Performance Improvement of Multi-User MC-CDMA System Using Discrete Hartley Transform Mapper

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ABSTRACT

Future wireless communication systems must be able to accommodate a large number of users and simultaneously to provide the high data rates at the required quality of service. In this paper a method is proposed to perform the N-Discrete Hartley Transform (N-DHT) mapper, which are equivalent to 4-Quadrature Amplitude Modulation (QAM), 16-QAM, 64-QAM, 256-QAM, … etc. in spectral efficiency. The N-DHT mapper is chosen in the Multi Carrier Code Division Multiple Access (MC-CDMA) structure to serve as a data mapper instead of the conventional data mapping techniques like QPSK and QAM schemes. The proposed system is simulated using MATLAB and compared with conventional MC-CDMA for Additive White Gaussian Noise, flat, and multi-path selective fading channels. Simulation results are provided to demonstrate that the proposed system improves the BER performance and reduce the constellation energy as compared with the conventional system.

Keywords: Discrete Hartley Transform (DHT), Hartley Mapper, MC-CDMA
INTRODUCTION

Multi-carrier CDMA (MC-CDMA) is a strong candidate for next generation wireless communication due to its high spectral efficiency, large system capacity, high flexibility in data rate and easy implementation using the fast Fourier transform (FFT) device without increasing the transmitter and receiver Complexities [1]. Therefore, MC-CDMA formed by combining orthogonal frequency division multiplexing (OFDM) with code division multiple accesses (CDMA) became significant research topics [2, 3]. The former is well suited for high data rate applications in frequency selective fading channels and the later is a multiplexing technique where number of users is simultaneously available to access a channel [4], thus the combining of these two systems results especially in high spectral efficiency, the multiple access capability, robustness in the case of frequency selective channels, simple one-tap equalization, narrow-band interference rejection and high flexibility of the MC-CDMA. The outlined potential properties of the MC-CDMA represent the fundamental reasons, why MC-CDMA has been receiving a great attention over the last decade (e.g. [1, 5]) and has been considering being a promising candidate for the future advanced wireless communication systems.

Recently, a number of multi-carrier code division multiple access (MC-CDMA) systems have been proposed [6 – 8]. Among these systems, multi-carrier code division multiple access (MC-CDMA) combines time domain spreading and multi-carrier (MC) modulation. An interesting property of MC-CDMA systems is that the available channel bandwidth is divided into a set of equal width subchannels, and narrowband CDMA waveforms are transmitted over the subchannels. Due to this configuration, MC-CDMA system is capable of supporting high data rate services over hostile radio channels. Another interesting property of MC-DS-CDMA systems is that the modulation and demodulation can be implemented using fast Fourier transform (FFT) [9]. The performance of MC-CDMA system degrades mainly due to the multiple access interference (MAI) [9, 10].

The effect of a carrier frequency offset on the performance of MC-DS-CDMA was considered in [11]. And the effect of chip waveform shaping on the performance of MC-CDMA systems was studied in [12]. In [13], the performance of generalized MC-CDMA over fading channels was studied.

In the downlink, DS-CDMA relies on the orthogonality of spreading codes to separate different users. However, inter-chip interference (ICI) destroys the orthogonality among users, which causes Multi-user interference (MUI). Since MUI is essentially caused by the multi-path channel, linear chip-level equalization, followed by correlation with the desired user’s spreading code, suppresses MUI [14].

The QAM has been considered in the structure of the MC-CDMA system as a mapping technique in order to increase the rate of transmission, as QAM requires less bandwidth at the cost of increased transmission power.

Recently, many researches are introduced to use the Discrete Hartley transform (DHT) in image processing and communication systems (e.g. [15 – 20]), where in [15] the DHT is used for Content Based Image Retrieval (CBIR), and in [16 – 19] the DHT
is used to generate the sub carriers in OFDM system instead of FFT. In [20] A DHT-based frequency-domain equalizer (FEQ) is presented to eliminate the real-to-complex transformation (R2CT) for the discrete Hartley transform (DHT) based discrete multi-tone (DMT) systems.

In this paper the DHT is to be used in different manner to improve the performance of the MC-CDMA system where an N-Hartley based mapper is designed to serve as data mapper instead of the QAM mapper in the conventional system with different bit per hertz efficiency. This mapper is equivalent to the QAM mapper from the spectral efficiency point of view.

**DISCRETE HARTLEY TRANSFORM (DHT)**

The Hartley transform is an integral transformation that maps a real-valued temporal or spatial function into a real-valued frequency function via the kernel, \( \text{cas}(\nu x) = \cos(\nu x) + \sin(\nu x) \). This novel symmetrical formulation of the traditional Fourier transform, attributed to Ralph Vinton Lyon Hartley in 1942, leads to a parallelism that exists between the function of the original variable and that of its transform. Furthermore, the Hartley transform permits a function to be decomposed into two independent sets of sinusoidal components; these sets are represented in terms of positive and negative frequency components, respectively. This is in contrast to the complex exponential, \( \exp(j\omega x) \), used in classical Fourier analysis. For periodic power signals, various mathematical forms of the familiar Fourier series come to mind. For aperiodic energy and power signals of either finite or infinite duration, the Fourier integral can be used. In either case, signal and systems analysis and design in the frequency domain using the Hartley transform may be deserving of increased awareness due necessarily to the existence of a fast algorithm that can substantially lessen the computational burden when compared to the classical complex-valued Fast Fourier Transform (FFT).[21,22]

The major difference between the two transform algorithms is the real function (\( \text{cas} \)) in the Hartley transform and the complex exponential term in the Fourier transform. Since real arithmetic is much simpler than complex computation, the FHT is faster than the FFT and requires fewer floating-point operations, which implies faster runtimes and less computer memory to process a signal in comparison to a typical FFT algorithm. Furthermore, the complex Fourier spectrum can be obtained from the Hartley transform. Finally, the FHT uses fewer operations to transform a given signal, so there are fewer round-off errors [23].

The Hartley transform received little attention until its discrete version, the *discrete* Hartley transform (DHT), was introduced in the early 1980s by Wang and Bracewell [24]. Like other discrete transforms such as the DFT or the DCT, the DHT can be implemented efficiently through a factorization of the transform matrix. This results in fast algorithms that are closely related to the FFT, and in fact, the *fast Hartley transform (FHT)* can be computed via the FFT, and vice versa, the FFT can be implemented via the FHT [24].

The forward and inverse discrete Hartley transform pair is given by [25]:

\[
\text{cas}(\nu x) = \cos(\nu x) + \sin(\nu x)
\]
The signal $x(n)$ is considered to be real-valued, so that also the transform is real-valued. As with the DFT, the sequence $X_H(k)$ is periodic with period $N$.

Note that apart from the prefactor $1/N$ the DHT is self-inverse, which means that the same computer program or hardware can be used for the forward and inverse transform. This is not the case for the DFT, where a real-valued signal is transformed into a complex spectrum.

Many researches were proposed to compute the Discrete Hartley Transform using Fast Hartley Transform algorithm (FHT) [26 – 28]. The discrete Hartley transform for a length $N$ sequence $h(n) = [h_0, h_1, ….., h_{N-1}]$ can be computed in matrix form as [28]:

$$H(k) = [T] \times h(n) \quad \text{… (5)}$$

Where $[T]$ is the Discrete Hartley Transform matrix.

As we know that DHT has identical inverse so IDHT and DHT matrices are same, (i.e. $[T] = [T]^{-1}$) so

$$h(n) = [T]^{-1} \times H(k) = [T] \times H(k) \quad \text{… (6)}$$

Where
THE PROPOSED MC-CDMA SYSTEM WITH HARTLEY BASED MAPPER

The 4-Hartley Mapper

The 4-Hartley mapper is equivalent to the 4QAM mapper from the spectral efficiency point of view i.e. increase the bit per hertz by a factor of 2. The algorithm of the proposed 4-Hartley mapper is given in the following steps:

**Step 1:** compute the DHT matrix \([T]\) with selected dimension \((N*N)\) as given in eq. (7).

\[
[T] = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & \text{cas}(1) & \text{cas}(2) & \cdots & \text{cas}(N-1) \\
1 & \text{cas}(2) & \text{cas}(4) & \cdots & \text{cas}(2(N-1)) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \text{cas}(N-1) & \text{cas}(2(N-2)) & \cdots & \text{cas}((N-1)(N-1)) \\
\end{bmatrix} \tag{7}
\]

**Step 2:** The input data is taken as a column vector of length \(N^2\).

\[d = [d_0, d_1, d_2, \ldots, d_{N^2}]^T.\]

**Step 3:** convert the data in step 2 to 2D matrix of dimension \((N,N)\).

\[r_1 = \begin{bmatrix}
x_{1,1} & x_{1,2} & \cdots & x_{1,N-1} & x_{1,N} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
x_{N,1} & x_{N,2} & \cdots & x_{N,N-1} & x_{N,N} \\
\end{bmatrix} \tag{8}
\]

where \(x_{i,j} = d_{iN+j}\), \(x_{N,j} = d_{N+j}\), \(x_{N,N} = d_{N^2}\)

**Step 4:** Take the DHT for each column using eq. (5); then divide the 1\(^{st}\) element in each column of the resultant matrix by a factor of \(N\), after that normalize all elements by a factor of \(\sqrt{N}\) to reduce the energy of the mapper. The resultant matrix \(r_2\) will be of dimension \((N,N)\)

\[r_2 = \begin{bmatrix}
X_{1,1} & X_{1,2} & \cdots & X_{1,N-1} & X_{1,N} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
X_{N,1} & X_{N,2} & \cdots & X_{N,N-1} & X_{N,N} \\
\end{bmatrix} \tag{9}
\]

**Step 5:** Generate the complex matrix by taking the 1\(^{st}\) column real and the 2\(^{nd}\) column imaginary, also take the 3\(^{rd}\) column real and the 4\(^{th}\) column imaginary and so on to obtain a new matrix \(r_3\) of dimension \((N,N/2)\) with complex elements as given in eq.(10) and eq.(11) below.

\[r_{3(i,j)} = r_{2(u,v)} + jr_{2(u,v+1)} \quad 0 \leq u \leq N, 0 \leq v \leq N-1 \tag{10}\]

thus \(r_3\) will be
\[
\begin{bmatrix}
X_{1,1} + jX_{1,2} & X_{1,3} + jX_{1,4} & \cdots & X_{1,N-1} + jX_{1,N} \\
\vdots & \vdots & \ddots & \vdots \\
X_{N,1} + jX_{N,2} & X_{N,3} + jX_{N,4} & \cdots & X_{N,N-1} + jX_{N,N}
\end{bmatrix}
\]

as seen from eq.(10) the number of columns is divided by 2 then the resultant matrix \( r_3 \) is of dimension \((N\times N/2)\).

**Step 6:** Convert the complex matrix \( r_3 \) in step 5 to one dimensional vector \( D \) of length \( N^*\left(\frac{N}{2}\right) \).

\[
D = [s_1 \ s_2 \ \cdots \ \cdots \ s_k]^{T}
\]

where \( k=N^*N/2 \)

Where

\( s_1 = X_{1,1} + jX_{1,2} \), \( s_2 = X_{1,3} + jX_{1,4} \), and \( s_k = X_{N,N-1} + jX_{N,N} \)

Figure (1) gives a signal flow diagram that explains the procedure for the above proposed mapper.

From the above procedure we note that the total number of the data points is divided by 2 and then the bit per hertz is 2 and that equivalent to the 4-QAM mapper.
Multi-Level Hartley Mappers: 16-Hartley and 64-Hartlety Mappers.

The algorithm of 16 and 64-Hartley mappers are the same of the 4-Hartley mapper except in the 1st three steps as given below:

**Step 1:** compute the DHT matrix \([T]\) with selected dimension \((N \times N)\) as given in eq. (7).

**Step 2:** The input data must be a vector of length \((2N^2)\) in case of 16-Hartley mapper, \((3N^3)\) in case of 64-Hartley mapper and so on for higher level mappers.

**Step 3:** Convert this column to 2D matrix of dimension \((N^2, 2)\) in case of 16-Hartley mapper and \((N^2, 3)\) in case of 64-Hartley mapper

\[
\mathbf{D} = [s_1, s_2, \ldots, s_{N \times N/2}]^T
\]
Step 4: Convert the data in step 3 from binary to decimal, to obtain column vector of length \(N^2\):

\[ d = [d_0 \ d_1 \ d_2 \ldots \ d_{N^2}]^T \]

all elements in this vector will be between (0 and 3) in case of 16-Hartley mapper and between (0 and 7) in case of 64-Hartley mapper and normalize this data by dividing it by \(k\) (here \(k\) is equal to the square root of the maximum number in the decimal data i.e. square root of 3 in case of 16-Hartley mapper and square root of 7 in case of 64-Hartley mapper).

Step 5: Convert the data in step 4 to 2D matrix (as in step 3 in section 3.1)

Step 6: perform the Discrete Hartley Transform and the normalization steps (as in step 4 in section 3.1).

Step 7: Generate the complex matrix using eq.(10) (as in step 5 in section 3.1); this matrix \(r_3\) is of dimension \((N,N/2)\).

Step 8: Convert the complex matrix in step 7 to one dimensional vector \(D\) (as in step 6 in section 3.1).

\[ D = [\ldots \ldots \ldots s_k]^{T} \quad where \ k = N*N/2 \]

At the end of this step, the mapping is done and the complex valued symbols now ready for spreading.

In the above procedure; steps (3-4) divide the number of data points by two in case of the 16- Hartley mapper, and also step (7) divides the number of data points by two i.e. this process divides the number of data points by four and that is equivalent to 16QAM map from spectral efficiency point of view. In case of the 64- Hartley mapper steps (3-4) divides the number of data points by three and also step (7) divides the number of data points by two i.e. this process divides the number of data points by six and that is equivalent to 64QAM map from spectral efficiency point of view.

Simply, the Hartley demapper can be performed by taking the inverse operations to the above steps.

Now the Hartley map is complete, after that, the Hartley based mappers are used in the construction of the MC-CDMA system instead of the conventional QAM mappers, as shown in Figure (2).

**Proposed Realization of Hartley Mapping Based MC-CDMA System**

In this paper we propose and describe a technique for \((N\)-Hartley\) mapper based MC-CDMA where \(N = 4, 16, 64, \ldots\) etc. which is equivalent to 4-QAM, 16-QAM, 64 QAM, \ldots etc. The block diagram of proposed \(N\)-Hartley based MC-CDMA transceiver is similar to that of conventional MC-CDMA transceiver as shown in Figure (2). Instead of QAM mapper used in conventional MC-CDMA where the mapper is used to increase the bit per hertz, a new Hartley transform mapper is implemented in this design. It works as a good interleaver and provides a high immunity against ICI due to implementation of orthogonality twice: in the mapper and in the sub-carrier modulation. As a result, proposed system gives a significant
reduction in Bit Error Rate (BER) as compared with conventional QAM based systems for various numbers of active users. Another important result obtained here is that, proposed Hartley mapper has constellation of very small energy as compared with that of QAM mapper.

Each user data mapped into complex symbols to increase the bit per hertz using the proposed mapper instead of the conventional QAM mapper, and then these symbols can be simultaneously processed at the spreading step before MC modulation, Walsh-Hadamard (WH) spreading sequences will be considered. Walsh code sequences that used are obtained from the Hadamard matrix which is a square matrix where each row in the matrix is orthogonal to all other rows, and each column in the matrix is orthogonal to all other columns. The Hadamard matrix $H_n$ is $N \times N$ matrix, where $N = 2^n$. These can be generated by the core matrix.

$$H_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$ \quad \ldots (12)

and applying the Hadamard transform successively using the Kronecker product recursion.

$$H_n = H_{n-1} \otimes H_1$$ \quad \ldots (13)

$$H_n = \frac{1}{\sqrt{2^n}} \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{bmatrix}$$ \quad \ldots (14)

The spreading code used in the simulation is 16 chips Walsh-Hadamard code generated using $(16 \times 16)$ Hadamard matrix.
Three types of channels are used: Additive White Gaussian Noise (AWGN) channel with several Signal to Noise Ratio (SNR) values, multi-path Raleigh distributed flat fading, and multi-path Raleigh distributed selective-fading. The selective-fading channel is simulated as 1-D FIR filter that adds multi-path effect and AWGN to transmitted symbols. Two ray channels were assumed in simulations with a second path gain of –2 dB, and a maximum delay for the second path of 3 samples for several values of signal-to-noise ratio (SNR). The channel is assumed to be slowly varying, which doesn’t change within a packet frame. Thus, the estimation is done with the long preambles at the beginning of the frame. The channel frequency response is estimated by using training pilots and received sequences as follows:

\[ H_{\text{estimated}}(k) = \frac{\text{Received Training Sample}(k)}{\text{Transmitted Training Sample}(k)} \quad k = 0, 1, 2, \ldots \]

The channel frequency response is used to compensate the channel effects on the data, and the estimated data is found using the following equation:

\[ \text{Data}_{\text{estimated}}(k) = (H_{\text{estimated}})^{-1} * \text{Data}_{\text{received}}(k), \quad k = 0, 1, 2, \ldots \]

For more details, Figure (3) shows K-user MC-CDMA transmitter with Hartley mapper evaluation. In this system the data of each user is mapped into complex symbols using Hartley mapper described in the previous section, before introduce to the CDMA-OFDM System. The receiver can be designed using inverse process.
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SIMULATION AND BER PERFORMANCE ANALYSIS OF THE PROPOSED SYSTEM

In this section, the results of bit error performance simulations for Hartley-based MC-CDMA are provided and compared with the conventional MC-CDMA under different channel conditions where the systems are simulated by MATLAB. AWGN channels, flat fading channels and selective fading channels are considered during simulations. In case of selective fading channel, two ray channels were assumed in simulations with a second path gain of –2 dB, and a maximum delay for the second path of 3 samples for several values of signal-to-noise ratio (SNR). The system parameters that used through simulations are given below in Table (1).
Table (1) simulation parameters.

<table>
<thead>
<tr>
<th>System</th>
<th>MC-CDMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>modulation</td>
<td>4-Hartley, 16-Hartley, 4-QAM, and 16-QAM</td>
</tr>
<tr>
<td>Active users</td>
<td>4, 6</td>
</tr>
<tr>
<td>Harley matrix size</td>
<td>8 by 8</td>
</tr>
<tr>
<td>Spreading Code</td>
<td>Walsh Code with code length = 16</td>
</tr>
<tr>
<td>IFFT bins</td>
<td>512</td>
</tr>
<tr>
<td>Guard interval</td>
<td>Cyclic prefix (0.25*IFFT size)</td>
</tr>
<tr>
<td>Channel Conditions</td>
<td>AWGN, Flat and Selective fading channel</td>
</tr>
</tbody>
</table>

The constellation of the proposed 16-Hartley mapper is shown in Figure (4) (a), this constellation is plotted by taking all the input symbols to see the symbols of maximum energy; where as the 16-QAM mapper constellation is shown in Figure (4) (b) where this constellation is plotted using the predetermined 16 symbols of this mapper. By comparing the two constellations one can see that the energy of the proposed mapper is lower than that of the conventional mapper (about 40% energy saving).

Figure (5) shows the power spectral density, of the MC-CDMA signal for both QAM and Hartley mapper based MC-CDMA systems at the transmitter output, the power density is normalized over $2\pi$ radian per samples since we use the default sampling frequency determined by Matlab. It is clearly seen from Figure (5) that power spectrum of Hartley based MC-CDMA is still as good as conventional QAM based MC-CDMA and it was not affected by the new mapping replacement.

(a) 16-Hartley Map
(b) 16-QAM Map

Figure (4) Constellation diagrams of the 16-Hartley, and 16-QAM mappers: (a) 16-Hartley mapper (b) 16-QAM mapper.
The BER performance is simulated by computing the errors in the received data due to channel effects when compared with the original data (ex. \(p_e=10^{-4}\) means that one bit in error for every 10000 bits). This process repeated for several values of SNR. Because 4-Hartley mapping is equivalent to 4-QAM mapping and 16-Hartley mapping is equivalent to 16-QAM mapping from spectral efficiency point of view (the same for higher levels mapping), the performances of the 4-Hartley and 16-Hartley mapping based MC-CDMA in different channel conditions and with different numbers of active users are compared with the performances of 4-QAM and 16-QAM mapping based MC-CDMA respectively.

The BER performances of the proposed 16-Hartley mapper based MC-CDMA and conventional 16-QAM mapper based MC-CDMA systems in AWGN channel are shown in Figure (6). From which it can be noted that Hartley-based MC-CDMA has an SNR gain of 3.5 dB compared with conventional system to achieve BER of \(10^{-3}\) for 4 and 6 active users.

Figure (7) shows that the proposed MC-CDMA with 16-Hartley mapper gives an improvement in SNR of about 4 dB to achieve a BER performance of \(10^{-4}\) for 4 and 6 active users as compared with the standard MC-CDMA system with 16QAM mapper in flat fading channel with AWGN.

Figure (5) power Spectrum of QAM and Hartley based MC-CDMA:
(a) 4-Hartley based MC-CDMA (b) 4-QAM based MC-CDMA.
Figure (6) Performance of Hartley-based and QAM-based MC-CDMA for AWGN channel.

Figure (8) shows the BER performance of 4-Hartley mapper based MC-CDMA in a two-ray Rayleigh distributed multi-path selective fading channel with AWGN simulation. The second path assumes a path gain of -2dB, and a delay of three samples. From simulation it is seen that BER performance of 4-Hartley mapper based MC-CDMA is still better than standard QAM-based MC-CDMA, where an error of $10^{-4}$ is achieved at SNR gain of about 6 dB.

The same channel conditions are used in the simulation of the proposed MC-CDMA system but with 16-Hartley mapper and the BER Performance compared with that of the standard system with 16QAM mapper as shown in Figure (9).

Figure (7) Performance of Hartley-based and QAM-based MC-CDMA in flat fading channel.
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Figure (8) Performance of the 4-Hartley-based and 4-QAM based MC-CDMA in frequency selective fading (Two ray channel with 2nd path gain -2dB and a maximum delay for the 2nd path of 3 samples).

Figure (9) Performance of the 16-Hartley and 16-QAM based MC-CDMA systems in frequency selective fading (Two ray channel with 2nd path gain -2dB and a maximum delay for the 2nd path of 3 samples).
CONCLUSIONS
In this paper, N-Hartley mapper based MC-CDMA system is designed and simulated. The Discrete Hartley Transform (DHT) is used to design a new data mapper equivalent to the QAM mapper from the spectral efficiency point of view. This mapper used in the structure of the MC-CDMA system to serve as data mapping technique and increase the bit per hertz instead of the QAM technique in the conventional system. The proposed Hartley-based MC-CDMA has a better performance than conventional MC-CDMA system.

The results shows an SNR gain of 3.5 dB, 4 dB and 6 dB in AWGN, flat fading and frequency selective channels, respectively, under the same conditions of the channel parameters. The use of N-Hartley based mapper in MC -CDMA maintains a spectrum shape and efficiency as good as the standard QAM based MC –CDMA and reduces the constellation energy. This mapper provides a high immunity against ICI due to implementation of orthogonality twice: in the mapper and in the sub-carrier modulation. As a result, proposed system gives a significant reduction in Bit Error Rate (BER) as compared with conventional QAM based systems for various numbers of active users. Another important result obtained here is that, proposed Hartley mapper has constellation of small energy as compared with that of QAM mapper which results in (about 40%) energy saving.

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