Slot Ventilated Room with Heated Obstruction
Dr. Waheed S. Mohammed, **Dr. Wissam A. Mohammed and ***Asst. Lect. AbdulJabbar Muttair Ahmed

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Abstract
A numerical study of two-dimensional turbulent buoyant recirculating flow within mechanically ventilated rooms is reported. The study involves the solution of elliptic partial differential equations for the conservation of mass, momentum, energy, turbulent energy and its dissipation rate in a finite difference form. These equations were solved together with algebraic expressions for the turbulent viscosity and heat diffusivity, using (K-\( \varepsilon \)) turbulence model. A modified version of a two-dimensional elliptic computer code was used to simulate the complex flows inside a slot-ventilated room. The present study demonstrates that the flow behavior depends on several parameters, such as airflow rate, size and temperature of the heated obstruction. Each of these parameters was modeled separately to understand their affects on the airflow characteristic inside ventilated room.

* Mechanical Engineering Department.
** Technical Education Department.
*** Technical Education Department.
### Symbols

- \( a_E \), \( a_W \), \( a_N \), \( a_S \): Coefficient of general finite-difference equation.
- \( A_e, A_w, A_n, A_s \): Area of finite-difference cell boundaries.
- \( B \): The source term in general finite-difference equation.
- \( C_1, C_2, C_D \): Coefficient in turbulence Model.
- \( D_e, D_w, D_n, D_s \): Diffusion terms.
- \( E \): Empirical constant.
- \( F_e, F_w, F_n, F_s \): Convection coefficients.
- \( G_K \): Kinetic energy generation by shear.
- \( G_B \): Kinetic energy generation by buoyancy.
- \( g_I \): Tensor notation for Gravitational Acceleration.
- \( H \): Height of heated obstruction.
- \( H \): Room height.
- \( h_I \): Height of inlet opening.
- \( K \): Turbulence kinetic energy.
- \( L \): Room length.
- \( P \): Static pressure.
- \( Pe \): Finite-difference cell Peclet number.
- \( P_j \): Jaytillaka’s P-parameter.
- \( R_\Phi \): Residual source.
- \( S_\Phi \): General source term.
- \( T \): Temperature.
- \( U_i, U_j \): Tensor notation for Velocity components.
- \( X_i, X_j \): Tensor notation for Principle directions.
- \( X, Y \): Cartesian coordinates.
- \( Y \): Normal distance from wall.
- \( Y^* \): Dimensionless distance from the wall.

### Greek symbols

- \( B \): Thermal expansion Coefficient.
- \( \Phi \): The general dependent Variable.
- \( \varepsilon \): The dissipation rate of Turbulence.
- \( \sigma_K, \sigma_\varepsilon \): Turbulence Prandtl number for Diffusion of \((K)\) and \((\varepsilon)\).
- \( \sigma_s, \sigma_t \): Laminar and turbulent Prandtl number.
- \( \nu_t \): Turbulent viscosity.
- \( \nu \): Kinematic viscosity.
- \( \tau_w \): Wall shear stress.
- \( \Gamma \): Diffusion coefficient.
- \( \nabla \): Gradient (vector operator).
- \( \delta_x, \delta_y \): Inter-nodes distance.
- \( K \): Von Karman constant.
- \( \Lambda \): Convergence criteria.
- \( M \): Dynamic viscosity.
- \( P \): Density.

### Subscripts

- \( \text{Eff} \): Effective values.
- \( \text{In} \): Inlet value.
- \( L \): Laminar flow condition.
- \( \text{Ref} \): Reference value.
- \( t \): Turbulent flow condition.
- \( \tau \): Friction velocity

### Superscripts

- \( \ast \): Previous iteration value.
- \( \cdot \): Correction to starred Values
- \( + \): Dimensionless value
1. Introduction
The air conditioning of a room serves several purposes to remove or supply heat in order to maintain a comfortable temperature level and to supply the room with a given amount of fresh air. Therefore, the practice of creating a controlled climate in indoor space (e.g. in home, schools, hospitals, hotels, commercial offices and even factories) is no longer luxury, but an essential part of the modern living and working. The four important factors for human comfort are temperature, humidity, purity and motion of the air [1]. In recent years a number of new ventilation systems and strategies have been introduced to improve air quality, thermal comfort and energy efficiency. This has changed the nature of current airflow problem [2]. Ventilation is defined as “controlled supply and exhaust of air to improve the indoor air quality”. There are four different ventilation principles used to control the air distribution within ventilation rooms (unidirectional flow ventilation, mixing ventilation, displacement ventilation, and local exhaust ventilation)[3]. Recent studies have emphasized the importance of inter zone air movement in building and demonstrated the need for better understanding of this movement in any attempt to predict the air distribution system efficiency or thermal performance of the building. The environmental conditions such as airflow velocity and temperature distribution within a conditioner space may be obtained by flow visualization and measurements respectively. These kinds of investigations are usually elaborate, expensive to undertake, and sometimes inaccurate. Recent research turned toward an alternative approach based on the numerical solution of the governing flow equations for mass, momentum, and scalar properties. This is because recent advances in numerical calculation procedures for fluid flow and heat transfer have brought computer usage within reasonable cost limits for practical design problems. Hjertager and Magnussen [4] used the Patankar and Spalding algorithm [5] to solve the finite difference equations for velocity components and continuity. They also used the (K-ε) turbulence model. This numerical study is performed to predict unsteady, turbulent flow in a ventilated box-shaped empty room. Nielsen et al. [6] carried out a study to assess the accuracy of the numerical procedure in predicating two-dimensional isothermal Awbi [7] applied a numerical solution to predict the air movement in square plan room ventilated by a continuous slot diffuser across the width of the ceiling and at a distance 1.2 m from a wall discharging towards the far wall. Two-dimensional non-isothermal solutions were used and this produced realistic predictions of the vertical velocity and temperature profiles when compared with measurements in the room. Davidson [8] investigated displacement ventilation room using finite difference numerical procedure together with a hybrid scheme. Comparison was made using experiment conducted in a water box model. The Boussinsque approximation was used for the buoyancy effect in the vertical component of the momentum. He concluded that the flow was well predicated in comparison with experiment. The present work was aimed to study the behavior of buoyant flow in ventilated room in the presence of a heated obstruction. The effect of flow rate, size and temperature of heated obstruction on such behavior will be included. Such flows are turbulent, recirculating and elliptic in nature. The study will be carried by numerical simulation of the governing
equations (i.e. mass, momentum, energy, turbulence kinetic energy and its dissipation rate) using a finite difference technique with a hybrid differencing scheme to solve these equations. The \((K-\varepsilon)\) model of turbulence was employed in these simulations. Momentum and temperature field will be numerically computed for such flows using a procedure similar to the SIMPLE algorithm of Patanker and Spalding. The computational procedure will then be modified to simulate the present problem. These modifications may be summarized by the geometrical shape, size of a heat obstruction, buoyancy effect and the boundary conditions. The results of the flow field, temperature distribution will then be presented accordance with human comfort inside a ventilated room.

3. Mathematical Formulation

The present problem governs by the principles of conservation of mass, momentum and energy. These principle can be expressed in terms of (PDE'S) for incompressible, two dimensional, turbulent flow as follow [9].

i. Continuity Equation
\[
\frac{\partial U_i}{\partial X_i} = 0
\]

ii. Momentum Equation
\[
\frac{\partial}{\partial X_i} \left( U_j U_i \right) = -\frac{\partial}{\partial X_j} \left( \rho \right) + g_i
\]

\[
\frac{\partial}{\partial X_i} \left( v_{\text{eff}} \left( \frac{\partial U_j}{\partial X_j} + \frac{\partial U_i}{\partial X_i} \right) \right)
\]

iii. Energy Equation
\[
\frac{\partial}{\partial X_i} (U_i T) = -\frac{\partial}{\partial X_i} \left( \Gamma_{\text{eff}} \frac{\partial T}{\partial X_i} \right)
\]

Here \(v_{\text{eff}}\) is the effective viscosity
\[v_{\text{eff}} = v + v_t\]
The main difficulty in solving these equation is the determination of the turbulent viscosity \((v_t)\), therefore it can be expressed by “turbulence model”

3.1 Turbulence model
In present study the \((K-\varepsilon)\) turbulence model has been used. The (PDE) for the turbulent kinetic energy \((K)\) and its dissipation rate \((\varepsilon)\) are as follow [9]:

\[
\frac{\partial}{\partial X_i} \left(U_i K - \left( \frac{v + \frac{\varepsilon}{\sigma_k}}{\sigma_k} \right) \frac{\partial K}{\partial X_i} \right) = G_K + G_B - \varepsilon
\]

\[
\frac{\partial}{\partial X_i} \left(U_i \varepsilon - \left( \frac{v + \frac{\varepsilon}{\sigma_\varepsilon}}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial X_i} \right) = \frac{\varepsilon}{K} \left[ C_1(G_K + G_B) - C_2 \varepsilon \right]
\]

where \(\sigma_k, \sigma_\varepsilon\) are turbulent Prandtl number for \((K)\) and \((\varepsilon)\).

\[G_K = v_t \left( \frac{\partial U_i}{\partial X_i} + \frac{\partial U_j}{\partial X_j} \right) \frac{\partial U_i}{\partial X_i} = \text{Kinetic energy generation by shear} \]

\[G_B = -\frac{v_t g \beta}{\sigma_\varepsilon} \frac{\partial T}{\partial Y} = \text{Kinetic energy generation by buoyancy} \]

The turbulent viscosity \((v_t)\) can now be
\[
c_0 \quad c_1 \quad c_2 \quad c_3
\]

<table>
<thead>
<tr>
<th>(c_0)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
<th>(c_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>1.44</td>
<td>1.92</td>
<td>1.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

calculated from the values of \((K)\) and \((\varepsilon)\).

\[v_t = \frac{C_D K^2}{\varepsilon}\]

where \(C_1, C_2, C_D\) are the turbulence model constant and given by [9]:

3.2 Wall Function
The wall function formulation of Lunder and Spalding [10] which is used to provide predictions of the flow within the viscous layer adjacent to the wall, assumes that the flow in this region behaves locally as a one- dimensional Couette flow. The variation of the resultant velocity \((U_p)\) in the near wall region is assumed to follow a logarithmic law. For a smooth wall this law can be expressed in the form :

\[U_p = \frac{v_{\text{eff}}}{v} X \ln \left( \frac{X}{\delta_x} \right) + A\]
\[ U_{p^+} = \frac{U_{p^+}}{U_t} = \frac{1}{\kappa} \ln \left( \frac{Y^+}{Y} \right) \]

where:

\[ U_{t} = \frac{\tau_w}{\rho C_D \frac{1}{2} K^{1/2}} \]

and

\[ Y^+ = \frac{C_D^{1/4} \rho K^{1/2} Y_p}{\mu} \]

Large temperature gradients are frequently present in the near wall region of turbulent flows. The distribution of temperature of this region is assumed to be analogous to that for velocity \((U_{p^+})\). It may be written in the form:

\[ T_{p^+} = \sigma_\tau (U_{p^+} + P_1) \]

where \((P_1)\) is the viscous sublayer resistance, which is given by \([11]\)

\[ P_1 = 9.0 \left[ \frac{\sigma}{\sigma_t} - 1 \right] \left[ \frac{\sigma}{\sigma_t} \right]^{1/4} \]

where \(\sigma\) and \(\sigma_t\) are the laminar and turbulent Prandtl number respectively.

### 3.3 The General Form of the (PDE’S)

The equations to be solved could be expressed in the general form

\[ \frac{\partial}{\partial x_i} \left( \rho \Phi U_i \right) = \frac{\partial}{\partial x_i} \left( \Gamma \frac{\partial \Phi}{\partial x_i} \right) + S_\Phi \]

where:

\[ \frac{\partial}{\partial x_i} \left( \rho \Phi U_i \right) = \text{Convection term} \]

\[ \frac{\partial}{\partial x_i} \left( \Gamma \frac{\partial \Phi}{\partial x_i} \right) = \text{Diffusion term} \]

\[ S_\Phi = \text{Source term} \]

Table (1) diffusion coefficients and source for each variable.

### 4. Numerical Solution Procedure

The numerical integration technique used in the present study is “Finite Difference Technique” which includes

The adoption of a staggered grid for the velocity components ensures the calculation of the pressure differences acting on the staggered central volumes. So the momentum equations become

\[ a_p U_p = \left( \sum_{E,W,N,S} a U \right) + b + A_u (P_p - P_E) \]

\[ a_p V_p = \left( \sum_{E,W,N,S} a V \right) + b + A_u (P_p - P_N) \]

The resulting velocity components will not satisfy the continuity equation so, the

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \Phi )</th>
<th>( \Gamma_\Phi )</th>
<th>( S_\Phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( x )-direction momentum</td>
<td>( U )</td>
<td>( \nu \tau )</td>
<td>(-P_x + \nabla (\nu \tau \nabla U) + \varepsilon_x )</td>
</tr>
<tr>
<td>( y )-direction momentum</td>
<td>( V )</td>
<td>( \nu \tau )</td>
<td>(-P_y + \nabla (\nu \tau \nabla U) + \varepsilon_y (T - T_{wall}) )</td>
</tr>
<tr>
<td>Temperature</td>
<td>T</td>
<td>( \nu \tau )</td>
<td>( G_{K} - C + G_{H} )</td>
</tr>
<tr>
<td>Turbulence energy</td>
<td>K</td>
<td>( \nu \tau )</td>
<td>( C_{1} \frac{e}{K} (G_{K} + G_{H}) - C_{2} )</td>
</tr>
<tr>
<td>Dispersion Rate</td>
<td>e</td>
<td>( \nu \tau )</td>
<td>( C_{1} \frac{e}{K} (G_{K} + G_{H}) - C_{2} \frac{e^2}{K} )</td>
</tr>
</tbody>
</table>
pressure-correction equation is used which can be written as

\[ a_p P'_p = \left( \sum_{E,W,N,S} a P' \right) + b \]

where

\[ b = (\rho AU'_w) + (\rho AU'_e) - (\rho AV'_w) - (\rho AV'_e) \]

(b) represent the residual in the continuity equation (mass source). When (b = 0) that means the velocity components satisfy the continuity equation.

The (PDE'S) must be solved to obtain (U, V, T, K, ε, P') for all the grid nodes in the field because of non-linearity and strongly interlink. Therefore, SIMPLE, iterative procedure of Patankar and Spalding is used to solve these equations. Line by line technique with block-adjustment procedure and (TDMA) is used for solving (PDE'S).

### 4.1 Convergence of the Numerical Solution

The convergence criterion for the numerical solution applied in the present study is based on the residual \( R_\Phi \) of any variable (\( \Phi \)) at node (P). The numerical solution reaches the convergence when

\[ \sum |R_\Phi| < R_\Phi^\lambda \]

for all field (\( \lambda \) convergence criteria). In the present study \( \lambda = 10^{-3} \).

### 5. Computations

One of the most important aims for ventilating a space with air is to maintain an acceptable air motion and temperature in the working of occupation zone. A slot-ventilated room has been widely used...
to give such conditions. This room with heated obstruction is shown in Figure (1).
The flow generated in such a geometry is a two dimensional turbulent recirculation buoyant one. The effect of ventilation flow rate on the flow characteristic in the occupation zone was studied first, followed by the effect of the size and temperature of the heated obstruction.

5.1 The Geometry Considered and Its Boundary Conditions
The geometrical arrangement that has been tested in the present work has a dimensions of (5.5 m) wide and (2.4 m) high with inlet opening \( w_i = 0.1 \text{ m} \) and outlet opening \( w_o = 0.3 \text{ m} \). The values for velocity component, temperature, turbulent kinetic energy and its dissipation rate at all the solid boundaries are given in table (2). Figure (2) shows the computational grid employed for the present solution of slot-ventilated room with a heated obstruction utilized (28 × 27) grids with an expansion between adjacent nodes on both the x and y directions.

5.2 Result and Discussion
There are many factors that affect the flow characteristic and temperature distribution inside the occupation zone of a slot-ventilated room. The results of the effect of some of these factors are discussed below.

5.2.1 The Effect of Ventilation Rate
The effect of ventilation rate has been studied for several flow rates (ie 5, 10, 16 and 21) air changes per hour (ACH). Figure (3a-b) shows the results of both 5 ACH and 10 ACH. The cold air is directly drops down and leads to high velocity near the heated obstruction. Figure (4a) shown an acceptable flow pattern since the vertical temperature gradient is less than 3 °C as specified by ASHRAE [12]. The results of the other two air changes (16 and 21) ACH are shown in Figure (5a-b). The increasing of ACH to 16 and beyond that causes the cold air to attach to the ceiling and cause a wall-jet flow. Figure (6a-b) shows that the flow pattern is not acceptable in this case since the vertical temperature gradient is greater than 3°C. Therefore as the ventilation rate is increased, the flow pattern and flow characteristic give an unacceptable level of comfort.

5.2.2 Effect of the Size of the Heated Obstruction
The results of the effect of the size of the heated obstruction represented as \( h/H = 0.11, 0.13 \) and \( 0.4 \) with airflow rate equal to 5-ACH are shown in Figure (7). Figure (7a) shows the results of \( h/H = 0.11 \) in which the jet is attached to the ceiling and flow as a wall jet. This physically means that the cold jet will exit directly to the outlet with a little heat exchange inside the comfort zone. Figure (8a) shows the temperature contour of this flow pattern and indicates that a high temperature gradient exists in the comfort zone. Figure (7b) shows the result of \( h/H = 0.13 \). In this figure the jet is dropped down attaching the left wall. The increasing effect of the heated obstruction size causes the flow above the heated obstruction to rise upward making two recirculation zones inside the comfort region. This effect is clearly shown in the temperature contour of Figure (8b). Figures (7c & 8c) show the increasing effect of heated obstruction size of \( h/H = 0.4 \). The heated obstruction in these figures works as divides the flow into two regions. The temperature distribution
within the comfort zone is acceptable in both zones.

5.2.3 Effect of Temperature of the Heated Obstruction

The effect of temperature of the heated obstruction on the jet characteristic and temperature distribution inside the ventilation is presented in Figures (9-10). The values of heated obstruction temperature considered were (17, 25 and 35 °C). This means that a temperature difference of (2, 10 and 20 °C) between the temperature of the heated obstruction and the inlet flow. The heated obstruction size is taken (h/H = 0.11) in these figure which gives ACH of 5. It is clear from Figure (9) that as the temperature of the heated obstruction increases the flow pattern changes from the wall jet to a two-region recirculation zone due to buoyancy effect.

6. Conclusions

A numerical study was conducted to predict the influence of ventilation flow rate on the flow characteristic in the occupation zone and the effects of the size and temperature of a heated obstruction on a slot ventilated room. The hybrid differencing scheme was used in formulating the relevant finite-difference coefficient for all variables. The line by line technique was adopted for this case. The computed flow are presented in graphical form as flow pattern and temperature contour to show the effect of ventilation rate on the air movement and temperature distribution inside the room. The study also include the effect of the size and temperature of the heated obstruction.

The following remarks are deduced.

i. Increasing the airflow rate beyond 16-ACH will produce an uncomfortable condition inside the room.

ii. The presence of heat obstruction affects the flow pattern inside the room. For the same inlet velocity and temperature difference the best size of heated obstruction is h/H (0.4) which gives acceptable air movement and temperature distribution inside the comfort zone.

iii. The most critical effects are those of temperature differences. When the size of the heated obstruction is h/H (0.11) and flow rate equal to (5 ACH), the best temperature difference between the inlet and heated obstruction was found to be 20 °C.

iv. Finally, the present computational procedure can be regarded as a good tool to simulate complex buoyant flow inside the room with different sizes of the heated obstruction.

References


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Fig. 7 Flow pattern of buoyant flow (different size of the heat obstruction h/w) for 3-ACH (a) h/w = 0.11 (b) h/w = 0.13 (c) h/w = 0.4

Fig. 8 Flow pattern of buoyant flow for (a) 16-ACCH (b) 21-ACCH

Fig. 9 Temperature contours of buoyant flow for (a) 16-ACCH (b) 21-ACCH

Fig. 10 Temperature contours of buoyant flow (different size of the heated obstruction h/w) for 3-ACCH (a) h/w = 0.11 (b) h/w = 0.13 (c) h/w = 0.4
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Fig. 8: Flow pattern for different heat obstruction temperatures of 5-ΔT and heated obstruction size h/T = 0.11 (a) 17 °C (b) 25 °C (c) 35 °C

Fig. 10: Temperature contours for different heated obstruction temperatures of 5-ΔT and heated obstruction size h/T = 0.11 (a) 17 °C (b) 25 °C (c) 35 °C

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