Design of a Nonlinear Fractional Order PID Neural Controller for Mobile Robot based on Particle Swarm Optimization

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Received on:9/3/2016 & Accepted on:19/7/2016

ABSTRACT:
The goal of this paper is to design a proposed non-linear fractional order proportional-integral-derivative neural (NFOPIDN) controller by modifying and improving the performance of fractional order PID (FOPID) controller through employing the theory of neural network with optimization techniques for the differential wheeled mobile robot multi-input multi-output (MIMO) system in order to follow a desired trajectory. The simplicity and the ability of fast tuning are important features of the particle swarm optimization algorithm (PSO) attracted us to use it to find and tune the proposed non-linear fractional order proportional-integral-derivative neural controller’s parameters and then find the best velocity control signals for the wheeled mobile robot. The simulation results show that the proposed controller can give excellent performance in terms of compared with other works (minimized mean square error equal to 0.131 for Eight-shaped trajectory and equal to 0.619 for Lissajous- curve trajectory as well as minimum number of memory units needed for the structure of the proposed NFOPIDN controller (M=2 for Eight-shaped trajectory and M=4 for Lissajous- curve trajectory) with smoothness of linear velocity signals obtained between (0 to 0.5) m/sec.

Keywords: Non-linear Fractional Order proportional-integral-derivative Neural Controller; Particle Swarm Optimization Algorithm; Trajectory Tracking; Mobile Robot.

INTRODUCTION

Wheeled mobile robots (WMR) systems have drawn a lot of interest for the researchers recently because they are found in many applications in industry, transportations, military, security settings and other fields due to their ability of handling complex visual and information processing for artificial intelligence, loading capability and they can work in dangerous and hazardous environment. Wheeled mobile robot suffer from non-holonomic constraints (pure rolling without side slipping motion) meaning that mobile robots can move only in the direction normal to the axis of the driving wheels [1]. Trajectory tracking control is important topic of the (WMR) it means to apply control signals to the (WMR) in such a way that the (WMR) follow a curve that connects its actual position and orientation with the goal position and orientation of the predefined trajectory (desired or reference trajectory). In general, Trajectory tracking control is still active region of research because as we mentioned above that (WMR) have found in various industrial applications [2].

The Motivation for this work is the problems in the mapping and localization; cognition trajectory planning; path-tracking and motion control; therefore, different trajectory tracking control approaches for (WMR) have been proposed so as to achieve the best performance for the wheeled
mobile robot including high speed, high tracking accuracy (minimized tracking error), low energy consumption and smoothness of velocity control signal obtained, such as fuzzy logic trajectory tracking controller [3 and 4]; sliding mode controller [5 and 6]; back-stepping technique [7 and 8]; nonlinear PID neural trajectory tracking controller [9, 10 and 11] and cognitive trajectory tracking neural controller [12 and 13].

The fundamental essence of the contribution novelty for this paper can be described by the points listed below:

- Modifying and improving the performance of fractional order PID (FOPID) controller by employing the theory of neural network techniques.
- The analytically derived control law based non-linear fractional order PID neural (NFOPIDN) controller has considerably numerical accuracy in terms of obtaining best actual signal and leading to minimizing tracking pose error of the wheeled mobile robot with the minimum number of memory units (M) needed based PSO algorithm.
- Validation of the controller adaptation performance through change the initial pose state.
- Verification of the controller capability of tracking different types of continuous gradient trajectories.

The remainder of the paper is organized as follows: Section two is a description of the kinematics model of the non-holonomic wheeled mobile robot. In section three, the proposed non-linear fractional order (PID) neural (NFOPIDN) controller is derived. The particle swarm optimization algorithm is explained in section four. In section five, the simulation results and discussion of the proposed controller are described finally the conclusions are presented in section six.

**Kinematic Mobile Robot Model**

Figure (1) shows the non-holonomic wheeled mobile robot platform which it is a vehicle with two wheels mounted on the same axis and two omni-directional castor wheels mounted in the front and rear of the vehicle. The two castor wheels carry the mechanical structure and keep the platform more stable. The wheeled mobile robot is driven by two independent DC analogue motors that are used as actuators of the left and right wheels for motion and orientation. These two wheels have the same radius denoted by \( r \) which is separated by the distance \( L \). The center of gravity of the (WMR) is located at point \( c \), center of axis connected the two driven wheels[14]. The location of the mobile robot in the global coordinate frame \([O, X, Y]\) can be represented by the vector as follow:

\[
q = (x_c, y_c, \theta)^T
\]  

...(1)
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![Diagram of Non-holonomic mobile robot model](image)

**Figure (1): Non-holonomic mobile robot model.**

$x_c$ and $y_c$ are the coordinates of point $c$ in the global coordinate frame, and $\theta$ is the robotic orientation angle measured from X-axis then these three generalized coordinates are described the mobile robot’s configuration. The wheeled mobile robot has the non-holonomic constraint that the driving wheels are purely roll without slipping [15]. This non-holonomic constraint can be written as in equation (2):

$$-\dot{x}(t)\sin \theta(t) + \dot{y} \cos \theta(t) = 0$$  

... (2)

The equations of the (WMR) in the global frame can be described in kinematics equations (3, 4 and 5) [16 and 17]:

$$\dot{x}(t) = V_l(t) \cos \theta(t)$$  

... (3)

$$\dot{y}(t) = V_l(t) \sin \theta(t)$$  

... (4)

$$\dot{\theta}(t) = V_w(t)$$  

... (5)

Where: $V_l$ is the linear velocity. $V_w$ is the angular velocity.

In the computer simulation, the form of the pose equations can be represented in equations (6, 7 and 8):

$$\theta(k)= \frac{1}{L^2} [V_l(k) - V_R(k)] \Delta t + \theta(k-1)$$  

... (6)

$$x(k)=0.5[V_R(k) + V_L(k)] \cos \theta(k) \Delta t + x(k-1)$$  

... (7)

$$y(k)=0.5[V_R(k) + V_L(k)] \sin \theta(k) \Delta t + y(k-1)$$  

... (8)

Where:

$x(k)$, $y(k)$ and $\theta(k)$ are the pose at the $k^{th}$ step.

**Non-Linear Fractional Order Pid Neural (Nfopidn) Controller Design**

A proportional-integral-derivative controller (PID controller) is widely used in industrial control systems because it is simple, reliable, strong robustness in broad operating condition and its parameters can be adjusted easily and separately. An enhanced (PID controller) has been designed in this section (non-linear fractional order PID neural controller) and the method to control the (MIMO) differential wheeled mobile robot depends on the available information of the unknown nonlinear system and the control objectives.

The particle swarm optimization (PSO) algorithm has been utilized to generate the optimal parameters for the (NFOPIDN) controller so as to get the best velocity control actions which try to...
minimize the tracking pose error of differential WMR. The block diagram of the proposed controller is shown in the figure (2).

Figure (2): Non-linear fractional order proportional-integral-derivative neural controller structure for mobile robot model.

The feedback (NFOPIDN) controller is essential because it is important to stabilize and control the tracking pose error of the mobile robot system when the pose of the wheeled mobile robot is drifted from the desired pose. The error and control action signals are denoted in (FOPID) controller as $e(t)$ and $u(t)$ respectively and they are associated as shown in the equation (9) [18]:

$$u(t) = Kp e(t) + K_i D^{-\sigma} e(t) + K_d D^{\alpha} e(t)$$  \ldots (9)

Where:
- $Kp$ is the proportional constant.
- $Ki$ is the integration constant.
- $Kd$ is the differentiation constant.
- $D^{-\sigma}$ is the fractional integral operator.
- $D^{\alpha}$ is the fractional differential operator.

By taking the Laplace transform of fractional derivative and fractional integral of $e(t)$ are given as equation (10 and 11) respectively [18]:

$$L[D^{\alpha} e(t)] = S^{\alpha} E(s) - [D^{\alpha} e(t)]_{t=0}$$  \ldots (10)

$$L[D^{-\sigma} e(t)] = S^{-\sigma} E(s)$$  \ldots (11)

After taking Laplace transform of equation (9) and submitting equations (10 and 11) in equation (9), the transfer function of (FOPID) controller will be as equation (12):

$$G_c(s) = Kp + Ki S^{-\sigma} + Kd S^{\alpha}$$  \ldots (12)
The first step in the design of the proposed (NFOPIDN) controller is converted the continuous-time (FOPID) control equation to discrete-time (FOPID) control equation by using the generating function technique with three steps:

- Pre-warped Tustin transform is used as equation (13) that converted equation (12) from s-domain to z-domain.

\[ S^\alpha = \left( \frac{w_c}{\tan \left( \frac{w_c T}{2} \right)} \right) \times \left( \frac{1-Z^{-1}}{1+Z^{-1}} \right)^\alpha = \Omega^\alpha \times \left( \frac{1-Z^{-1}}{1+Z^{-1}} \right)^\alpha \text{ } \ldots (13) \]

Where:
- \( T \): sampling period of the system (0.2 sec).
- \( w_c \): is gain crossover frequency of the open – loop transfer function and it is taken from [19] and equal to (15 rad/sec).
- \( \Omega : \frac{w_c}{\tan \left( \frac{w_c T}{2} \right)} = 1.0637 \text{ } \ldots (14) \)
- \( \alpha \): is order of derivative

- Power series expansion (PSE) is used as the resulted expression which is acquired in the term of \( z \) with a limited (minimum) memory must necessarily use for any practical discrete-time controller.
  - The (PSE) of the expression in the right hand side of equation (13) must be calculated in order to check the stability of the controller as shown in figure (3) as follows:
    1- If the poles and zeros are inside the unit circle of the \( z \) plane the system is stable.
    2- If the poles and zeros are outside the unit circle of the \( z \) plane the system is unstable.
    3- If the poles and zeros are on the border of the unit circle of the \( z \) plane the system is critically stable.

![Figure (3): Z-plane pole and zero locations.](image)

After applying the generating function technique with the three steps, the proposed control law can be driven as follows:

- The derivative term \( S^\alpha \) of equation (12) can be expanded as follows:

Assuming \( w = z^{-1} \) in equation (12) yield:

\[ \Omega^\alpha \times \left( \frac{1-w}{1+w} \right)^\alpha = \Omega^\alpha \sum_{j=0}^{\infty} f_j(\alpha)w^j \text{ } \ldots (15) \]

where:

\[ f_j(\alpha) = \frac{1}{j!} \times \left. \frac{d^j}{dw^j} \right|_{w=0} \left( \frac{1-w}{1+w} \right)^\alpha \text{ } \ldots (16) \]

By substitution of \( w = z^{-1} \) in equation (15) yield:

\[ S^\alpha = \Omega^\alpha \sum_{j=0}^{\infty} f_j(\alpha)z^{-j} \text{ } \ldots (17) \]

Where the coefficients \( f_j(\alpha) \) are calculated from equation (16).

It can be shown (using Maple) that the first few coefficients in (16) are as follow:

\[ f_0(\alpha) = 1, \quad f_1(\alpha) = -2 \times \alpha, \quad f_2(\alpha) = 2 \times \alpha^2, \quad f_3(\alpha) = -\frac{4}{3} \times \alpha^3 - \frac{2}{3} \times \alpha \]
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The integral term \( S^{-\sigma} \) of equation (12) can be expanded as follows:

\[
S^{-\sigma} = \left( \frac{1}{\sigma} \right) \times S^{1-\sigma} 
\]

… (18)

\[
S^{-\sigma} = S^{-1} \times S^{1-\sigma} 
\]

… (19)

Where:

\( \sigma \) : is the order of integral.

By applying a Tustin method in equation (19), then:

\[
S^{-\sigma} = \Omega^{-1} \times \left( \frac{1+z^{-1}}{1-z^{-1}} \right) \times \Omega^{1-\sigma} \times \left( \frac{1-z^{-1}}{1+z^{-1}} \right)^{1-\sigma} 
\]

… (20)

\[
S^{-\sigma} = \Omega^{-\sigma} \left( \frac{1+z^{-1}}{1-z^{-1}} \right) \sum_{j=0}^{\infty} f_j(1-\sigma)z^{-j} 
\]

… (21)

Where the term \( \left( \frac{1-z^{-1}}{1+z^{-1}} \right)^{1-\sigma} \) can be calculated in the same manner as \( \left( \frac{1-z^{-1}}{1+z^{-1}} \right)^{\alpha} \) in equations (13) and (15).

The term \( f_j(1-\sigma) \) is the again which can be calculated as in equation (16).

Then, substitution of equations (17) and (21) in equation (12) yield the general control law becomes:

\[
C_d(z) = kp + kd \sum_{j=0}^{\infty} f_j(\alpha)z^{-j} + ki \left( \frac{1+z^{-1}}{1-z^{-1}} \right) \sum_{j=0}^{\infty} f_j(1-\sigma)z^{-j} 
\]

… (22)

Where:

\[
\begin{align*}
kp &= Kp \\
kd &= Kd \times \Omega^\alpha \\
ki &= Ki \times \Omega^{-\sigma}
\end{align*}
\]

… (23)

The infinite numbers of memory units are needed for its realization and consequently the computational cost is increased by increasing the time. In other words, in practice the upper bound of \( \infty \) as in (22) cannot be considered equal to infinity. By restricting the number of memory units to \( M \) the equation (22) will be:

\[
C_d(z) = kp + kd \sum_{j=0}^{M} f_j(\alpha)z^{-j} + ki \left( \frac{1+z^{-1}}{1-z^{-1}} \right) \sum_{j=0}^{M} f_j(1-\sigma)z^{-j} 
\]

… (24)

By multiplying each side of equation (24) by \( (1-z^{-1}) \), then the equation will be:

\[
(1-z^{-1})C_d(z) = (1-z^{-1})kp + (1-z^{-1})kd \sum_{j=0}^{M} f_j(\alpha)z^{-j} + ki(1+z^{-1}) \sum_{j=0}^{M} f_j(1-\sigma)z^{-j} 
\]

… (25)

\[
(1-z^{-1})U(z) = (1-z^{-1})kpE(z) + (1-z^{-1})kdE(z) \sum_{j=0}^{M} f_j(\alpha)z^{-j} + ki(1+z^{-1})E(z) \sum_{j=0}^{M} f_j(1-\sigma)z^{-j} 
\]

… (26)

Finally, the discrete-time equation of fractional order (PID) controller will be:

\[
U(k) = u(k-1) + kp(e(k) - e(k-1)) + kd \sum_{j=0}^{M} f_j(\alpha)(e(k) - e(k-j - 1)) + ki \sum_{j=0}^{M} f_j(1-\sigma)(e(k) - e(k-j)) + e(k-j - 1) + e(k-j - 1) 
\]

… (27)

Based on the discrete-time equation of fractional order (PID), the proposed control law for the (NFOPIDN) controller is driven as follows:

\[
V_{R,L}(k) = V_{R,L}(k-1) + kp_\gamma(e_\gamma(k) - e_\gamma(k-1)) + kd_\gamma \sum_{j=0}^{M} f_j(\alpha)(e_\gamma(k) - e_\gamma(k-j - 1)) + ki_\gamma \sum_{j=0}^{M} f_j(1-\sigma)(e_\gamma(k) - e_\gamma(k-j)) + e_\gamma(k-j - 1) 
\]

… (28)

Where \( x, y, \theta \).

The tuning (NFOPIDN) input vector include \( e_x(k), e_y(k-1), e_x(k-j) \) and \( e_\gamma(k-j - 1) \) for control signals. The proposed of the feedback control law can be represented by equations (29 and 30):

\[
V_R(k) = V_R(k-1) + O_x + O_y 
\]

… (29)

\[
V_L(k) = V_L(k-1) + O_\theta + O_y 
\]

… (30)

The outputs of neural networks are \( O_x, O_y \) and \( O_\theta \). The sigmoid function is used as equation (31).
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\[ O_\gamma = \frac{2}{1+e^{-\text{net}_\gamma}} - 1 \]  \hspace{1cm} \text{(31)}

\( \text{net}_\gamma \) is calculated from equation (32):

\[
\text{net}_\gamma = k_p f_\gamma (e_\gamma (k) - e_\gamma (k - 1)) + k_d \sum_{j=0}^{M} f_j (e_\gamma (j) - e_\gamma (j - 1)) + k_i \sum_{j=0}^{M} f_j (1 - \sigma) (e_\gamma (k - j) + e_\gamma (k - j - 1))
\]  \hspace{1cm} \text{(32)}

The control parameters of the (NFOPIDN) controller, \( k_p, k_i, k_d, \alpha \) and \( \sigma \) can be adjusted using particle swarm optimization technique with minimum number of memory (M) needed and figure (4) is the (NFOPIDN) controller for WMR.

Particle Swarm Optimization Algorithm

The update velocity and position equations for each particle can be written as follow [20]:

\[
V_{i}^{ii+1} = w \cdot V_{i}^{ii} + c_1 \cdot r_1 \cdot (p_{best}^{ii} - X_{i}^{ii}) + c_2 \cdot r_2 \cdot (g_{best}^{ii} - X_{i}^{ii})
\]  \hspace{1cm} \text{(33)}

\[
X_{i}^{ii+1} = X_{i}^{ii} + V_{i}^{ii+1}
\]  \hspace{1cm} \text{(34)}

Where:

- pop: the population number.
- \( ii \): the current iteration \( \ldots \ldots \text{Nit.} \)
- \text{Nit}: is max no. of iterations.
- \( w \): the inertia factor.
- \( c_1 \): the cognitive parameter.
- \( c_2 \): the social parameter.
- \( r_1, r_2 \): an independent random numbers with uniform probability between 0 and 1.
- \( V \): the velocity of the particle.
- \( X \): the position of the particle.
- \( p_{best} \): is the best previous weight of \( i^{th} \) particle.
- \( g_{best} \): is the best particle among all the particles in the population.

The steps of PSO for (NFOPIDN) controller can be described by the following flowchart in figure (5).

The non-linear fractional order proportional-integral-derivative neural (NFOPIDN) controller with eleven weights parameters and the matrix is written as an array to form a particle. First, particles are initialized randomly and updated in accordance with the following equations in order to tune the NFOPIDN parameters [10]:
Figure (4): Non-linear fractional order proportional-integral-derivative neural controller structure.
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Start

Initialize each particle with the random position vector \((k_p, k_i, k_d, \alpha, \sigma)\) and zero velocity.

Let \(N_{it}=\) max no. of iterations, \(pop=\) max no. of particles, \(i=\) the current particle and \(it=\) the current iteration.

\[ c_1, c_2 = 1.49618, w=0.7298 \]

Evaluate the (MSE) value for \((i)\) particle position as in equation (45)

For particle \((i)\) set local best cost =current objective function
Local best position = current position

Yes
\(i < pop\)

No

Set global best objective function = min (for all local best objective functions)

\(it=1\)

Update the particles velocity and position (the parameters of each particle) as in equations (35, 36, 37, 38, 39, 40, 41, 42, 43, and 44)

Evaluate the (MSE) value for \((i)\) particle position as in equation (45)

\(i+1\)

For this particle set local best objective function =current objective function

Yes
\(i <= pop\)

No

If current cost < local best objective function

Yes

No

If current cost < global best objective function

Yes

Set global best objective function =current objective

\(ii=1\)

If \(ii < Nit\)

Yes

No

The best solution = global best particle

End

Figure (5): Flowchart of (PSO) for Non-linear fractional order (PID) neural controller structure.
\[ \Delta K_{y,l}^{ii+1} = \Delta K_{y,l}^{ii} + c_1 \cdot r_1 \cdot (p_{best}^{ii} - K_{y,l}^{ii}) + c_2 \cdot r_2 \cdot (g_{best}^{ii} - K_{y,l}^{ii}) \quad \ldots (35) \]
\[ K_{y,l}^{ii+1} = K_{y,l}^{ii} + \Delta K_{y,l}^{ii+1} \quad \ldots (36) \]
\[ \Delta K_{i,l}^{ii+1} = \Delta K_{i,l}^{ii} + c_1 \cdot r_1 \cdot (p_{best}^{ii} - K_{i,l}^{ii}) + c_2 \cdot r_2 \cdot (g_{best}^{ii} - K_{i,l}^{ii}) \quad \ldots (37) \]
\[ K_{i,l}^{ii+1} = K_{i,l}^{ii} + \Delta K_{i,l}^{ii+1} \quad \ldots (38) \]
\[ \Delta K_{d,l}^{ii+1} = \Delta K_{d,l}^{ii} + c_1 \cdot r_1 \cdot (p_{best}^{ii} - K_{d,l}^{ii}) + c_2 \cdot r_2 \cdot (g_{best}^{ii} - K_{d,l}^{ii}) \quad \ldots (39) \]
\[ K_{d,l}^{ii+1} = K_{d,l}^{ii} + \Delta K_{d,l}^{ii+1} \quad \ldots (40) \]
\[ \Delta \sigma_{l}^{ii+1} = \Delta \sigma_{l}^{ii} + c_1 \cdot r_1 \cdot (p_{best}^{ii} - \sigma_{l}^{ii}) + c_2 \cdot r_2 \cdot (g_{best}^{ii} - \sigma_{l}^{ii}) \quad \ldots (41) \]
\[ \sigma_{l}^{ii+1} = \sigma_{l}^{ii} + \Delta \sigma_{l}^{ii+1} \quad \ldots (42) \]
\[ \Delta \sigma_{r}^{ii+1} = \Delta \sigma_{r}^{ii} + c_1 \cdot r_1 \cdot (p_{best}^{ii} - \sigma_{r}^{ii}) + c_2 \cdot r_2 \cdot (g_{best}^{ii} - \sigma_{r}^{ii}) \quad \ldots (43) \]
\[ \sigma_{r}^{ii+1} = \sigma_{r}^{ii} + \Delta \sigma_{r}^{ii+1} \quad \ldots (44) \]

Where:

\[ y = x, y, \theta \]

The number of dimensions is equal to eleven because there are three (NFOPIDN) and each one has three parameters in addition to the two parameters of derivative and integral orders parameters. The proposed mean square error (MSE) function is used as follows:

\[ E = \frac{1}{N} \sum_{k=1}^{N} (e_x(k))^2 + (e_y(k))^2 + (e_\theta(k))^2 + (V_{ref}(k) - V_r(k))^2 + (V_{hopt}(k) - V_l(k))^2 \quad \ldots (45) \]

\[ N: \text{max no. of samples.} \ k: \text{current sample.} \ V_{r,opt}: \text{right linear reference velocity.} \ V_{l,opt}: \text{left linear reference velocity.} \ V_r: \text{linear velocity of the right wheel.} \ V_l: \text{linear velocity of the left wheel.} \]

**Simulation Results and Discussion**

The proposed controller is proved by simulation program using MATLAB package. The simulation is executed off-line by tracking a reference pose with the (Eight-shaped) and (Lissajous-curve) trajectories in the motion control of the WMR. The specifications of the mobile robot type (Eddie model) are picked from [21]: max. linear velocity =1 m/sec, max. angular velocity =2.222 rad/sec and 0.2 sec is the sampling time. The non-linear fractional order PID neural controller scheme in figure (2) is applied to the WMR model and it is used the learning PSO algorithm flowchart for finding and tuning the controller’s parameters. The first stage of operation is to define the parameters of the PSO algorithm: Population of particle is equal to (35) for the (Eight-shaped) trajectory and equal to (25) for the (Lissajous-curve) trajectory and the number of iteration is equal to 100 for both trajectories. Each particle has 11 weights because there are eleven parameters of (NFOPIDN) controller. The acceleration constant \( c_1 \) and \( c_2 \) are equal to (1.49618). The weight factor is equal to (0.7298). \( r_1 \) and \( r_2 \) are random values between 0 and 1.

**CASE STUDY#1:**

The reference (Eight-shaped) path can be described by the following equations (46, 47 and 48):

\[ x_r(t) = \sin\left(\frac{t}{10}\right) \quad \ldots (46) \]
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\[ y_r(t) = \sin \left( \frac{t}{20} \right) \]  \[ \theta_r(t) = 2 \tan^{-1} \left( \frac{\Delta y_r(t)}{\sqrt{\Delta x_r(t)^2 - (\Delta y_r(t))^2 + \Delta x_r(t)}} \right) \]  \[ \theta_r(0) = \frac{\pi}{6} \text{rad} \]  \[ x_0 = [0.25, 0, 1.0472] \]  \[ \text{The figures (6 and 7) exhibited the excellent position and orientation tracking performance as compared with other works such as [10 and 11].} \]

The optimized off-line tuning based on particle swarm optimization algorithm is used for finding and tuning the control gains parameters of the controller which has demonstrated, as shown in table (1).

\[ \text{Figure (6): Reference path and output mobile robot trajectory. Figure (7): Reference orientation and output mobile robot orientation.} \]

\[ \text{Figure (8): The right and left wheel velocity. Figure (9): The linear and angular velocity.} \]
Finally, according to equation (23) the parameters of the (NFOPIDN) controller are described in table (2).

### Table (1): Parameters of the proposed controller using (PSO).

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$K_{p\theta}$</th>
<th>$K_{i\theta}$</th>
<th>$K_{d\theta}$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0800</td>
<td>0.0317</td>
<td>-0.1787</td>
<td>0.9043</td>
<td>0.1396</td>
<td>0.3213</td>
<td>0.1535</td>
<td>0.0491</td>
<td>0.0222</td>
<td>0.2864</td>
<td>0.6936</td>
</tr>
</tbody>
</table>

Table (2): The parameters of the nonlinear (FOPID) neural controller.

<table>
<thead>
<tr>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$k_{p\theta}$</th>
<th>$k_{i\theta}$</th>
<th>$k_{d\theta}$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0800</td>
<td>0.0303</td>
<td>-0.1818</td>
<td>0.9043</td>
<td>0.1337</td>
<td>0.3270</td>
<td>0.1535</td>
<td>0.0470</td>
<td>0.0225</td>
<td>0.2864</td>
<td>0.6936</td>
</tr>
</tbody>
</table>

The performance index for different values of memory $M=1, 2, 3, 4, 5$ and $6$ can be shown in the figure (10). This figure shows that the best number of memory units for the (Eight-shaped) trajectory is equal to 2.

**Figure (10): The performance indexes for different values of M (1, 2, 3, 4, 5 and 6)**

Figures (11-a and b) show that the best number of memory units needed against minimum value of (MSE) and the values of the fractional derivative ($\alpha$) and fractional integration ($\sigma$) orders.

Figure (11): Memory units against minimum (MSE): (a) with the value of fractional derivative ($\alpha$); (b) with the value of fractional integration orders ($\sigma$).
CASE STUDY #2:

Another reference path (Lissajous-curve) that can be described by the equations (49, 50 and 51):

\[ x_r(t) = -\sin\left(\frac{2\pi t}{45}\right) \quad \text{... (49)} \]
\[ y_r(t) = \cos\left(\frac{2\pi t}{15}\right) \quad \text{... (50)} \]
\[ \theta_r(t) = 2 \tan^{-1}\left(\frac{\Delta y_r(t)}{\sqrt{\left(\Delta x_r(t)\right)^2 - \left(\Delta y_r(t)\right)^2 + \Delta x_r(t)}}\right) \quad \text{... (51)} \]

For the simulation objective, the desired trajectory is described in equations (49 and 50) and the desired orientation angle is expressed in equation (51). The differential wheeled mobile robot starts from initial posture \( q(0) = [0.15, 1, 0.6981] \).

The figures (12 and 13) showed the excellent pose tracking performance as compared with other works such as [10 and 11]. The simulation results showed the effectiveness of the proposed controller by showing its ability to generate smoothness control input velocities without spikes as well as without saturation state action.

![Figure 12: Reference path and output mobile robot trajectory.](image1)

![Figure 13: Reference orientation and output mobile robot orientation.](image2)

![Figure 14: The right and left wheel velocity.](image3)

![Figure 15: The linear and angular velocity.](image4)

The signals described in figure (14) shows that the low power is required to drive the DC motors of the mobile robot model. Figure (15) shows that the mean linear velocity of the mobile robot and the angular velocity.

The optimized–off-line-tuning based on particle swarm optimization algorithm is used for finding and tuning the parameters of the (NFOPIDN) controller which has demonstrated as shown in table (3).
Table (3): Parameters of the nonlinear controller using (PSO).

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$K_{p1}$</th>
<th>$K_{i1}$</th>
<th>$K_{d1}$</th>
<th>$K_{p2}$</th>
<th>$K_{i2}$</th>
<th>$K_{d2}$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1596</td>
<td>-0.7845</td>
<td>-0.0335</td>
<td>0.9888</td>
<td>0.7047</td>
<td>2.0595</td>
<td>-0.2983</td>
<td>0.2513</td>
<td>-0.0932</td>
<td>0.0017</td>
<td>0.5590</td>
</tr>
</tbody>
</table>

Finally, according to equation (23) the parameters of the (NFOPIDN) controller are described in table (4).

Table (4): Final parameters of the nonlinear (FOPID) neural controller.

<table>
<thead>
<tr>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$k_d$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1596</td>
<td>-0.7578</td>
<td>-0.0335</td>
<td>0.9888</td>
<td>0.6807</td>
<td>2.0597</td>
<td>-0.2983</td>
<td>0.2427</td>
<td>-0.0932</td>
<td>0.0017</td>
<td>0.5590</td>
</tr>
</tbody>
</table>

The performance index for different values of $m=1, 2, 3, 4, 5$ and $6$ can be shown in the figure (16). This figure shows that the best number of memory units for the (Lissajous- curve) trajectory is equal to 4.

![Figure (16): The performance index for different values of M (1, 2, 3, 4, 5 and 6)](image)

Figures (17-a and b) show that the best number of memory units needed with minimum value of (MSE) and the values of the fractional derivative ($\alpha$) and fractional integration($\sigma$) orders.

![Figure (17): Memory units against minimum (MSE): (a) with the value of fractional derivative ($\alpha$); (b) with the value of fractional integration orders ($\sigma$).](image)
CONCLUSIONS
The Matlab simulation results on the off-line tuning PSO for the nonlinear fractional order PID neural controller are presented in this work for the mobile robot type Eddie model which shows precisely that the proposed control algorithm has the following characteristics:

- Fast and stable off-line tuning control parameters with a minimum number of memory units which is needed (M=2 for Eight-shaped trajectory) and (M=4 for Lissajous curve trajectory) for the structure of the proposed (NFOPIDN) controller.
- Effective minimization capability of mean square tracking errors to follow a reference path.
- Efficiency of generating smooth suitable velocity action between (0 to 0.5) m/sec, without spikes and no saturation state in the action.

REFERENCES


