st. conditions a certain gas wt. 0.14 lb/ft³ calculate the density, sp.vol., sp.gr. relative to air weighing 0.075 lb/ft³.
sol.:

EX.(4)- A large plate moves with speed \( v_0 \) over a stationary plate on a layer of oil (see Fig.). If the velocity profile is that of a parabola, with the oil at the plates having the same velocity as the plates, what is the shear stress on the moving plate from the oil? If a linear profile is assumed, what is the shear stress on the upper plate?
sol.:

EX.(5)- A square block weighing 1.1 KN and 250mm on an edge slides down an incline on a film of oil 6.0\( \mu \)m thick (see Fig.) Assuming a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7 mPa.s.
sol.:
The basic properties of a static fluid is pressure (P). The pressure exerted by a static fluid depends only upon: the depth of the fluid (h), the density of the fluid (ρ), and acc. of gravity (g). As the fluid is at rest:

- a static fluid can have no shearing force acting on it, and that
- any force between the fluid and the boundary must be acting at right angles to the boundary.
- For an element of fluid at rest, the element will be in equilibrium - the sum of the components of forces in any direction will be zero.
- The sum of the moments of forces on the element about any point must also be zero.

Pressure always acts inward normal to any surface, and hence has dimensions of force per unit area, or [ML^{-1}T^{-2}], in the Metric system of units, pressure is expressed as "pascals" (Pa) or N/m^2. Standard atmospheric pressure is 101.3 kPa or 14.69 psi.

Pressure is formally defined to be  \( P = \frac{F}{A} \)

**Pascal’s Law for Pressure At A Point**

**Pascal’s Law**: Pressure at any point is the same in all directions. This is known as Pascal’s Law and applies to fluids at rest., as shown in Figs.2,3

\[ p_x = p_y = p_z \]

(Proof that pressure acts equally in all directions.)

Fig.2 Triangular prismatic element of fluid

Fig.3 Pressure at a point has the same magnitude in all directions, and is called isotropic.
Fluid Pressure Calculation: Consider cubic element of the vertical column of fluid (constant $\rho$ homogeneous fluid), as shown in Fig.4

- Assume a stationary column of fluid of height ($h_2$).
- Assume the cross-sectional area $dA$ is uniform $A=A_0=A_1=A_2$
- The pressure above fluid is $P_o$ (atmospheric pressure).
- Pressure, $P$ acting on an element at height $h_1$ is equal at all direction.

Then, to calculate $P_2$ as following:

2nd law Newton’s is given as $F = m. g$..............(1)

But $m = \rho V$ & $V = A h_2$ $P = F / A$ Fig.4 Pressure in a static fluid

$\therefore P_2 = \rho A h_2 g / A$..........................(2)

$\therefore P_2 = \rho g h_2$.....................................(3) In SI unit this is Fundamental Equation and pressure is given by this equation gauge pressure

However to get the total pressure $P_2$ on $A_2$ is given by: $P_{total} = P_2 + P_o$.........(4)

To calculate $P_1$ at $h_1$ is given as $P_1 = \rho g h_1 + P_o$

Therefore to calculate pressure difference between two points 1 & 2 is:

$P_2 - P_1 = (h_2 \rho g + P_o) - (h_1 \rho g + P_o) = (h_2 - h_1) \rho g$

$\therefore \Delta P = \rho g \Delta h$...............(5) in SI Unit N/m$^2$

$\therefore \Delta P = \rho g \Delta h / g_c$............(6) in English Units psi

Note:- Static fluid pressure does not depend on the shape, total mass or surface area of the liquid as shown in Figs5

Fig.5 Pressure in vessels of various shapes

The term pressure is sometimes associated with different terms as shown in Figure:

Absolute pressure = Gauge pressure + Atmospheric pressure

Absolute pressure=Atmospheric pressure-Vacuum pressure

The pressure in some times is given in head unit as $m$, $mm$, ft., $mmH_2O$...etc,

Example: a pressure of 500 $K N m^{-2}$ in the height a column of water of density, $\rho = 1000 kg/m^3$

sol: $P = \rho g h \ h = P / \rho g \ h = 50.95m$ of water in term m.Hg, $\rho = 13.6*10^3 Kg/m^3$

$h = 3.75$ m-Hg
**Pressure Measurement By Manometer**

**Piezometer Tube** The simplest manometer is a tube, open at the top, which is attached to a vessel or a pipe containing liquid at a pressure (higher than atmospheric) to be measured. This simple device is known as a piezometer tube. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is gauge pressure: \( P_A = \gamma h_1 = \rho g h_1 \) This method can only be used for liquids (i.e. not for gases) and only when the liquid height is convenient to measure. It must not be too small or too large & pressure changes must be detectable.

**manometer** : The manometer is an important device for measuring pressure difference and it is modified of piezometer, it can be used for measurement of comparatively high pressure and of both gauge and vacuum pressures. It consist different types as:

- simple U-tube
- differential manometer
- inclined manometer
- multifluid manometer
- inverted manometer
- micromanometer

**Simple U-tube Manometer**
- Assume fluid A immiscible with B.
- \( \rho_A \) heavier than \( \rho_B \) such as Hg.
- \( P_a \) is exerted in one arm of U-tube and \( P_b \) in the other arm.
- difference in \( P_a - P_b \)
- Then to derive relationship between \( P_a - P_b \) as following:

Pressure at point 1 is \( P_a \)
Pressure at point 2 is \( P_2 = P_a + (Z + R) g \rho_B \)
By hydrostatic balance at the same elevation \( P_2 = P_3 \)
\( P_4 = Z \rho_B g \) \( P_5 = P_6 \)
These statement Eqns. can be summarized by the Eqn.: \( P_a + (Z + R) g \rho_B - R \rho_A g - Z g \rho_B = P_b \) simplification of this Eqn. gives:

\[ P_a - P_b = R_m g (\rho_A - \rho_B) \] in SI Unit where \( R \rightarrow \) is called reading manometer.

**Differential manometer**
A differential manometer can be used to measure the difference in pressure between two containers or two points in the same system

\( P_2 = P_3 \) to get expression for the pressure difference between \( A \) and \( B \):

\[ p_A - p_B = \gamma_A h_2 + \gamma_3 h_3 - \gamma_1 h_1 \]

In the common case when \( A \) and \( B \) are at the same elevation \( h_1 = h_2 + h_3 \) and the fluids in the two containers are the same \( \gamma_1 = \gamma_3 \) one may show that the pressure difference registered by a differential manometer is given by

\[ \Delta p = \left( \frac{\rho_m}{\rho} - 1 \right) \rho g h \]
where \( \rho_m \) is the density of the manometer fluid, \( \rho \) is the density of the fluid in the system, and \( h \) is the manometer differential reading.

**Inclined-tube manometer**

As shown in Figure, the differential reading is proportional to the pressure difference. If the pressure difference is very small, the reading may be too small to be measured with good accuracy. To increase the sensitivity of the differential reading, one leg of the manometer can be inclined at an angle \( \theta \), and the differential reading is measured along the inclined tube. As shown in Fig., \( h_2 = l_2 \sin \theta \), and hence:

\[
 p_A - p_B = \gamma_1 l_2 \sin \theta + \gamma_3 h_3 - \gamma_1 h_1
\]

**Multifluid manometer**

The pressure in a pressurized tank is measured by a multifluid manometer, as is shown in the figure. Show that the air pressure in the tank is given by

\[
 P_{\text{air}} = P_{\text{atm}} + g \left( \rho_{\text{mercury}} h_3 - \rho_{\text{oil}} h_2 - \rho_{\text{water}} h_1 \right)
\]

**The inverted manometer**

As shown in Figure inverted manometer is used for measuring pressure differences in liquids. The space above the liquid in the manometer is filled with air, which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.

**Vertical Micromanometer**

Micromanometer is a modified form of a U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one limb of the manometer, as shown in Fig. For any variation in pressure, the change in the liquid level in the reservoir will be so small that it may be neglected, and the pressure is indicated by the height of the liquid in the other limb.

Let: X-X be the datum line in the reservoir when the single column manometer is not connected to the pipe. We consider that the man. is connected to a pipe containing light liquid under very high pressure, the n how read the pressure?
Let: $h_1 =$ height of the centre of the pipe above X-X,
$h_2 =$ rise of heavy liquid (after experiment) in the right limb
$\partial h =$ fall of heavy liquid in the reservoir
$h =$ P in the pipe,
A = Cross-sectional area of the reservoir
a = Cross-sectional area of the tube (right limb)
$S_1, S_2 =$ Sp.Gr. of light and heavy liquid respectively

Therefore: by quantity balance \( A \times \partial h = a \times h_2 \)

let P above Z-Z is given as:
in the lift limb $= h + (h_1 + \partial h)S_1$
in the right limb $= (h_2 + \partial h)S_2$

Equating P, to give, $h = \frac{a \times h_2}{A} \left( S_2 - S_1 \right) + h_2S_2 - h_1S_1$

But when $a/A$ very small, \( \therefore h = h_2S_2 - h_1S_1 \)

**Mechanical Gauges:**
I is used for medium & high pressure measurement, include the following type:-
- Bourdon tube pressure gauge.
- Diaphragm gauge, and
- Vacuum gauge.

Bourdon tube pressure gauge.: It is used for measuring high as well as low pressure. A simple form of this gauge is shown in Fig. the Bourdon tube are generally made of bronze or nickel steel.

Diaphragm gauge,: This type of gauge employs a metallic disc or diaphragm instead of a bent tube. This disc or diaphragm is used for actuating the indicating device,. as shown in Fig.

Vacuum gauge.: Bourdon gauges can be used to measure vacuum instead of pressure.
FLUID FLOW- 2nd Year CHEMICAL ENG. DEPT.
LECTURE-THREE
DIMENSIONAL ANALYSIS

Dimensional Analysis: It is a mathematical technique which makes use of the study of the dimensions for solving several engineering problems, such as in Fluid, Heat, and Mass transfer. Each physical phenomena can be expressed by an equation giving relationship between different quantities, such quantities are dimensional and non-dimensional.

Dimensional analysis has become an important tool for analyzing fluid flow problems. It is specially useful in presenting experimental results in a concise form.

Uses of Dimensional Analysis:
1. To test the dimensional homogeneity of any equation of fluid flow or others phenomena in transport phenomena.
2. To derive rational formula.
3. To derive Eqn. expressed in terms of non-dimensional parameters to show the relative significance of each parameters.
4. To plane model tests.

Advantages:
1. It express the functional relationship between the variables in dimensionless groups.
2. It reduces the No. of variables to dimensionless groups.
3. Design curves.
4. It enables getting up a theoretical Eqn.in a simplified dimensional form.
5. It is provides partial solutions.

Dimensions: The various physical quantities used in fluid phenomenon can be expressed in terms of:
- Fundamental Dimension or quantities are; Mass →[M], Length→[l], Time→[T], & Temperature→[θ], therefore a quantity expressed dimensionally in [M] [L] [T] [θ]
- OR [F] [L] [T] [θ] as shown in Table 1.1 p.3 in vol.1 5ed. or Table 7.1
- Derived Quantities are expressed in terms of the fundamental dim. (ex. area→L^2, acceleration→LT^-2…..etc.

Ex.1 Determine the dimensions of :- Discharge, Kinematic viscosity, Sp. Wt., Force.

Dimensional Homogeneity: It states that every term in an Eqn.when reduced to fundamental dimensions must contain identical powers of each dimensions. In dimensional homogeneous equation, only quantities having the same dimensions can be added, subtracted or equated, i.e L.H.S=R.H.S.

For ex. P=ρgh  Dimensions L.H.S=ML^-1T^-2, & R.H.S=ML^3 x LT^-2 x L=ML^1T^-2 therefore L.H.S=R.H.S

Ex.2 Determine the dimensions of  E in the dimensional Homogeneous Eqn. as

\[ E = mc^2 \left[ \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \right] - 1 \]

Ans. E=ML^2T^-2 has dimension Energy.

Methods of Dimensional Analysis: Rayligh's or (power series) , & Buckingham;s

Rayligh's Methods: It is used for determining the expression for a variable (X) which depends upon maximum three or four variables only. In case the No. of independent variables (x_1, x_2, ..), becomes more than four, then it is very difficult to find the expression for the X. In this method a functional relationship of some variables is expressed in the form of an exponential Eqn. which must be dimensionally homogeneous. The following procedure is expressed as:
Assume X is a variable which depends on  x_1, x_2, x_3,.....x_n, then:
i. First of all, write the functional relationship with the given data, i.e.
\[ X = f(x_1, x_2, x_3, \ldots, x_n) \] ............(1)

ii. Now write the equation in terms of a constant with exponents i.e. powers a, b, c, ..., i.e.,
\[ X = C(x_1^a, x_2^b, x_3^c \ldots, x_n^a) \] ............(2)
C \rightarrow \text{constant}, a, b, c, ..., n are arbitrary powers. are obtained by comparing the powers of the fundamental dimensions on both sides, (simultaneous Eqns.).

iii. Now substitute the values of these exponents in the main equation, and simplify it as dimensionless groups., fro ex. Rynold's No. or others.

\textbf{Ex.3}: Find an expression for the drag force (F) on smooth sphere of diameter (D), moving with a uniform velocity (v), in an fluid a density (\( \rho \)), & dynamic viscosity (\( \mu \)).

\textbf{Sol.}
Write the F.D in term of MLT as following:
\[ F \rightarrow \text{MLT}^{-2}, \quad D \rightarrow \text{L}, \quad v \rightarrow \text{LT}^{-1}, \quad \rho \rightarrow \text{ML}^{-3}, \quad \mu \rightarrow \text{ML}^{-1}\text{T}^{-1} \]
Write the function mathematically as,
\[ F = f(D, v, \rho, \mu) \]
Write the functional with powers exponents as,
\[ F = C(D^a, v^b, \rho^c, \mu^d) \]
Write the function in term of MLT as,
\[ \text{MLT}^{-2} = [C(LT^{-1})^b \text{ML}^{-3}x^c \text{ML}^{-1}T^{-1}] \]
Now by the principle of dimensional homogeneity, equating the power of M, L, T on both sides of the equation;
\begin{align*}
M & : 1 = c + d \quad \text{------------------(i)} \\
L & : 1 = a + b - 3c - d \quad \text{------------------(ii)} \\
T & : -2 = -b - d \quad \text{---------------(iii)}
\end{align*}

There are 4 unknowns (a, b, c, d) but 3 Eqns. Therefore, it is not possible to find the values of a, b, c & d. To find, 3 of them can be expressed in terms of 4th variable which \textbf{is most important}. There the role of \( \mu \) is vital one and hence a, b, c are expressed in terms of d (i.e power to viscosity). Therefore : c=1-d, from i, b=2-d, from iii then putting these values in ii, to get a=2-d
Sub. these values of exponent Eqn., to get,
\[ F = C[D^{2-d} \cdot v^{2-d} \cdot \rho^{1-d} \cdot \mu^d] \]
\[ \therefore F = \rho D^2 v^2 \phi(\frac{\mu}{\rho v D}) \ldots \text{OR} \ldots \frac{F}{\rho v^2 D^2} = \phi(\text{Re}^{-1}) \]
Therefore obtain 2 D. groups.

\textbf{Prove: Dimensionless?}

\textbf{Ex.4}: The efficiency (\( \eta \)) of fan depends on the density (\( \rho \)), the dynamic viscosity (\( \mu \)) of the fluid, the angular velocity (\( \omega \)), diameter (D), of the rotor and the discharge (Q). Express \( \eta \) in terms of dimensionless parameters.

\textbf{Ans:}
\[ \eta = \phi(\frac{\mu}{\rho \omega D^2}, \frac{Q}{\omega D}) \]

\textbf{Buckingham's \( \pi \)-Theorem:}
Buckingham's method is an improvement over the Rayligh's method. It states that “ If there are \( n \) variables in a dimensionally homogeneous equation, and if these variables contain \( m \) fundamental dimensions such as (MLT) they may be grouped into \( n-m \) non-dimensional independent \( \Pi \)-terms” i.e \( \Pi = n-m \), where \( \Pi \) dimensionless groups.
Mathematically, if a dependent variable X1 depends upon independent variables (X2, X3, X4, ......... Xn), the functional equation may be written as:
\[ X_1 = k (X_2, X_3, X_4, \ldots \ldots \ldots X_n) \]

This equation may be written in its general form as; \[ f(X_1, X_2, X_3, \ldots \ldots \ldots X_n) = 0 \]

In this equation, there are \( n \) variables. If there are \( m \) fundamental dimensions, then according to Buckingham’s \( \Pi \)-theorem; \( f_1 (\Pi_1, \Pi_2, \Pi_3, \ldots \ldots \ldots \Pi_{n-m}) = 0 \)

The Buckingham’s \( \Pi \)-theorem is based on the following procedure:

1. First of all, write the functional relationship with the given data. (dep. & independent variables). i.e \( f(X_1, X_2, X_3, \ldots \ldots X_n) = 0 \)
2. Then write the equation in its general form.
3. Now choose \( m \) repeating variables (or recurring set) and write separate expressions for each \( \Pi \)-term. Every \( \Pi \)-term will contain the repeating variables and one of the remaining variables. Just the repeating variables are written in exponential form. i.e \( f(\Pi_1, \Pi_2, \Pi_3, \ldots \ldots \ldots \Pi_{n-m}) = 0 \), let \( X_2, X_3, X_4 \) are rep. var., then; \( \Pi_1 = X_2^{a_1}, X_3^{b_1}, X_4^{c_1}, X_1 \), and so on, \( \Pi_2, \Pi_3, \ldots \ldots, \Pi_{n-m} \)
4. With help of the principle of dimensional homogeneity find out the values of powers \( a, b, c, \ldots \ldots \) by obtaining simultaneous equations.
5. Now substitute the values of these exponents in the \( \Pi \)-terms.
6. After the \( \Pi \)-terms are determined, write the functional relation in the required form, as dimensionless group.

**Selection of Repeating Variables:** The following points should be kept in view while selecting \( m \) repeating variables:

1. The variables should be such that none of them is dimension les.
2. No two variables should have the same dimensions, for ex. \( D/h \)
3. Independent variables should, as far as possible, be selected as repeating variables.
4. Each of the fundamental dimensions must appear in at least one of the \( m \) variables.
5. It must not possible to form a dimensionless group from some or all the variables within the repeating variables. If it were so possible, this dimensionless group would, of course, be one of the \( \Pi \)-term.
6. In general the selected repeating variables should be expressed as the following: (1) representing the flow characteristics, for ex. \( Q \), (2) representing the geometry, for ex. \( D \) and (3) representing the physical properties of fluid, for ex. \( \rho \)

Note: The choice of rep. var., in most of fluid flow problems, may be:

(i) \( l, v, \rho \) (ii) \( d, v, \rho \) (iii) \( l, v, \mu \) (iv) \( d, v, \mu \)

**Ex.5:** The resistance (R) experience by a partially submerged body depends upon the velocity, \( u \), length of the body \( l \), viscosity of the fluid, \( \mu \), density, \( \rho \), and \( g \). Obtain a dimensionless expression for \( R \)?

Sol. in Glass room. Ans.: \( R = l^2 u^2 \rho \phi \left( \frac{\rho ul}{\mu}, \frac{u}{\sqrt{lg}} \right) \)

**Ex.6:** Using Buckingham's, show that the velocity through circular orifice is given by:

\[ u = \sqrt{2gH \phi \left( \frac{D}{H}, \frac{\mu}{\rho u H} \right)} \]

Where \( H \)=Heads causing flow, \( D \)=Diameter of the orifice, \( \mu \)=viscosity, \( \rho \)=density and \( g \)=acceleration gravity.

Sol. show in glass room.

**Note:** Read Examples in Text-book Vol.1 Chem. Eng. & Read some important dimensionless group in Table 1.3
### Table 7.1. Quantities used in Fluid Mechanics and Heat Transfer and their Dimensions

<table>
<thead>
<tr>
<th>L.No</th>
<th>Quantity</th>
<th>Dimensions M-L-T System</th>
<th>Dimensions P-L-T System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>Fundamental Quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Mass, ( M )</td>
<td>( M )</td>
<td>( FL^{-1}T^0 )</td>
</tr>
<tr>
<td>2.</td>
<td>Length, ( L )</td>
<td>( L )</td>
<td>( L )</td>
</tr>
<tr>
<td>3.</td>
<td>Time, ( T )</td>
<td>( T )</td>
<td>( T )</td>
</tr>
<tr>
<td>(b)</td>
<td>Geometric Quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Area, ( A )</td>
<td>( L^2 )</td>
<td>( L^2 )</td>
</tr>
<tr>
<td>5.</td>
<td>Volume, ( V )</td>
<td>( L^3 )</td>
<td>( L^3 )</td>
</tr>
<tr>
<td>6.</td>
<td>Moment of inertia</td>
<td>( L^4 )</td>
<td>( L^4 )</td>
</tr>
<tr>
<td>(c)</td>
<td>Kinematic Quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Linear velocity, ( v ), ( \vec{v} )</td>
<td>( LT^{-1} )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>8.</td>
<td>Angular velocity, ( \omega ), rotational speed, ( \Omega )</td>
<td>( T^{-1} )</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td>9.</td>
<td>Acceleration, ( a )</td>
<td>( LT^{-2} )</td>
<td>( LT^{-2} )</td>
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<tr>
<td>10.</td>
<td>Angular acceleration, ( \alpha )</td>
<td>( T^{-2} )</td>
<td>( T^{-2} )</td>
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<tr>
<td>11.</td>
<td>Discharge, ( Q )</td>
<td>( LT^{-1} )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>12.</td>
<td>Gravity, ( g )</td>
<td>( LT^{-2} )</td>
<td>( LT^{-2} )</td>
</tr>
<tr>
<td>13.</td>
<td>Kinematic viscosity, ( \nu )</td>
<td>( LT^{-1} )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>14.</td>
<td>Stream function, ( \psi ), circulation, ( \Gamma )</td>
<td>( LT^{-1} )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>15.</td>
<td>Vorticity, ( \Omega )</td>
<td>( T^{-1} )</td>
<td>( T^{-1} )</td>
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<tr>
<td>(d)</td>
<td>Dynamic Quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>Force, ( F )</td>
<td>( MLT^{-2} )</td>
<td>( P )</td>
</tr>
<tr>
<td>17.</td>
<td>Density, ( \rho )</td>
<td>( ML^{-3} )</td>
<td>( FL^{-1}T^2 )</td>
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<tr>
<td>18.</td>
<td>Specific heat, ( c )</td>
<td>( ML^{-1}T^{-1} )</td>
<td>( FL^{-2}T^0 )</td>
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<tr>
<td>19.</td>
<td>Dynamic viscosity, ( \mu )</td>
<td>( ML^{-1}T^{-1} )</td>
<td>( FL^{-1}T^0 )</td>
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<td>20.</td>
<td>Pressure, ( P )</td>
<td>( ML^{-1}T^{-2} )</td>
<td>( FL^{-2}T^0 )</td>
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<tr>
<td>21.</td>
<td>Shear stress, ( \tau )</td>
<td>( ML^{-1}T^{-2} )</td>
<td>( FL^{-2}T^0 )</td>
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<tr>
<td>22.</td>
<td>Modulus of elasticity, ( E, )</td>
<td>( ML^{-1}T^{-2} )</td>
<td>( FL^{-2}T^0 )</td>
</tr>
<tr>
<td>23.</td>
<td>Power, ( P )</td>
<td>( MLT^{-2} )</td>
<td>( PL )</td>
</tr>
<tr>
<td>(e)</td>
<td>Thermodynamic Quantities</td>
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<td></td>
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<tr>
<td>24.</td>
<td>Temperature, ( T )</td>
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<td>( 0 )</td>
</tr>
<tr>
<td>25.</td>
<td>Thermal conductivity, ( k )</td>
<td>( ML^{-2}T^{-1} )</td>
<td>( FT^{-1} )</td>
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<tr>
<td>26.</td>
<td>Entropy, ( S )</td>
<td>( ML^{-1}T^{-1} )</td>
<td>( LT^{-1} )</td>
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<tr>
<td>27.</td>
<td>Enthalpy per unit mass, ( H )</td>
<td>( ML^{1}T^{-1} )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>28.</td>
<td>Energy, ( E )</td>
<td>( ML^{1}T^{2} )</td>
<td>( PL )</td>
</tr>
<tr>
<td>29.</td>
<td>Internal energy per unit mass</td>
<td>( ML^{1}T^{2} )</td>
<td>( PL )</td>
</tr>
<tr>
<td>30.</td>
<td>Heat transfer</td>
<td>( ML^{1}T^{2} )</td>
<td>( PL )</td>
</tr>
</tbody>
</table>

### Table 1.3. Some important dimensionless groups

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name of group</th>
<th>In terms of other groups</th>
<th>Definition</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ar )</td>
<td>Archimedes</td>
<td>( \rho L \rho_g d^3 )</td>
<td>Gravitational settling of particle in fluid</td>
<td></td>
</tr>
<tr>
<td>( Db )</td>
<td>Deborah</td>
<td>( \frac{\varepsilon}{\varepsilon^*} )</td>
<td>Flow of viscoelastic fluid</td>
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<tr>
<td>( Eu )</td>
<td>Euler</td>
<td>( P )</td>
<td>Pressure and momentum in fluid</td>
<td></td>
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<tr>
<td>( Fo )</td>
<td>Fourier</td>
<td>( \frac{D_{Th} D_t}{T^2} )</td>
<td>Unsteady state heat transfer/mass transfer</td>
<td></td>
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<tr>
<td>( Fr )</td>
<td>Froude</td>
<td>( \frac{g}{d} )</td>
<td>Fluid flow with free surface</td>
<td></td>
</tr>
<tr>
<td>( Ga )</td>
<td>Galileo</td>
<td>( \frac{\rho L \rho_g d^3}{\mu^2} )</td>
<td>Gravitational settling of particle in fluid</td>
<td></td>
</tr>
<tr>
<td>( Gr )</td>
<td>Grashof</td>
<td>( \frac{g^2 \beta \Delta T}{\mu^2} )</td>
<td>Heat transfer by natural convection</td>
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<tr>
<td>( Gz )</td>
<td>Graetz</td>
<td>( \frac{G_{Re} \rho \mu}{k \delta} )</td>
<td>Heat transfer to fluid in tube</td>
<td></td>
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<tr>
<td>( He )</td>
<td>Hedstrom</td>
<td>( \frac{Re \mu^2}{\mu^2} )</td>
<td>Flow of fluid exhibiting yield stress</td>
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<tr>
<td>( Le )</td>
<td>Lewis</td>
<td>( \frac{k C_{Pr} D}{D} )</td>
<td>Simultaneous heat and mass transfer</td>
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<td>( Ma )</td>
<td>Mach</td>
<td>( \frac{M}{u} )</td>
<td>Gas flow at high velocity</td>
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<tr>
<td>( Nu )</td>
<td>Nusselt</td>
<td>( \frac{h}{k} )</td>
<td>Heat transfer in fluid</td>
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<tr>
<td>( Pe )</td>
<td>Peclet</td>
<td>( Re \cdot Pr )</td>
<td>Fluid flow and heat transfer</td>
<td></td>
</tr>
<tr>
<td>( Re )</td>
<td>Reynolds</td>
<td>( \frac{D_{H} D}{u} )</td>
<td>Fluid flow and mass transfer</td>
<td></td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl</td>
<td>( \frac{C_{Pr} \mu}{k} )</td>
<td>Heat transfer in flowing fluid</td>
<td></td>
</tr>
<tr>
<td>( Sc )</td>
<td>Schmidt</td>
<td>( \frac{\mu}{\rho \theta} )</td>
<td>Fluid flow involving viscous and inertial forces</td>
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</tr>
<tr>
<td>( Sh )</td>
<td>Sherwood</td>
<td>( \frac{h}{D} )</td>
<td>Mass transfer in fluid</td>
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</tr>
<tr>
<td>( St )</td>
<td>Stanton</td>
<td>( Re \cdot Pr \cdot Pr^{-1} )</td>
<td>Heat transfer in flowing fluid</td>
<td></td>
</tr>
<tr>
<td>( We )</td>
<td>Weber</td>
<td>( \frac{u \rho L \theta^2}{\sigma} )</td>
<td>Fluid flow with interfacial forces</td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>Friction factor</td>
<td>( \frac{1}{D} )</td>
<td>Fluid drag at surface</td>
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</tr>
<tr>
<td>( Np )</td>
<td>Power number</td>
<td>( \frac{p}{\rho N^2 \delta^4} )</td>
<td>Power consumption for mixers</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Home work Problems in Vol.1 + Tut. Sheet