Chapter one:

MAGNETOSTATIC FIELDS

1. The basic laws (Biot-Savart's and Ampere's) that govern magnetostatic fields are discussed. Biot-Savart's law, which is similar to Coulomb's law, states that the magnetic field intensity $dH$ at $r$ due to current element $IdI$ at $r'$ is

$$dH = \frac{I \, dl \times R}{4\pi R^3} \quad \text{(in A/m)}$$

where $R = r - r'$ and $R = |R|$. For surface or volume current distribution, we replace $IdI$ with $K \, dS$ or $J \, dv$ respectively; that is,

$$i \, dl = K \, dS = J \, dv$$

2. Ampere's circuit law, which is similar to Gauss's law, states that the circulation of $H$ around a closed path is equal to the current enclosed by the path; that is,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = \int \mathbf{J} \cdot d\mathbf{S}$$

or

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \text{(third Maxwell's equation to be derived).}$$

When current distribution is symmetric so that an Amperian path (on which $H = H_\phi$ is constant) can be found, Ampere's law is useful in determining $H$; that is,

$$H_\phi \oint dl = I_{enc} \quad \text{or} \quad H_\phi = \frac{I_{enc}}{l}$$

3. The magnetic flux through a surface $S$ is given by

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S} \quad \text{(in Wb)}$$

where $\mathbf{B}$ is the magnetic flux density in Wb/m². In free space,

$$\mathbf{B} = \mu_0 \mathbf{H}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m = permeability of free space.
4. Since an isolated or free magnetic monopole does not exist, the net magnetic flux through a closed surface is zero:

\[ \mathbf{\Psi} = \oint \mathbf{B} \cdot d\mathbf{S} = 0 \]

or

\[ \nabla \cdot \mathbf{B} = 0 \quad \text{(fourth Maxwell’s equation to be derived).} \]

5. At this point, all four Maxwell’s equations for static EM fields have been derived, namely:

\[ \nabla \cdot \mathbf{D} = \rho_v \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = 0 \]
\[ \nabla \times \mathbf{H} = \mathbf{J} \]

6. The magnetic scalar potential \( V_m \) is defined as

\[ \mathbf{H} = -\nabla V_m \quad \text{if } \mathbf{J} = 0 \]

and the magnetic vector potential \( \mathbf{A} \) as

\[ \mathbf{B} = \nabla \times \mathbf{A} \]

where \( \nabla \cdot \mathbf{A} = 0 \). With the definition of \( \mathbf{A} \), the magnetic flux through a surface \( S \) can be found from

\[ \mathbf{\Psi} = \oint_L \mathbf{A} \cdot d\mathbf{l} \]

where \( L \) is the closed path defining surface \( S \). Rather than using Biot-Savart’s law, the magnetic field due to a current distribution may be found using \( \mathbf{A} \), a powerful approach that is particularly useful in antenna theory. For a current element \( IdI \) at \( r' \), the magnetic vector potential at \( r \)

\[ \mathbf{A} = \int \frac{\mu_0 I}{4\pi R} d\mathbf{l}, \quad R = |r - r'| \]

**Chapter two:**

**MAGNETIC FORCES, MATERIALS, AND DEVICES**

1. The Lorentz force equation

\[ \mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) = m \frac{d\mathbf{u}}{dt} \]
relates the force acting on a particle with charge $Q$ in the presence of EM fields. It expresses the fundamental law relating EM to mechanics.

2. Based on the Lorentz force law, the force experienced by a current element $Idl$ in a magnetic field $B$ is

$$dF = Idl \times B$$

From this, the magnetic field $B$ is defined as the force per unit current element.

3. The torque on a current loop with magnetic moment $m$ in a uniform magnetic field $B$ is

$$T = m \times B = lS\mathbf{a}_{n} \times B$$

4. A magnetic dipole is a bar magnet or a small filamental current loop; it is so called due to the fact that its $B$ field lines are similar to the $E$ field lines of an electric dipole.

5. When a material is subjected to a magnetic field, it becomes magnetized. The magnetization $M$ is the magnetic dipole moment per unit volume of the material. For linear material,

$$M = \chi_{m}H$$

where $\chi_{m}$ is the magnetic susceptibility of the material.

6. In terms of their magnetic properties, materials are either linear (diamagnetic or paramagnetic) or nonlinear (ferromagnetic). For linear materials,

$$B = \mu H = \mu_{\omega}B_{0} = \mu_{\omega}(1 + \chi_{m})H = \mu_{\omega}(H + M)$$

where $\mu = $ permeability and $\mu_{\omega} = \mu/\mu_{0} =$ relative permeability of the material. For nonlinear material, $B = \mu(H)H$, that is, $\mu$ does not have a fixed value; the relationship between $B$ and $H$ is usually represented by a magnetization curve.

7. The boundary conditions that $H$ or $B$ must satisfy at the interface between two different media are

$$B_{1n} = B_{2n}$$

$$(H_{1} - H_{2}) \times a_{n12} = K \quad \text{or} \quad H_{1} = H_{2}, \quad \text{if} \ K = 0$$

where $a_{n12}$ is a unit vector directed from medium 1 to medium 2.

8. Energy in a magnetostatic field is given by

$$W_{m} = \frac{1}{2} \int B \cdot H \, dv$$

For an inductor carrying current $I$

$$W_{m} = \frac{1}{2} LI^{2}$$

Thus the inductance $L$ can be found using

$$L = \frac{\int B \cdot H \, dv}{I^{2}}$$
9. The inductance $L$ of an inductor can also be determined from its basic definition: the ratio of the magnetic flux linkage to the current through the inductor, that is,

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

Thus by assuming current $I$, we determine $B$ and $\psi = \int B \cdot dS$, and finally find $L = N\psi/I$.

10. The magnetic pressure (or force per unit surface area) on a piece of magnetic material is

$$P = \frac{F}{S} = \frac{1}{2} BH = \frac{B^2}{2\mu_0}$$

where $B$ is the magnetic field at the surface of the material.

**Chapter three:**

**MAXWELL'S EQUATIONS**

1. In this chapter, we have introduced two fundamental concepts: electromotive force (emf), based on Faraday's experiments, and displacement current, which resulted from Maxwell's hypothesis. These concepts call for modifications in Maxwell's curl equations obtained for static EM fields to accommodate the time dependence of the fields.

2. Faraday's law states that the induced emf is given by $(N = 1)$

$$V_{\text{emf}} = -\frac{\partial \Psi}{\partial t}$$

For transformer emf, $V_{\text{emf}} = -\int \frac{\partial B}{\partial t} \cdot dS$

and for motional emf, $V_{\text{emf}} = (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$.

3. The displacement current

$$I_d = \int \mathbf{J}_d \cdot d\mathbf{S}$$

where $J_d = \frac{\partial \mathbf{D}}{\partial t}$ (displacement current density), is a modification to Ampere's circuit law. This modification attributed to Maxwell predicted electromagnetic waves several years before it was verified experimentally by Hertz.

4. In differential form, Maxwell's equations for dynamic fields are:
Each differential equation has its integral counterpart that can be derived from the differential form using Stokes's or divergence theorem. Any EM field must satisfy the four Maxwell's equations simultaneously.

\[
\nabla \cdot \mathbf{D} = \rho_v \\
\nabla \cdot \mathbf{B} = 0 \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}
\]

Chapter four:

**ELECTROMAGNETIC WAVE PROPAGATION**

1. The wave equation is of the form

\[
\frac{\partial^2 \Phi}{\partial t^2} - u^2 \frac{\partial^2 \Phi}{\partial z^2} = 0
\]

with the solution

\[\Phi = A \sin (\omega t - \beta z)\]

where \(u\) = wave velocity, \(A\) = wave amplitude, \(\omega\) = angular frequency (=2\(\pi\)\(f\)), and \(\beta\) = phase constant. Also, \(\beta = \omega/u = 2\pi/\lambda\) or \(u = f\lambda = \lambda/T\), where \(\lambda\) = wavelength and \(T\) = period.

2. In a lossy, charge-free medium, the wave equation based on Maxwell’s equations is of the form

\[\nabla^2 \mathbf{A}_s - \gamma^2 \mathbf{A}_s = 0
\]

where \(\mathbf{A}_s\) is either \(\mathbf{E}_s\) or \(\mathbf{H}_s\), and \(\gamma = \alpha + j\beta\) is the propagation constant. If we assume \(E_s = E_{s0}(z) \mathbf{a}_z\), we obtain EM waves of the form

\[
\mathbf{E}(z, t) = E_{s0}e^{-\alpha z}\cos (\omega t - \beta z) \mathbf{a}_z \\
\mathbf{H}(z, t) = H_{s0}e^{-\alpha z}\cos (\omega t - \beta z - \theta_{\eta}) \mathbf{a}_z
\]
where \( \alpha \) = attenuation constant, \( \beta \) = phase constant, \( \eta = |\eta|/\theta \) = intrinsic impedance of the medium. The reciprocal of \( \alpha \) is the skin depth \( (\delta = 1/\alpha) \). The relationship between \( \beta \), \( \omega \), and \( \lambda \) as stated above remain valid for EM waves.

3. Wave propagation in other types of media can be derived from that for lossy media as special cases. For free space, set \( \sigma = 0 \), \( \varepsilon = \varepsilon_0 \), \( \mu = \mu_0 \); for lossless dielectric media, set \( \sigma = 0 \), \( \varepsilon = \varepsilon_0 \), \( \mu = \mu_0 \); and for good conductors, set \( \sigma = \infty \), \( \varepsilon = \varepsilon_0 \), \( \mu = \mu_0 \) or \( \sigma/\omega \varepsilon \rightarrow 0 \).

4. A medium is classified as lossy dielectric, lossless dielectric or good conductor depending on its loss tangent given by

\[
\tan \theta = \frac{|J_z|}{|J_{x\omega}|} = \frac{\sigma}{\omega \varepsilon} = \frac{\varepsilon''}{\varepsilon'}
\]

where \( \varepsilon = \varepsilon' - j\varepsilon'' \) is the complex permittivity of the medium. For lossless dielectrics \( \tan \theta \ll 1 \), for good conductors \( \tan \theta \gg 1 \), and for lossy dielectrics \( \tan \theta \) is of the order of unity.

5. In a good conductor, the fields tend to concentrate within the initial distance \( \delta \) from the conductor surface. This phenomenon is called skin effect. For a conductor of width \( w \) and length \( \ell \), the effective or ac resistance is

\[
R_{ac} = \frac{\ell}{\sigma w \delta}
\]

where \( \delta \) is the skin depth.

6. The Poynting vector, \( \mathcal{P} \), is the power-flow vector whose direction is the same as the direction of wave propagation and magnitude the same as the amount of power flowing through a unit area normal to its direction.

\[
\mathcal{P} = \mathbf{E} \times \mathbf{H}, \quad \mathcal{P}_{av} = |\mathcal{P}| = 1/2 \Re (\mathbf{E} \times \mathbf{H})
\]

7. If a plane wave is incident normally from medium 1 to medium 2, the reflection coefficient \( \Gamma \) and transmission coefficient \( \tau \) are given by

\[
\Gamma = \frac{E_{2r}}{E_{1i}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{E_{t2}}{E_{1i}} = 1 + \Gamma
\]

The standing wave ratio, \( s \), is defined as

\[
s = \frac{1 + |\Gamma|}{1 - |\Gamma|}
\]

8. For oblique incidence from lossless medium 1 to lossless medium 2, we have the Fresnel coefficients as

\[
\Gamma_i = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i}, \quad \eta_i = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i}
\]
for parallel polarization and

\[
\Gamma_1 = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i}, \quad \tau_1 = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i},
\]

for perpendicular polarization. As in optics,

\[
\theta_r = \theta_i, \\
\frac{\sin \theta}{\sin \theta_i} = \frac{\beta_1}{\beta_2} = \sqrt{\frac{\mu_2 \varepsilon_1}{\mu_1 \varepsilon_2}}
\]

Total transmission or no reflection \((\Gamma = 0)\) occurs when the angle of incidence \(\theta_i\) is equal to the Brewster angle.