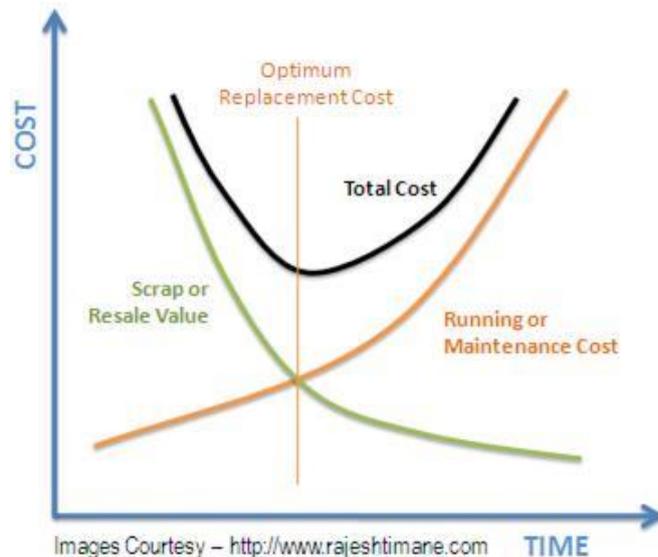


# Replacement Theory



The Replacement Theory in Operations Research is used in the decision making process of replacing a used equipment with a substitute; mostly a new equipment of better usage. The replacement might be necessary due to the deteriorating property or failure or breakdown of particular equipment. The 'Replacement Theory' is used in the cases like; existing items have out-lived, or it may not be economical anymore to continue with them, or the items might have been destroyed either by accident or otherwise. The above discussed situations can be solved mathematically and categorized on some basis like:

- Items that deteriorate with time e.g. machine tools, vehicles, equipment buildings etc.
- Items becoming out-of-date due to new developments like ordinary weaving looms by automatic, manual accounting by tally etc.
- Items which do not deteriorate but fail completely after certain amount of use like electronic parts, street lights etc (*Group Replacement*) and
- The existing working staff in an organization gradually diminishing due to death, retirement, retrenchment & otherwise (*Staff Replacement*).

## Replacement Policy for Equipment which Deteriorate Gradually

Let us see the fiUSDt case of gradual failure of items with time. Consider the example of a Motor Vehicle; the pattern of failure here is progressive in nature i.e. as the life of vehicle increases; its efficiency decreases. This results in additional expenditure in running or maintaining this vehicle and at the same time its resale value (also called as scrap value) also keeps on decreasing. The above case makes this situation a typical case for applying 'Replacement Theory'.

### Example:

A transport company purchased a motor vehicle for rupees 80000/-. The resale value of the vehicle keeps on decreasing from USD 70000/- in the first year to USD 5000/- in the eighth year while, the running cost in maintaining the vehicle keeps on increasing with USD. 3000/- in the first year till it goes to USD. 20000/- in the eighth year as shown in the below table. Determine the optimum replacement policy?

Figure: Yearly Scrap Value & Running Cost								
Year	1	2	3	4	5	6	7	8
Scrap Value	70 000	61 000	55 000	49 000	32 000	20 000	10 000	5 000
Running Cost	3 000	3 600	4 800	5 000	8 000	11 200	15 000	20 000

The MS-Excel Files of this Algorithm can be downloaded from the links provided further in this post. The cost of the equipment is to be entered in the cell B1 (as shown by the green cell with 80000). Now, enter the scrap values and the running costs as entered in the green columns C5 to C12 and D5 to D12. The algorithm will now automatically calculate the solution which is as shown in the below figure.

*Screen Shot of Replacement Theory Algorithm (MS Excel Sheet)*

	A	B	C	D	E	F	G	H
1	C =	80000						
2								
3	YEAR	C	S	Rn	$\Sigma Rn$	(C-S)	TC [ $\Sigma Rn + (C-S)$ ]	ATC [TC / YEAR]
4								
5	1	80000	70000	3000	3000	10000	13000	13000.00
6	2	80000	61000	3600	6600	19000	25600	12800.00
7	3	80000	55000	4800	11400	25000	36400	12133.33
8	4	80000	49000	5000	16400	31000	47400	11850.00
9	5	80000	32000	8000	24400	48000	72400	14480.00
10	6	80000	20000	11200	35600	60000	95600	15933.33
11	7	80000	10000	15000	50600	70000	120600	17228.57
12	8	80000	5000	20000	70600	75000	145600	18200.00

Images by – <http://www.rajeshitimane.com>

The answer can be fetched from the last column. See the pattern; the average total cost (ATC) at first starts dipping from USD. 13000/- till it reaches USD. 11850/- in the cell H8. From H9 it again starts increasing. This cost at which the ATC is lowest in a particular year (after which it starts increasing again) gives the optimum replacement period and cost of the vehicle.

**Solution:** The vehicle needs to be replaced **after four years** of its purchase wherein the cost of maintaining that vehicle would be lowest at an average of **USD 11850/-** per year.

## Clarification on the Methodology

There are two considerations here. First, the running cost ( $R_n$ ) is increasing every year at the same time the vehicle is depreciating in its value. This depreciation is '(C-S)' i.e. in the first year the scrap value of the vehicle is USD. 70000/- which was purchased for USD. 80000/- . So, the vehicle is depreciated by USD. 10000/- in year one and so on (see column F).

Thus the total cost in keeping this vehicle is this depreciation and its maintenance. The maintenance is made cumulative by adding previous years running cost to it every successive year. Let's make this simple!

The depreciation is USD. 10000/- in the first, 19000/- in the second, 25000/- in the third and so on. See here, the vehicle is depreciated by **USD. 25000/-** "by" the third year and not "in" the third year. Note that the non-cumulative cost of depreciation "in" the third year would be USD. 6000/- [USD. 25000/ minus USD. 19000/, see the cells F6 and F7]

As, the depreciation in itself is a cumulative function here, we make the running cost cumulative also. That means the cost of maintaining the vehicle "by" the particular years. So, the cost of maintaining the vehicle "by" the third year is **USD. 11400/-** ( $D_5 + D_6 + D_7$  or  $3000 + 3600 + 4800$ ).

Hence the total cost incurred by the third year would be USD. 25000 + USD. 11400 = USD. 36400 (see cell G7). Finally, the "average cost" of keeping this vehicle for three years would be 36400 divided by 3 years i.e. USD. 12133.33 as can be seen from cell H7 and so on.

### Notations Used:

- **C** – (Capital) Cost of Equipment
- **S** – Scrap (or Resale) Value
- **$R_n$**  – Running (or Maintenance) Cost
- **$E R_n$**  – Cumulative Running Cost
- **(C-S)** – Depreciation
- **TC** – Total Cost
- **ATC** – Average Total Cost

# Group Replacement Theory

## Replacement of items that fail suddenly

There are certain items which do not deteriorate but fail completely after certain amount of use. These kinds of failures are analyzed by the method called as group replacement theory. Here, large numbers of items are failing at their average life expectancy. This kind of items may not have maintenance costs as such but they fail suddenly without any prior warning. Also, in case of sudden breakdowns immediate replacement may not be available. Few examples are fluorescent tubes, light bulbs, electronic chips, fuse etc.



Let's consider the example of street lights. We often see street-lights being repaired by the corporation staff using extendable ladders. If a particular light is beyond repairs, then it is replaced. This kind of policy of replacement is called as 'replacement of items as-and-when they fail' or '*Individual Replacement*'. On the other hand, if all the street lights in a particular cluster are replaced as and when they fail and also simultaneously in groups, then the policy is called as '*Group Replacement*'. It should be noted that, **group replacement does involve periodic simultaneous replacements along with individual replacements in between.**

It is found that replacing these random failing items simultaneously at specific intervals is economical as compared to replacing them only when an item fails. A long period between group replacements results in increase in cost of individual replacements, while frequent group replacements are definitely costly. There lies the need to balance this and find an optimum replacement time for optimum cost of replacement.

### Problem:

A factory has 1000 bulbs installed. Cost of individual replacement is USD. 3/- while that of group replacement USD 1/-per bulb respectively. It is decided to replace all the bulbs simultaneously at fixed interval & also to replace the individual bulbs that fail in between. Determine optimal replacement policy. Failure probabilities are as given below:

Week	1	2	3	4	5
Failure Probability (P)	0.10	0.25	0.50	0.70	1.00

**Solution:**

The probabilities given in the problem are cumulative i.e. till week 1, till week 2 etc. Individual probabilities would be 0.10 in 1<sup>st</sup> week, 0.15 (0.25-0.10) in 2<sup>nd</sup> week, and so on. (as shown in the below table)

**Policy-I: Individual Replacement**

**Step 1) Cost of Individual Replacements**

Table 1	Week	Probability	Life (=time x probability)
	1	0.10	1 x 0.10 = 0.10
	2	0.15	2 x 0.15 = 0.30
	3	0.25	3 x 0.25 = 0.75
	4	0.20	4 x 0.20 = 0.80
	5	0.30	5 x 0.30 = 1.50
<b>Mean Life</b>		<b>3.45</b>	

Individual Failures/week = Total Quantity / Mean Life = 1000 / 3.45 = 289.9

Individual Replacement Cost = (Individual Failures per week) x (Individual replacement cost)

= 289.9 x 3 = **USD. 869.6**

**Policy-II: Group Replacement**

**Step 2) Individual failures per week**

In the first week: 10 % (0.10) of the bulbs will fail out of 1000 bulbs i.e. 100

In the second week: 15 % of the bulbs will fail out of 1000 bulbs i.e. 150. Also, 10% of 100 replaced in the first week i.e. 10. TOTAL bulbs failed until second week = 160 (150+10)

Rest of the calculation is as shown in the below table:

Table 2	Wk	I*	II	III	IV	V	TOTAL
	1	100.00					100.00
	2	150.00	10.00				160.00
	3	250.00	15.00	16.00			281.00
	4	200.00	25.00	24.00	28.10		277.10
	5	300.00	20.00	40.00	42.15	27.71	429.86
	6	42.99**	41.57	70.25	32.00	30.00	216.80
	7	21.68	64.48	69.28	56.20	48.00	259.63
*10% of 1000, 15 % of 1000 etc.							
** 429.86 x 0.10 = 42.99							

### Step 3) Calculating the total cost & time of replacement:

Table 3	Wk	Cumulative Failures*	Individual Failure Cost (Rs 3/bulb)**	Group Failure Cost	Total Cost (TC)	Average Cost (TC/Wk)
	1	100.00	300.0	1000	1300.0	1300.0
	2	260.00	780.0	1000	1780.0	890.0
	3	541.00	1623.0	1000	2623.0	874.3
	4	818.10	2454.3	1000	3454.3	<b>863.6</b>
	5	1247.96	3743.9	1000	4743.9	948.8
	6	1464.76	4394.3	1000	5394.3	899.0
	7	1724.40	5173.2	1000	6173.2	881.9
* Adding the values of the last column of previous table						
** Values of previous column by Rs 3.						

Thus, replacing all the bulbs simultaneously at fixed interval & also to replace the individual bulbs that fail in between will be economical or optimal after 4 weeks (optimal interval between group replacements).

#### Interpretation:

1. The cost of only individual replacements is USD. 869.6 (As seen in the Policy-I)
2. The cost of combine policy i.e. group and individual replacement is USD. 863.6 (see last column of table 3)
3. Hence the Policy-II is the optimum replacement policy

Hence, the bulbs shall be replaced every four weeks individually as well as in groups which combine would cost USD. 863.6 per week (lesser than individual cost of USD. 869.6 per week)

**[Click the following link to Download the Excel Solver](#)**

**Group Replacement Algorithm**

**Practice Problem:**

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The following mortality rates have been observed for a special type of light bulbs. There are 1000 such bulbs in the concerned unit of the industry.

<b>Month</b>	1	2	3	4	5
<b>Monthly Failures (%)</b>	10	25	50	80	100

It costs USD 10 to replace an individual bulb that has burnt out. If the bulbs were replaced simultaneously, it would cost USD. 2.50 per bulb. It is proposed to replace all the bulbs at fixed interval, whether are not they have burnt out, and to continue replacing the burnt out bulbs as they fail. At what intervals of time should the manager replace all the bulbs? Decide the optimum replacement policy.

**Solution:**

Step 1) Download the Excel Solver

Step 2) Over-write the new problem values in the orange cells.

Step 3) Compare:

- Individual Cost = USD. 2985 (Cell D20)
- Individual & Group Cost = USD. 2550 (Cell Q10)

Hence, the bulbs need to be replaced completely after every two months along with individual replacements as and when they fail.

**PowerPoint Presentation**