Three-Dimensional Analysis of Extrusion in Complex Die by Using Upper Bound Theory

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Abstract

This paper depends on a new analytical method to calculate relative deformation energies for complex die shape by using upper bound theory which is an important method to solve technical problem in the field of metal deformation. This study divides the complex shape to a number of sectors for counting the deformation energies and then to find the total energy requirement for extrusion process in die with arbitrary linear function. This study uses (H) section which represents one of complex shapes to calculate the relative deformation energies for it, taking into account the effects of friction factor, relative die length and reduction of area.

The results of the present study are compared with M.KIUCHI study and found it in a very good agreement.

Keywords: complex die shape, upper bound theory, friction factor, relative die length, reduction area.

الخلاصة

البحث الحالي يعتمد طريقة تحليلية جديدة لحساب طاقات التشكيل النسبية للقوالب المعقدة الشكل باستخدام نظرية الحد الأعلى والتي هي طريقة مهنية في حل المشكلة التقنية في مجال تشكيل المعادن. الدراسة الحالية قسمت الشكل المعقد إلى عدد من المقطع لأجل حساب طاقات التشكيل ومن ثم إيجاد الطاقة الكلية اللازمة لعملية الطبق في القالب ذو أبعاد حجمية خطيية. الدراسة الحالية استخدمت مقطع (H) الذي يمثل أحد الأشكال المعقدة لحساب طاقة التشكيل النسبية له، أخذت في الاعتبار تأثيرات عامل الاحتكاك، طول القالب النسيبي ونسبة النقصان بمساحة القالب.

أجرى البحث الحالي مقارنة تالية مع نتائج بحث M.KIUCHI ووجد أنها في نطاق جيد جداً.

Nomenclature:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_f )</td>
<td>Die final area (mm(^2))</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>Die initial area (mm(^2))</td>
</tr>
<tr>
<td>( AM )</td>
<td>Friction factor</td>
</tr>
<tr>
<td>( L )</td>
<td>Die length (mm)</td>
</tr>
<tr>
<td>( J, n, z )</td>
<td>Polar coordinates system</td>
</tr>
<tr>
<td>( R_o )</td>
<td>Length of radius for regular circular shape (mm)</td>
</tr>
<tr>
<td>( V_o, V_f )</td>
<td>Velocity of the billet at inlet and final (mm/sec)</td>
</tr>
<tr>
<td>( V_y, V_z )</td>
<td>Velocity components in the Cartesian coordinates system</td>
</tr>
<tr>
<td>( V_x )</td>
<td></td>
</tr>
<tr>
<td>( W_{1,2,3,4} )</td>
<td>Power losses due to plastic deformation</td>
</tr>
</tbody>
</table>
Three-Dimensional Analysis of Extrusion in Complex Die by Using Upper Bound Theory

Introduction

The complex shape extrusion is considered very difficult process because the form of the 3-dimensional shape inside the die, beside to the effect of extrusion process parameters such that area reduction, relative die length, and friction factor, therefore; the study of extrusion process and determine the optimal conditions is very important to get a good accuracy in dimensions and surface finishing. Hani [1], the main objective of this study is to find the best angle for the extrusion die by linking the die with springs having different stiffness and the die having different coefficient of friction. The Model of they study consider the visco-plastic case for the material and the coefficient of friction between the die and the metal flow where the coefficient of friction value are (0.005, 0.008, 0.015).

Pathak ET. al. [2] In this study, experimental verification of a proposed extrusion die profile design approach, which aims to satisfy microstructural criteria at maximum production speed and minimum left out material in the die cavity, is presented. The design problem is formulated as a nonlinear programming problem, which is solved using genetic algorithm (GA). Selection of the processing parameters is carried out using Dynamic material modeling (DMM). Venketesan [3] study a new method has been proposed for optimum shape design of extrusion die. The Design problem is formulated as an unconstrained optimization problem. Here nontraditional optimization techniques like Simulated Annealing Algorithm and Particle Swarm Optimization are used to minimize the extrusion force by optimizing the extrusion ratio and die cone angle. Internal power of deformation is also calculated and results are compared. Also, Maity [4], the extrusion through mathematically contoured die plays a critical role in improvement of surface integrity of extruded product. There is gradual deformation which results in the uniform microstructure. In the present investigation non-dimensional extrusion pressure and optimum die length for cosine die profile has been obtained by three dimensional upper bound method using dual stream function method for different reductions.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WE_{1,2,3,4}</td>
<td>Power losses due to velocity discontinuities at inlet</td>
</tr>
<tr>
<td>WF_{1,2,3,4}</td>
<td>Power losses due to velocity discontinuities at outlet</td>
</tr>
<tr>
<td>WS_{1,2,3,4}</td>
<td>Power losses due to surface friction for H-shape die and suggested shape section (1,2,3,4)</td>
</tr>
<tr>
<td>WT_{1,2,3,4}</td>
<td>Total power for H-shape die and suggested shape section (1,2,3,4)</td>
</tr>
<tr>
<td>WT</td>
<td>Total power required to extrusion</td>
</tr>
<tr>
<td>REP</td>
<td>Relative total extrusion power</td>
</tr>
<tr>
<td>XL</td>
<td>Relative die length (L/Ro)</td>
</tr>
<tr>
<td>x,y,z</td>
<td>Cartesian coordinates system</td>
</tr>
<tr>
<td>σ₀</td>
<td>Yield stress (MPa)</td>
</tr>
<tr>
<td>ϕ_{1,2,3,4}</td>
<td>Angle of each sector at internal section (degree)</td>
</tr>
<tr>
<td>ψ_{1,2,3,4}</td>
<td>Angle of each sector at external section (degree)</td>
</tr>
</tbody>
</table>
Chitkara and CeliK[5], study previously developed analytical approach based on the upper bound theory, for the design of three-dimensional off-centric extrusion of arbitrarily shape dies was applied to the extrusion of T-Shape section from initially round billets with the experimental verifications. Pathak et. al. [6] study, die profile of the square tube extrusion process is optimized to produce micro structurally sound product at maximum production speed and minimum left out material in the die. The design problem is formulated as a nonlinear programming model.

This paper deals with extrude complex shape such as H cross section from circular cross section depending on the upper bound theory which considered a new approached in this field according to the literature.

**Theoretical Conception**

This paper depending on upper bound theory for power forming calculations to achieve the direct cold extrusion process by using FORTRAN program as shown in block diagram Fig.(1) such as,

**1- Plastic deformation energy (WI):**

Which obtained from plastic deformation inside the die because the increasing of strain rate. At three axis (x, y, z) as a result for decreasing the cross section between inlet and outlet, beside the effect of product shape changing. The power calculating according to three dimension strain as [7]:

\[
WI = \frac{2}{\sqrt{3}} \sum_{i,j=1}^{n} \sum_{r=1}^{m} (\varepsilon_{ij} - \varepsilon_{i}^{0})^2 d\sigma d\varphi dz
\]

**2- Friction energy (WS):**

Which obtained from the friction between the billet and die surface area. The extrusion process condition such as friction factor, design details (die length which lead to surface area) has affecting on this energy as shown below[7]:-

\[
WS = \frac{AM \cdot \sigma}{\sqrt{3}} \sum_{i,j=1}^{n} \sum_{r=1}^{m} (\varepsilon_{ij} - \varepsilon_{i}^{0})^2 d\sigma d\varphi dz
\]

**3- Discontinuity energy:**

Which obtained from changing the flow direction of billet at inlet and outlet of the die. The design details of die has affecting on this energy through the velocity changing region has arc edge shape lie not Sharpe edge shape, beside the increase this energy, as shown in equation below[7]:-

\[
WE = \frac{\sigma}{\sqrt{3}} \int_{0}^{\frac{\pi}{2}} \int_{0}^{L} (V_{x}^2 + V_{y}^2) d\varphi d\rho
\]

at the outlet cross section (z = L);

\[
WF = \frac{\sigma}{\sqrt{3}} \int_{0}^{\frac{\pi}{2}} \int_{0}^{L} (V_{x}^2 + V_{y}^2) d\varphi d\rho
\]

All of these energies were calculated in case of neglected the effect of hardening strain for plastic zone and the calculation achieved for all die parts which divided to sectors to avoidant complex shape, then the total energy for these sectors are calculated, where the effect of the parameters such as (area reduction, die length, friction factor) are consider [6]. Advanced program prepared for this purpose (energies calculation). Also, the three dimension kinematic velocity admissible field was determined for forming inside the die as shown in Fig. (2).

Fig.(3) illustrated then 3- dimension cross section die billet beside the sequence of shape transformation from circular at input to the shape at output die. According to the following assumption (to
determine the velocity field):

- A constant volume for billet.
- Die surface consist of many straight line (linear die) drawing from the input to output.
- Forming zone bounded by input and output surface. (S_i, S_o).
- Incompressible material [8].
- Divided the complex shape into simple parts to calculate energies for them.
- The shear stress ($\tau$) in billet investigated according to von-misses theory [9].

To calculate energies form required to calculate the strain rate and 3-dimension velocities components, then it must be found the Cartesian-coordinate of the arbitrary liner equation for die surface through identify inlet and outlet coordinate for lines which represent the surface (as shown in Fig.(1)), as below:

$$F (n \sin \phi, n \cos \phi, 0)$$
$$F' (\mu \sin \psi, \mu \cos \psi, L)$$

Then limited the Cartesian coordinate for each sector from complex shape (H) after this the total energy (Wt) were determined with considered the effect of extrusion process parameters according to the following equation:

$$W_{T1} = W_{I1} + W_{S1} + W_{E1} + W_{F1}$$
$$W_{T2} = W_{I2} + W_{S2} + W_{E2} + W_{F2}$$
$$W_{T3} = W_{I3} + W_{S3} + W_{E3} + W_{F3}$$
$$W_{T4} = W_{I4} + W_{S4} + W_{E4} + W_{F4}$$

Therefore the total energy are:

$$W_T = W_{T1} + W_{T2} + W_{T3} + W_{T4}$$

Where the relative extrusion power:

$$\text{REP} = 4 \ast W_T / \sigma_o$$

**Results and discussion**

Fig.(4) show the relationship between relative extrusion power and relative die length in which the relative extrusion power variation from high values then the curve in Fig.(4) decrease until reach minimum values which represented the optimum conditions for design process in which limiting the optimum relative die length value it is very important parameter in die design.

Because of increasing die length, the surface area was increase therefore the friction energy is increase too that lead to increasing in forming energies.

Observed increasing in area reduction rate at constrain the friction factor results increasing plastic forming energies because increasing strains rates which lead to increasing in total forming energy as shown in Fig. (5) so, its observed in this fig the optimum relative die length varying at each area reduction.

Figs. (5) and (6), illustrates the total energy was required to extrusion process increase when the friction is increase because the increasing the friction energy when effect to total energy calculation, and may by controlled effect of friction factor by controlling to the die design parameters, where in experimental field its must contain tapping on internal die surfaces to obtained good lubrication at increasing velocity of electrode carbon in electrical discharge machine (EDM).

The result in this study shows a very good agreement with Kiuchi [10] study for H-section extrusion with consideration of many parameters such as, reduction of area, die length and friction factor.

Figs. (6), and (7) shows the results comparisons between this study and M. Kiuchi study at area reduction rate (R.A=55%, 60%),
friction factor value (AM=1.0). From this comparison the total energy at optimum relative die length less than this in M. Kiuchi study at same relation length. That’s mean the present results more acceptable to the die designer because the consumption of forming energy for extrusion process, also, this study observed that the forming energy has large increasing when die length is increasing compass with M. Kiuchi study. Where this increasing depending on friction energy and relative die length which is considered in this study beside of the friction factor friction effect as more accurate than M. Kiuchi study. Where friction is very important parameter in design process therefore, the result was nearest to the actual case for extrusion process procedure.

In Figs.(8) and (9) the comparison between this study results and M. Kiuchi study were achieved at area reduction rate (RA = 70%, 80%), friction factor (AM = 1.0), the total energy for this study consider to the M. Kiuchi study where the total energy at relative die length less than M. Kiuchi study when same conditions are considered the optimum relative die length is very important parameter in design process (design of extrusion), also, for the extrusion required at this optimum relation die length table (1) shows the comparison between optimums relative die length and relative extrusion energies in this study and M. Kiuchi study.

Conclusions

The main conclusions obtain from the present work can be summarized as follows:

- The study predict the relative function per yield stress unit which can be applied for different extruded metal and predict the optimum die design which required for extrusion process with taking in the count the effects of different condition (reduction of area, friction factor and relative die length).
- The relative extrusion power increase with approximate rate (10.7 %) as a result of increasing the friction factor from (0.2-0.4) at reduction of area (50 %).
- The optimum relative die length decrease with approximate rate (23.6 %) due to increase the friction factor from (0.2-0.4) at reduction area (55%).
- The relative extrusion power increase with approximate rate (22.9 %) and the optimum relative die length increase with rate (20.9 %) as a results of increasing the reduction of area from (50 %-65 %).

References

Three-Dimensional Analysis of Extrusion in Complex Die by Using Upper Bound Theory

Square Section From Square Billet 
", International Conference on Extrusion and Benchmark, Dortmud, Germany, 16-17,September, 2009.


Table (1) shows the comparison between optimum relative die length and relative extrusion power of this study and M. Kiuchi study.

<table>
<thead>
<tr>
<th>R.A=55%</th>
<th>RELATIVE POWER</th>
<th>OPTIMUM RELATIVE DIE LENGTH</th>
<th>RELATIVE POWER</th>
<th>OPTIMUM RELATIVE DIE LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM=1.0</td>
<td>4.4</td>
<td>0.65</td>
<td>4.2638</td>
<td>0.9044</td>
</tr>
<tr>
<td>R.A=60%</td>
<td>4.8</td>
<td>0.7</td>
<td>4.6347</td>
<td>1.0098</td>
</tr>
<tr>
<td>AM=1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R.A=70%</td>
<td>5.1</td>
<td>0.8</td>
<td>5.4238</td>
<td>1.1782</td>
</tr>
<tr>
<td>AM=1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R.A=80%</td>
<td>6.1</td>
<td>0.95</td>
<td>6.3111</td>
<td>1.3023</td>
</tr>
<tr>
<td>AM=1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Start

Input: relative length \(L/R\), friction factor \(AM\) and reduction of area \(R.A.\%\)

Limited the function of strain rate \(\varepsilon_{ij}\) and velocity components \(V_x, V_y, V_z\)

Calculate the plastic deformation energy \((WI)\) of each sector

Calculate the friction energy \((WS)\) of each sector

Calculate the discontinuity energy \((WF, WE)\) of each sector

Calculate the total energy \((WT)\) of the extrusion die

Calculate the relative total energy of extrusion die \((WT/\sigma_0)\)

End

Triple integral by depending on numerical analysis method Gauss Legendre

Double integral by depending on numerical analysis method Gauss Legendre

Figure (1) Show the block diagram of Fortran program.
Three-Dimensional Analysis of Extrusion in Complex Die by Using Upper Bound Theory

Figure (2) Three dimension Kinematic velocity admissible filed

Figure (3) Three dimension cross section die billet
Figure (4) The relationship between the relative die length and the relative extrusion power at friction factor (AM=0.2)

Figure (5) The relationship between the relative die length and the relative extrusion power at friction factor (AM=0.4)
Figure (6) Show the result comparisons between this study and M. Kiuchi study at area reduction rate (R.A=55%) and friction factor (AM=0.1)

Figure (7) Show the result comparisons between this study and M. Kiuchi study at area reduction rate (R.A=60%) and friction factor (AM=0.1)
Three-Dimensional Analysis of Extrusion in Complex Die by Using Upper Bound Theory

Figure (8) Show the result comparisons between this study and M. Kiuchi study at area reduction rate (R.A=70%) and friction factor (AM=0.1)

Figure (9) Show the result comparisons between this study and M. Kiuchi study at area reduction rate (R.A=80%) and friction factor (AM=0.1)