

Strength of materials

1. General concepts

All structures, both natural and man-made, are composed of materials that are arranged and assembled in a way that will fulfill the purpose of the structure. Buildings, bridges, and bones are made from clay, steel, and calcium. These have a correspondingly wide range of behaviors under load

The **strength** of a material is the ability of that material to withstand an applied stress without failure. Different types of stress can be defined within this field like tensile stress, compressive stresses beside shear stresses. In addition materials could be failed by another types of failures like fatigue stress, thermal stress and/or creep failure. These all within the so called the strength of materials or mechanics of materials will be included to clarify how to deal with applied loads. Hence, strength of materials is a subject which deals with loads, deformations and the forces acting on the material.

2. Important definitions

The main objective of the study of the mechanics of materials is to provide the engineer with the means of analyzing and designing various machines and load bearing structures.

2.1 Stress and Strain

Stress (Intensity of force)

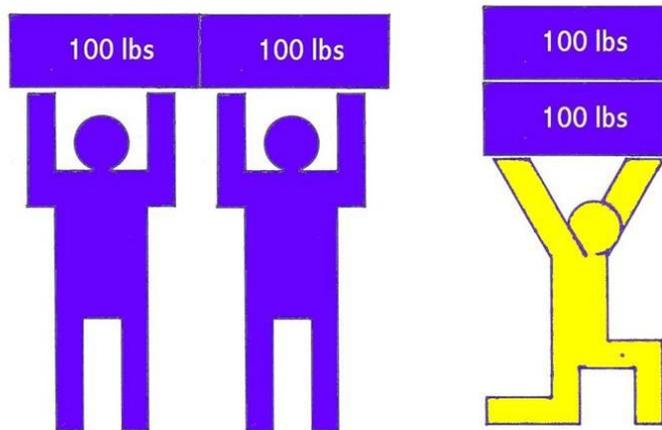


Fig. 1

Stress is the internal resistance offered by a unit area of the material from which a member is made to an externally applied load. Normal stress is determined using the following equation: $\sigma = \text{Applied Load} / \text{Original Cross Section Area} = P/A$

This has a unit of Pa or N/m^2 .

Type of stresses

1. Normal
2. Shear
3. Hydrostatic (uniform pressure)

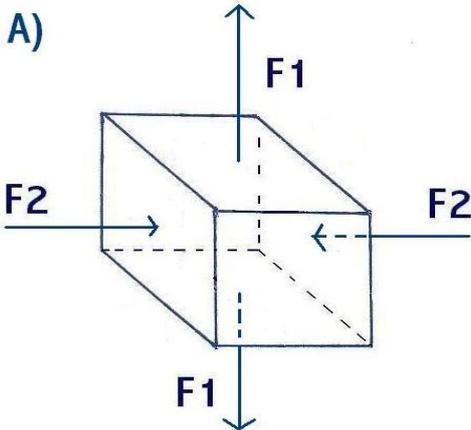


Fig. 2 A, Load perpendicular to area (Normal stress)

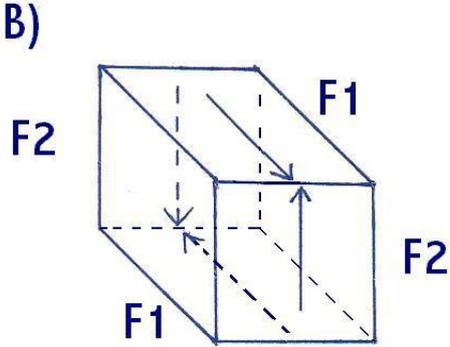


Fig. 2 B, Load parallel to area (Shear stress)

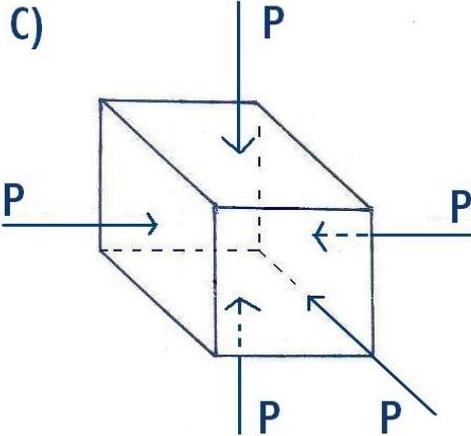


Fig. 2 C, Hydrostatic (uniform pressure)

Strain

Strain is the total deformation divided by the original length of the bar.

Normal or longitudinal strain is calculated using the following equation:

$$\varepsilon = \text{Changing in length} / \text{Original length} = \Delta L / L_0$$



Fig. 3

$$\text{Percentage Strain} = \Delta L / L_0 \times 100\%$$

It is to be expected that the tensile stress and strain cause positive increase in length (dimensions), whereas the compressive stress and strain translated to negative change or decrease in dimensions.

Generally materials can be classified into:

- (1) Elastic material which undergoes a deformation when subjected to an external loading such that, the deformation disappears on the removed of the loading, (Rubber).
- (2) A plastic material undergoes a continuous deformation during the period of loading and the deformation is permanent and the material does not regain its original dimensions on the removal of the loading, (Aluminum).
- (3) A rigid material does not undergo any deformation when subjected to an external loading, (Glass and Cast iron).

In other words, in many cases we are dealing with either ductile materials such as low carbon steel or copper, tough materials such as titanium alloys, and brittle materials such as ceramics and glasses. Fig. 4 explains the differences in the stress strain curve which may appear in different materials.

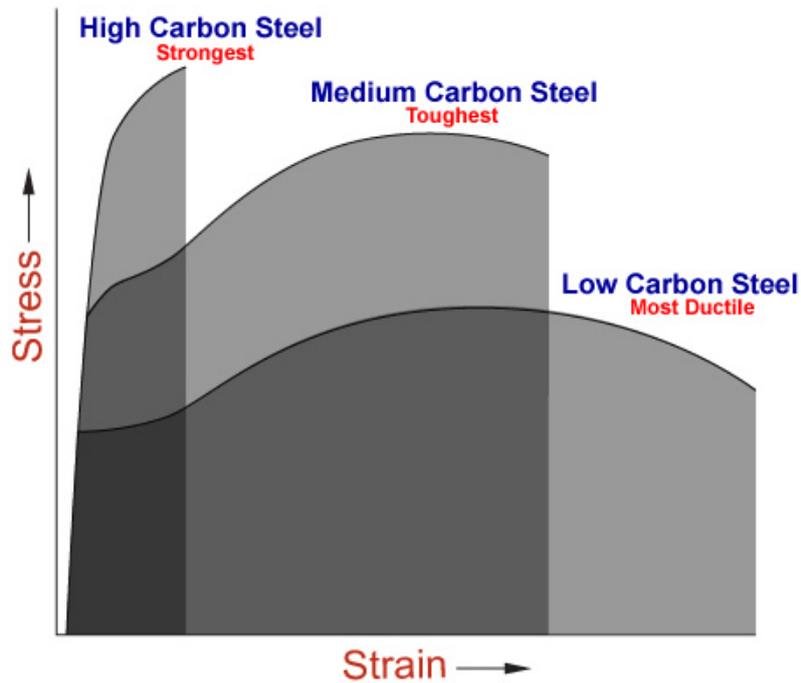


Fig. 4

2.2 Elasticity

Elasticity (or stretchiness) is the property of a material that returns to its original shape after the stress (e.g. external forces) that made it deform or distort is removed.

The relative amount of deformation is called the strain.

2.3 Plasticity

plasticity describes the deformation of a material undergoing non-reversible changes of shape in response to applied forces. For example, a solid piece of metal being bent or pounded into a new shape displays plasticity as permanent changes occur within the material itself. In engineering, the transition from elastic behavior to plastic behavior is called yield.

2.4 Toughness

Toughness is the ability of a material to absorb energy and plastically deform without fracturing. Material toughness is defined as the amount of energy per volume that a material can absorb before rupturing. It is also defined as the resistance to fracture of a material when stressed. Fig. 5 shows the relationship between toughness and strength for different materials. It can be seen that materials like ceramics have low toughness values in spite of they are so strong. On the other hand, rubbers are tough materials as well as they are weak in terms of strength. The area under the entire stress-strain curve from zero to rupture gives the so called modulus of toughness which is the energy per unit volume necessary to rupture the material under the test.

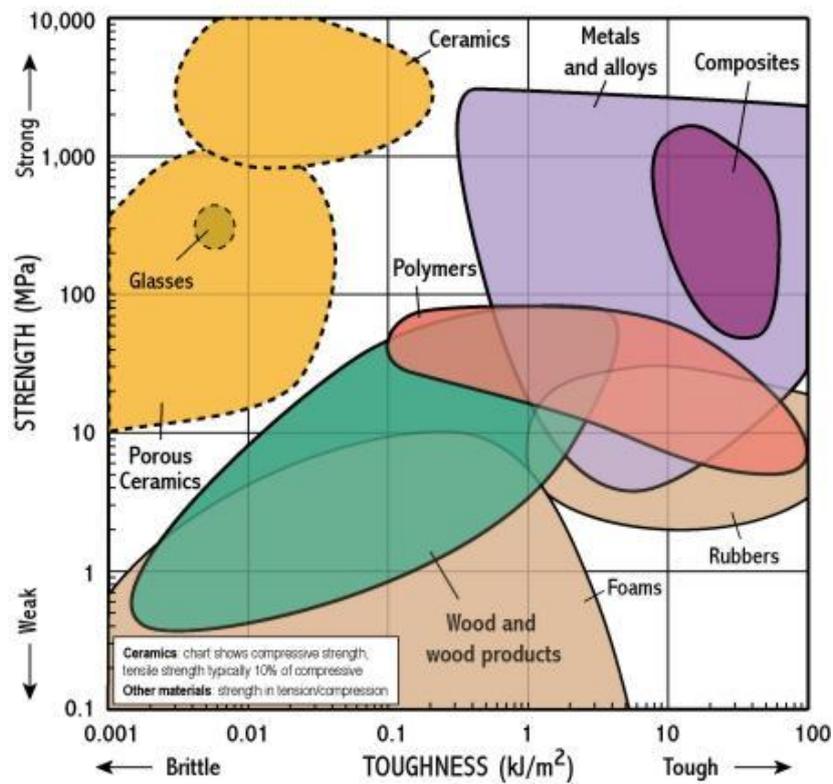


Fig. 5

2.4 Brittleness

A material is brittle if, when subjected to stress, it breaks without significant deformation (strain). Brittle materials absorb relatively little energy prior to fracture, even those of high strength. Breaking is often accompanied by a snapping sound. Brittle materials include most ceramics and glasses (which do not deform plastically) and some polymers, such as PMMA and polystyrene. Many steels become brittle at low temperatures, depending on their composition and processing.

2.5 Hardness

Hardness is the measure of how resistant solid matter is to various kinds of permanent shape change when a force is applied. Macroscopic hardness is generally characterized by strong intermolecular bonds, but the behavior of solid materials under force is complex; therefore there are different measurements of hardness: scratch hardness, indentation hardness, and rebound hardness. As an example, glass materials have high hardness number as compared with soft metals like Copper or Aluminum.

3. Stresses

3.1 stress at a point

Consider a body in equilibrium under point and traction loads as shown in figure 6.

After cutting the body along section AA, take an infinitesimal area ΔA lying on the surface consisting a point C.

The interaction force between the cut sections 1 & 2, through ΔA is ΔF . Stress at the point C can be defined as:

$$\sigma = \lim_{\Delta A \rightarrow 0} \Delta F / \Delta A$$

ΔF is resolved into ΔF_n and ΔF_s that are acting normal and tangent to ΔA .

Normal stress $\sigma_n = \lim_{\Delta A \rightarrow 0} \Delta F_n / \Delta A$

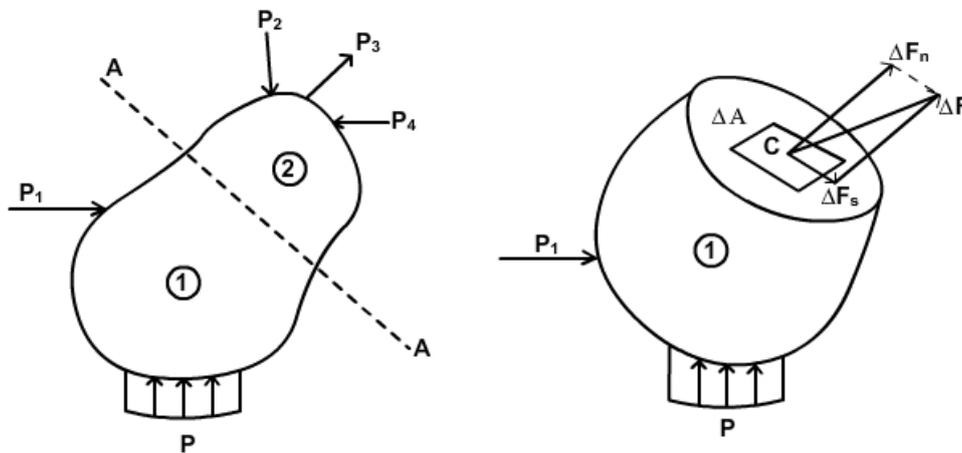


Fig. 6

3.2 Free body diagram

The first step towards solving an engineering problem is drawing the free body diagram of the element/structure considered.

Removing an existing force or including a wrong force on the free body will badly affect the equilibrium conditions, and hence, the analysis.

In view of this, some important points in drawing the free body diagram are discussed below.

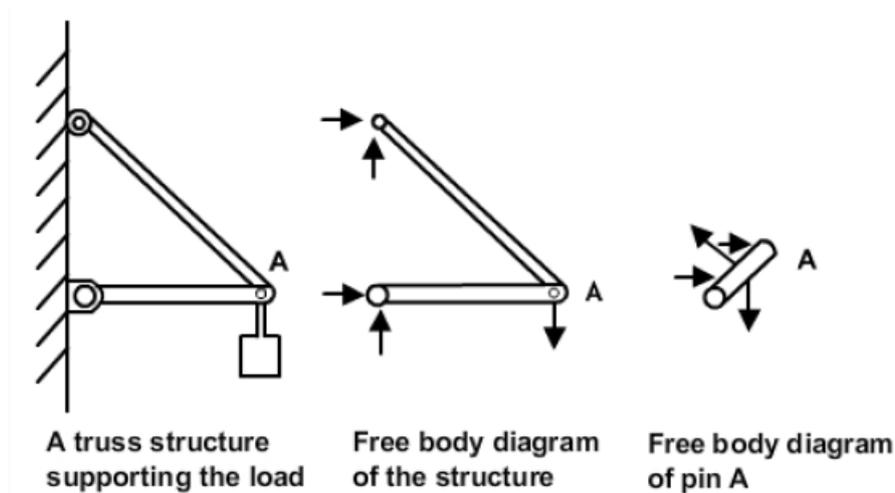


Fig. 7

At the beginning, a clear decision is to be made by the analyst on the choice of the body to be considered for free body diagram.

Then that body is detached from all of its surrounding members including ground and only their forces on the free body are represented.

The weight of the body and other external body forces like centrifugal, inertia, etc., should also be included in the diagram and they are assumed to act at the centre of gravity of the body.

When a structure involving many elements is considered for free body diagram, the forces acting in between the elements should not be brought into the diagram.

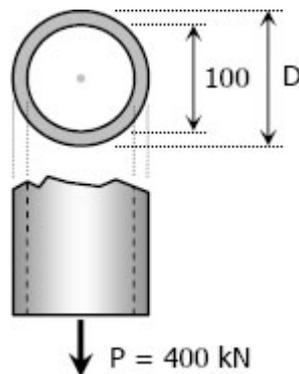
The known forces acting on the body should be represented with proper magnitude and direction.

If the direction of unknown forces like reactions can be decided, they should be indicated clearly in the diagram.

After completing free body diagram, equilibrium equations from statics in terms of forces and moments are applied and solved for the unknowns.

Problem 1

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m².



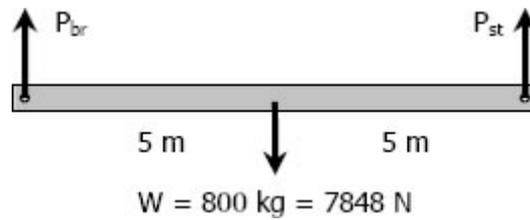
Solution: 119.35 mm

Problem 2

Given: Weight of bar = 800 kg

Maximum allowable stress for bronze = 90 MPa

Maximum allowable stress for steel = 120 MPa



Required: Smallest area of bronze and steel cables

Solution: $A_{br} = 43.6 \text{ mm}^2$, $A_{st} = 32.7 \text{ mm}^2$

4. Thermal Stresses

Temperature changes cause the body to expand or contract. Mechanical stress induced in a body when some or all of its parts are not free to expand or contract in response to changes in temperature. In most continuous bodies, thermal expansion or contraction cannot occur freely in all directions because of geometry, external constraints, or the existence of temperature gradients, and so stresses are produced. Such stresses caused by a temperature change are known as thermal stresses.

Problems of thermal stress arise in many practical design problems, such as those encountered in the design of steam and gas turbines, diesel engines, jet engines, rocket motors, and nuclear reactors. The high aerodynamic heating rates associated with high-speed flight present even more severe thermal-stress problems for the design of spacecraft and missiles.

The amount δT which gives the total thermal deflection (deformation due to temperature changes) is given by:

$$\delta T = \alpha L (T_f - T_i) = \alpha L \Delta T$$

where α is the coefficient of thermal expansion in $m/m^\circ C$, L is the length in meter, and T_i and T_f are the initial and final temperatures, respectively in $^\circ C$.

deformation due to equivalent axial stress;



Fig. 8

$$\delta T = \delta P$$

$$\delta P = P L / A E = \sigma L / E$$

$$\alpha L \Delta T = \sigma L / E$$

Which means : $\sigma = \alpha \Delta T E$

Where σ is the thermal stress in MPa, E is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of x as shown, the following calculations will be made:

$$\text{That means: } \delta T = x + \delta P$$

$$\text{Or that: } \alpha L \Delta T = x + \sigma L / E$$

Where σ represents the thermal stress.

Keep in mind that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

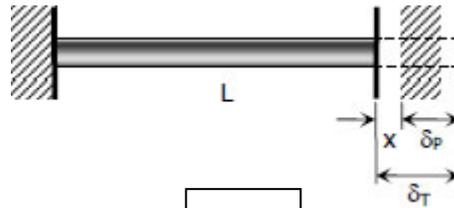
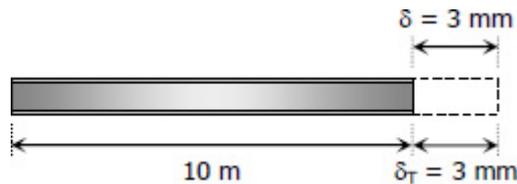


Fig. 9

Problem 3

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume $\alpha = 11.7 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$ and $E = 200 \text{ GPa}$.



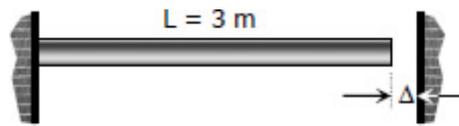
$$\delta T = \alpha L (T_f - T_i) = \alpha L \Delta T$$

Answer 1: $T_f = 40.64 \text{ }^\circ\text{C}$

Answer 2: $\sigma = 60 \text{ MPa}$

Problem 4

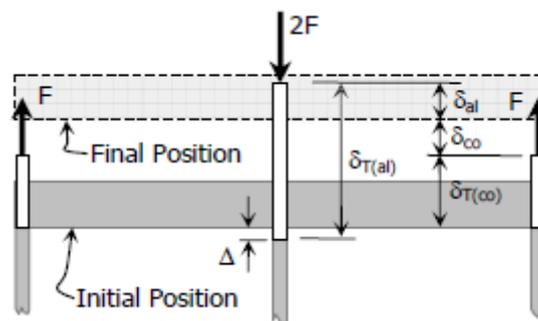
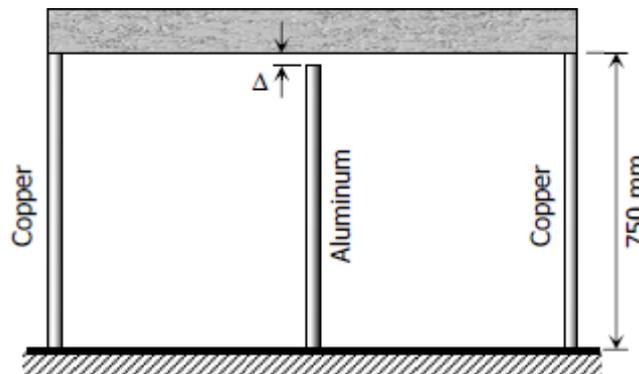
A bronze bar 3 m long with a cross sectional area of 320 mm² is placed between two rigid walls as shown. At a temperature of -20°C, the gap $\Delta = 25 \text{ mm}$. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use $\alpha = 18.0 \times 10^{-6} \text{ m}/(\text{m}\cdot^\circ\text{C})$ and $E = 80 \text{ GPa}$.



Answer : 50.6 °C

Problem 5:

There is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, $\Delta = 0.18 \text{ mm}$. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar, $A = 500 \text{ mm}^2$, $E = 120 \text{ GPa}$, and $\alpha = 16.8 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$. For the aluminum bar, $A = 400 \text{ mm}^2$, $E = 70 \text{ GPa}$, and $\alpha = 23.1 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$.



Answer :

$$\Sigma_{C_0} = 12.14 \text{ MPa}$$

$$\sigma_{Al} = 30.35 \text{ MPa}$$

5. Elastic constants

An elastic modulus, or modulus of elasticity, is the mathematical description of an object or substance's tendency to be deformed elastically (i.e., non-permanently) when a force is applied to it. The elastic modulus of an object is defined as the slope of its stress–strain curve in the elastic deformation region. As such, a stiffer material will have a higher elastic modulus.

$$\lambda = \text{Stress/Strain}$$

where lambda (λ) is the elastic modulus; stress is the restoring force caused due to the deformation divided by the area to which the force is applied; and strain is the ratio of the change caused by the stress to the original state of the object. If stress is measured in Pascal's, since strain is a dimensionless quantity, then the units of λ are Pascal's as well.

Since the denominator becomes unity if length is doubled, the elastic modulus becomes the stress induced in the material, when the sample of the material turns double of its original length on applying external force. While this endpoint is not

realistic because most materials will fail before reaching it, it is practical, in that small fractions of the defining load will operate in exactly the same ratio. Thus, for steel with a Young's modulus of 30 million psi, a 30 thousand psi load will elongate a 1 inch bar by one thousandth of an inch; similarly, for metric units, where a thousandth of the modulus in gigapascals will change a meter by a millimeter.

Specifying how stress and strain are to be measured, including directions, allows for many types of elastic moduli to be defined. The three primary ones are:

- * Young's modulus (E) describes tensile elasticity, or the tendency of an object to deform along an axis when opposing forces are applied along that axis; it is defined as the ratio of tensile stress to tensile strain. It is often referred to simply as the elastic modulus.

- * The shear modulus or modulus of rigidity (G or μ) describes an object's tendency to shear (the deformation of shape at constant volume) when acted upon by opposing forces; it is defined as shear stress over shear strain. The shear modulus is part of the derivation of viscosity.

- * The bulk modulus (K) describes volumetric elasticity, or the tendency of an object to deform in all directions when uniformly loaded in all directions; it is defined as volumetric stress over volumetric strain, and is the inverse of compressibility. The bulk modulus is an extension of Young's modulus to three dimensions.

Three other elastic moduli are Poisson's ratio, Lamé's first parameter, and P-wave modulus.

Young's modulus is a measure of the stiffness of an elastic material and is a quantity used to characterize materials. It is defined as the ratio of the uniaxial stress over the uniaxial strain in the range of stress in which Hooke's Law holds. In solid mechanics, the slope of the stress-strain curve at any point is called the tangent modulus. The tangent modulus of the initial, linear portion of a stress-strain curve is called Young's modulus, also known as the tensile modulus. It can be experimentally determined from the slope of a stress-strain curve created during tensile tests conducted on a sample of the material. In anisotropic materials, Young's modulus may have different values depending on the direction of the applied force with respect to the material's structure.

It is also commonly called the elastic modulus or modulus of elasticity, because Young's modulus is the most common elastic modulus used, but there are other elastic moduli measured, too, such as the bulk modulus and the shear modulus.

Young's modulus is named after Thomas Young, the 19th century British scientist. However, the concept was developed in 1727 by Leonhard Euler, and the first experiments that used the concept of Young's modulus in its current form were performed by the Italian scientist Giordano Riccati in 1782, predating Young's work by 25 years.

Young's modulus is the ratio of stress, which has units of pressure, to strain, which is dimensionless; therefore, Young's modulus has units of pressure.

The SI unit of modulus of elasticity (E, or less commonly Y) is the pascal (Pa or N/m² or m⁻¹·kg·s⁻²). The practical units used are megapascals (MPa or N/mm²) or gigapascals (GPa or kN/mm²). In United States customary units, it is expressed as pounds (force) per square inch (psi).

Young's modulus, E, can be calculated by dividing the tensile stress by the tensile strain in the elastic (initial, linear) portion of the stress-strain curve:

$$E = \text{tensile stress} / \text{tensile strain} = \sigma / \varepsilon = F / A_0 / \Delta L / L_0 = F L_0 / A_0 \Delta L$$

Where

E is the Young's modulus (modulus of elasticity)

F is the force exerted on an object under tension;

A₀ is the original cross-sectional area through which the force is applied;

ΔL is the amount by which the length of the object changes;

L₀ is the original length of the object.

$$F = - E A_0 \Delta L / L_0$$

$$E A_0 / L_0 = \text{Constant} \quad \text{and} \quad \Delta L = x$$

Then Hooke's law can be derived from this formula, which describes the stiffness of an ideal spring:

$$F = -k x$$

Stress strain curve

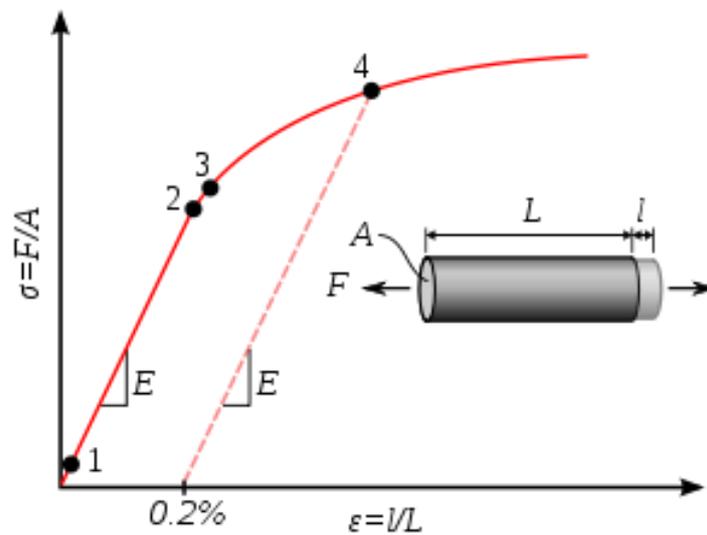


Fig. 8

Illustration of offset yield point. Key:

1: True elastic limit

2: Proportionality limit

3: Elastic limit

4: Offset yield strength, usually defined at $e=0.2\%$

S: Engineering stress

e: Engineering strain

A: Undeformed cross-sectional area

F: Uniaxial load

L: Underformed length

l: Elongation

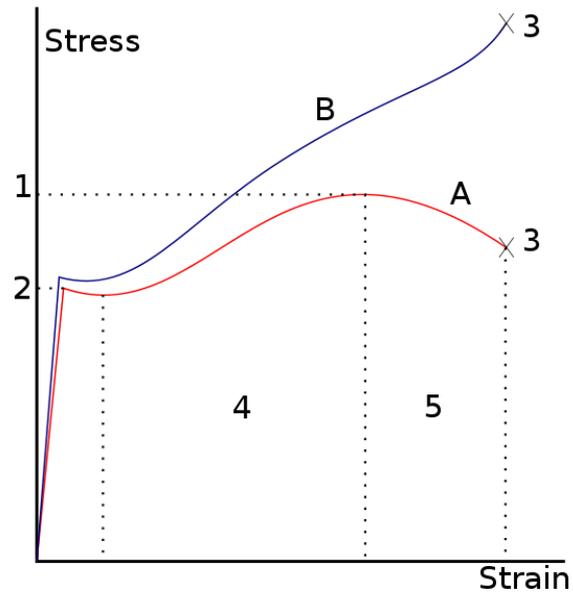


Fig. 9

Stress vs. Strain curve for structural steel. Reference numbers are:

- 1 - Ultimate Strength
- 2 - Yield Strength(elastic limit)
- 3 - Rupture
- 4 - Strain hardening region
- 5 - Necking region

- A: Apparent stress (F/A_0)
- B: Actual stress (F/A)

6. Plastic deformation

For most metallic materials, elastic deformation persists only to strains of about 0.005. As the material is deformed beyond this point, the stress is no longer proportional to strain (Hooke's law, ceases to be valid), and permanent, nonrecoverable, or plastic deformation occurs. Figure 10 a. plots schematically the tensile stress–strain behavior into the plastic region for a typical metal. The transition from elastic to plastic is a gradual one for most metals; some curvature results at the onset of plastic deformation, which increases more rapidly with rising stress.

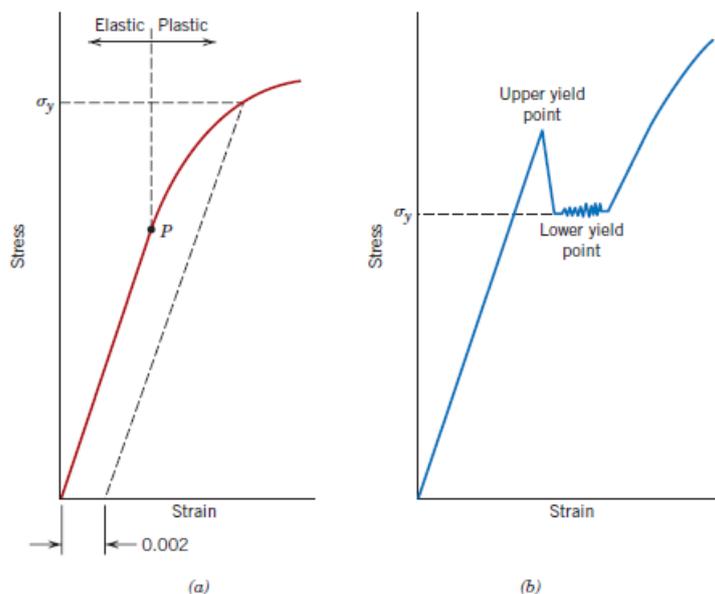


Fig. 10

From an atomic perspective, plastic deformation corresponds to the breaking of bonds with original atom neighbors and then reforming bonds with new neighbors as large numbers of atoms or molecules move relative to one another; upon removal of the stress they do not return to their original positions. The mechanism of this deformation is different for crystalline and amorphous materials. For crystalline solids, deformation

is accomplished by means of a process called slip, which involves the motion of dislocations as discussed in Section 7.2. Plastic deformation in noncrystalline solids (as well as liquids) occurs by a viscous flow mechanism.

TENSILE PROPERTIES

Yielding and Yield Strength

Most structures are designed to ensure that only elastic deformation will result when a stress is applied. A structure or component that has plastically deformed, or experienced a permanent change in shape, may not be capable of functioning as intended. It is therefore desirable to know the stress level at which plastic deformation begins, or where the phenomenon of yielding occurs. For metals that experience this gradual elastic–plastic transition, the point of yielding may be determined as the initial departure from linearity of the stress–strain curve; this is sometimes called the proportional limit, as indicated by point P in Figure 10 a. In such cases the position of this point may not be determined precisely. As a consequence, a convention has been established wherein a straight line is constructed parallel to the elastic portion of the stress–strain curve at some specified strain offset, usually 0.002. The stress corresponding to the intersection of this line and the stress–strain curve as it bends over in the plastic region is defined as the yield strength σ_y . This is demonstrated in Figure 10 a. Of course, the units of yield strength are MPa or psi. For those materials having a nonlinear elastic region, use of the strain offset method is not possible, and the usual practice is to define the yield strength as the stress required to produce some

amount of strain (e.g., $\epsilon=0.005$). Some steels and other materials exhibit the tensile stress–strain behavior as shown in Figure 10 b. The elastic–plastic transition is very well defined and occurs abruptly in what is termed a yield point phenomenon. At the upper yield point, plastic deformation is initiated with an actual decrease in stress. Continued deformation fluctuates slightly about some constant stress value, termed the lower yield point; stress subsequently rises with increasing strain. For metals that display this effect, the yield strength is taken as the average stress that is associated with the lower yield point, since it is well defined and relatively insensitive to the testing procedure. Thus, it is not necessary to employ the strain offset method for these materials. The magnitude of the yield strength for a metal is a measure of its resistance to plastic deformation. Yield strengths may range from 35 MPa (5000 psi) for a low-strength aluminum to over 1400 MPa (200,000 psi) for high-strength steels.

Tensile Strength

After yielding, the stress necessary to continue plastic deformation in metals increases to a maximum, point M in Figure 6.11, and then decreases to the eventual fracture, point F. The tensile strength TS (MPa or psi) is the stress at the maximum on the engineering stress–strain curve (Figure 6.11). This corresponds to the maximum stress that can be sustained by a structure in tension; if this stress is applied and maintained, fracture will result. All deformation up to this point is uniform throughout the narrow region of the tensile specimen. However, at this maximum stress, a small constriction or neck begins to form at some point, and all subsequent deformation is confined at

this neck, as indicated by the schematic specimen insets in Figure 11. This phenomenon is termed “necking,” and fracture ultimately occurs at the neck. The fracture strength corresponds to the stress at fracture. Tensile strengths may vary anywhere from 50 MPa (7000 psi) for an aluminum to as high as 3000 MPa (450,000 psi) for the high-strength steels. Ordinarily, when the strength of a metal is cited for design purposes, the yield strength is used. This is because by the time a stress corresponding to the tensile strength has been applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering design purposes.

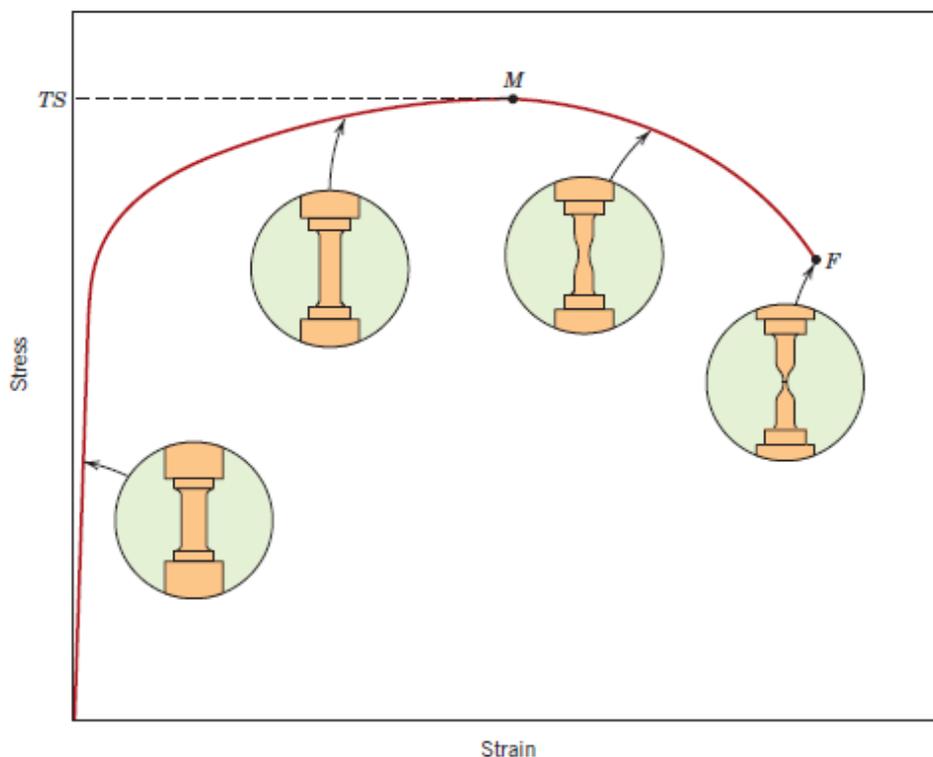


Fig. 11

Ductility

Ductility is another important mechanical property. It is a measure of the degree of plastic deformation that has been sustained at fracture. A material that experiences very little or no plastic deformation upon fracture is termed brittle. The tensile stress - strain behaviors for both ductile and brittle materials are schematically illustrated in Figure 12.

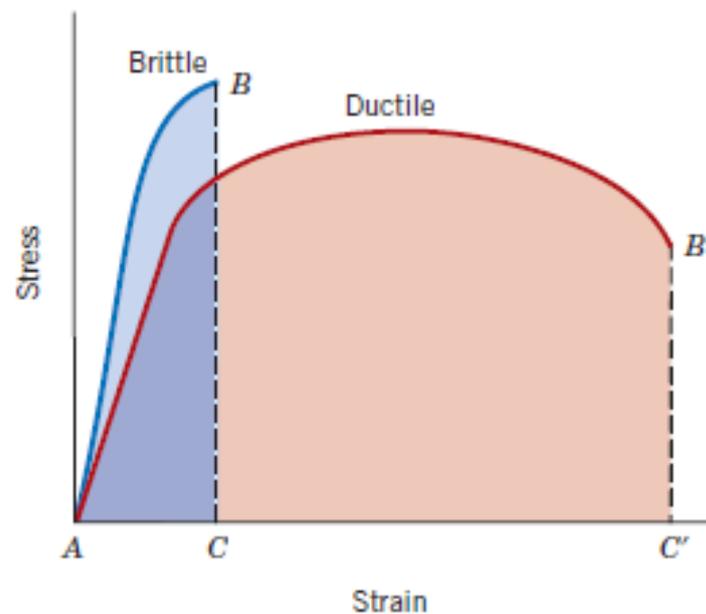


Fig. 12

Ductility may be expressed quantitatively as either percent elongation or percent reduction in area. The percent elongation %EL is the percentage of plastic strain at fracture, or

$$\%EL = \left(\frac{l_f - l_0}{l_0} \right) \times 100$$

where l_f is the fracture length and l_0 is the original gauge length as above. Inasmuch as a significant proportion of the plastic deformation at fracture is confined to the neck region, the magnitude of %EL will depend on specimen gauge length. The shorter l_0 the greater is the fraction of total elongation from the neck and, consequently, the higher the value of %EL. Therefore, l_0 should be specified when percent elongation values are cited; it is commonly 50 mm (2 in.).

Percent reduction in area %RA is defined as

$$\%RA = \left(\frac{A_0 - A_f}{A_0} \right) \times 100$$

where A_0 is the original cross-sectional area and A_f is the cross-sectional area at the point of fracture. Percent reduction in area values are independent of both l_0 and A_0 . Furthermore, for a given material the magnitudes of %EL and %RA will, in general, be different. Most metals possess at least a moderate degree of ductility at room temperature; however, some become brittle as the temperature is lowered. A knowledge of the ductility of materials is important for at least two reasons. First, it indicates to a designer the degree to which a structure will deform plastically before fracture. Second, it specifies the degree of allowable deformation during fabrication operations. We sometimes refer to relatively ductile materials as being “forgiving,” in the sense that they may experience local deformation without fracture should there be an error in the magnitude of the design stress calculation. Brittle materials are approximately considered to be those having a fracture strain of less than about 5%.

Thus, several important mechanical properties of metals may be determined from tensile stress–strain tests.

As with modulus of elasticity, the magnitudes of both yield and tensile strengths decline with increasing temperature; just the reverse holds for ductility—it usually increases with temperature. Figure 13 shows how the stress–strain behavior of iron varies with temperature.

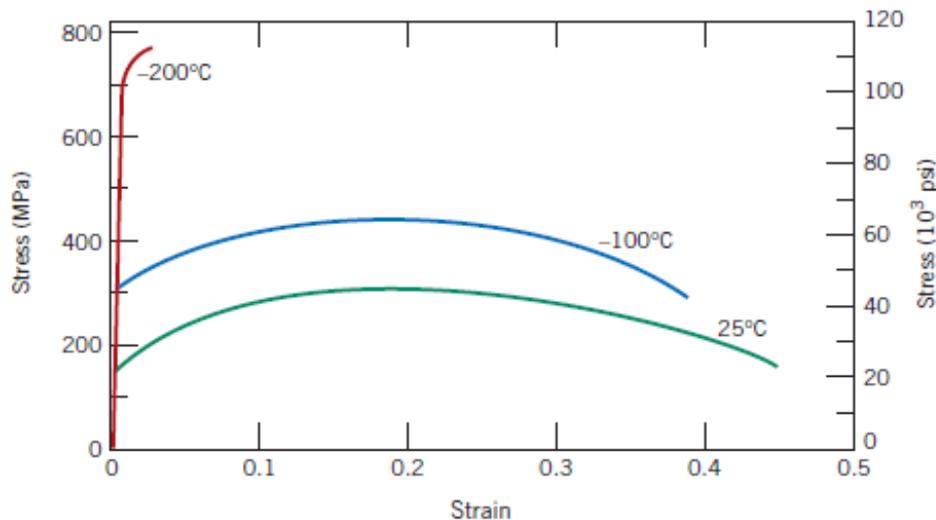


Fig. 13

Resilience

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered. The associated property is the modulus of resilience, which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding. Computationally, the

modulus of resilience for a specimen subjected to a uniaxial tension test is just the area under the engineering stress–strain curve taken to yielding (Figure 14), or

$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon$$

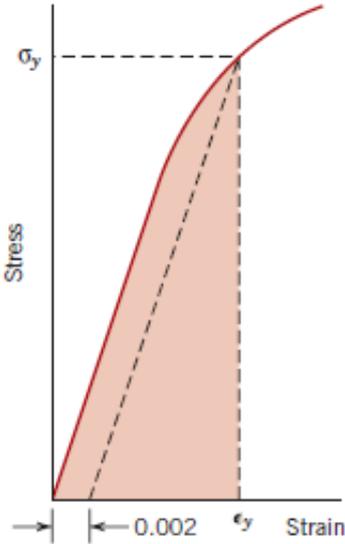


Fig. 14

PROPORTIONAL LIMIT (HOOKE'S LAW)

From the origin O to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called Hooke's Law that within the proportional limit, the stress is directly proportional to strain or



Robert Hooke

$$\sigma \propto \epsilon \text{ or } \sigma = k \epsilon$$

The constant of proportionality k is called the Modulus of Elasticity E or Young's Modulus and is equal to the slope of the stress-strain diagram from O to P. Then

$$\sigma = E \epsilon$$

ELASTIC LIMIT

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may be developed such that there is no permanent or residual deformation when the load is entirely removed.

ELASTIC AND PLASTIC RANGES

The region in stress-strain diagram from O to P is called the elastic range. The region from P to R is called the plastic range.

YIELD POINT

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

ULTIMATE STRENGTH

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

RAPTURE STRENGTH

Rapture strength is the strength of the material at rapture. This is also known as the breaking strength.

MODULUS OF RESILIENCE

Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from O to P, in Nm/m³. This may be calculated as the area under the stress-strain curve from the origin O to up to the elastic limit E (the shaded area in the figure). The resilience of the material is its ability to absorb energy without creating a permanent distortion.

MODULUS OF TOUGHNESS

Modulus of toughness is the work done on a unit volume of material as the force is gradually increased from O to R, in Nm/m³. This may be calculated as the area under the entire stress-strain curve (from O to R). The toughness of a material is its ability to absorb energy without causing it to break.

WORKING STRESS, ALLOWABLE STRESS, AND FACTOR OF SAFETY

Working stress is defined as the actual stress of a material under a given loading. The maximum safe stress that a material can carry is termed as the allowable stress. The allowable stress should be limited to values not exceeding the proportional limit. However, since proportional limit is difficult to determine accurately, the allowable stress is taken as either the yield point or ultimate strength divided by a factor of safety. The ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety.

AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the stress is proportional to strain and is given by

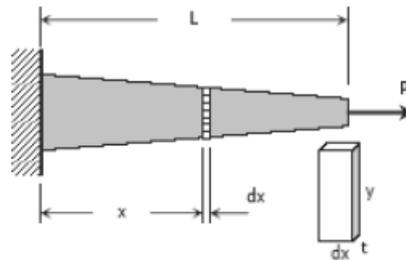
$$\sigma = E\varepsilon$$

since $\sigma = P / A$ and $\varepsilon = \delta / L$, then $P / A = E \delta / L$. Solving for δ ,

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit. If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.

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$$\delta = \frac{P}{E} \int_0^L \frac{dx}{A}$$

where $A = ty$ and y and t , if variable, must be expressed in terms of x .

For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is

$$\delta = \frac{\rho g L^2}{2E} = \frac{M g L}{2AE}$$

where ρ is in kg/m^3 , L is the length of the rod in mm , M is the total mass of the rod in kg , A is the cross-sectional area of the rod in mm^2 , and $g = 9.81 \text{ m/s}^2$.

STIFFNESS, k

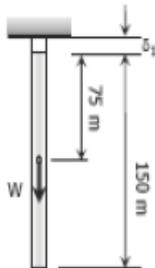
Stiffness is the ratio of the steady force acting on an elastic body to the resulting displacement. It has the unit of N/mm .

$$k = P / \delta$$

Problem 206

A steel rod having a cross-sectional area of 300 mm² and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m³ and $E = 200 \times 10^3$ MN/m², find the total elongation of the rod.

Solution 206



Let δ = total elongation

δ_1 = elongation due to its own weight

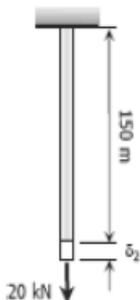
δ_2 = elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: $P = W = 7850(1/1000)3(9.81)[300(150)(1000)]$
 $P = 3465.3825$ N
 $L = 75(1000) = 75\,000$ mm
 $A = 300$ mm²
 $E = 200\,000$ MPa

$$\delta_1 = \frac{3465.3825(75000)}{300(200\,000)} = 4.33 \text{ mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: $P = 20 \text{ kN} = 20\,000$ N
 $L = 150 \text{ m} = 150\,000$ mm
 $A = 300$ mm²
 $E = 200\,000$ MPa

$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

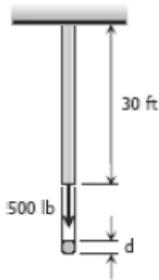
Total elongation:

$$\delta = 4.33 + 50 = 54.33 \text{ mm}$$

Problem 207

A steel wire 30 ft long, hanging vertically, supports a load of 500 lb. Neglecting the weight of the wire, determine the required diameter if the stress is not to exceed 20 ksi and the total elongation is not to exceed 0.20 in. Assume $E = 29 \times 10^6$ psi.

Solution 207



Based on maximum allowable stress:

$$\sigma = \frac{P}{A}$$

$$20\,000 = \frac{500}{\frac{1}{4}\pi d^2}$$

$$d = 0.0318 \text{ in}$$

Based on maximum allowable deformation:

$$\delta = \frac{PL}{AE}$$

$$0.20 = \frac{500(30 \times 12)}{\frac{1}{4}\pi d^2 (29 \times 10^6)}$$

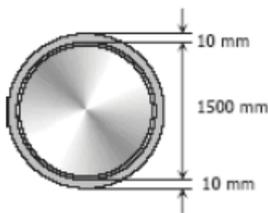
$$d = 0.0395 \text{ in}$$

Use the bigger diameter, $d = 0.0395 \text{ in}$

Problem 208

A steel tire, 10 mm thick, 80 mm wide, and 1500.0 mm inside diameter, is heated and shrunk onto a steel wheel 1500.5 mm in diameter. If the coefficient of static friction is 0.30, what torque is required to twist the tire relative to the wheel? Neglect the deformation of the wheel. Use $E = 200 \text{ GPa}$.

Solution 208



$$\delta = \frac{PL}{AE}$$

Where: $\delta = \pi(1500.5 - 1500) = 0.5\pi \text{ mm}$
 $P = T$
 $L = 1500\pi \text{ mm}$
 $A = 10(80) = 800 \text{ mm}^2$
 $E = 200\,000 \text{ MPa}$

$$0.5\pi = \frac{T(1500\pi)}{800(200\,000)}$$

$$T = 53\,333.33 \text{ N}$$

$$F = 2T$$

$$p(1500)(80) = 2(53\,333.33)$$

$$p = 0.8889 \text{ MPa} \rightarrow \text{internal pressure}$$

Total normal force, N :

$$N = p \times \text{contact area between tire and wheel}$$

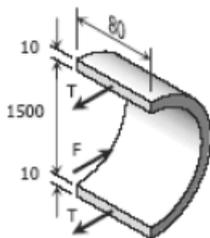
$$N = 0.8889 \times \pi(1500.5)(80)$$

$$N = 335\,214.92 \text{ N}$$

Friction resistance, f :

$$f = \mu N = 0.30(335\,214.92)$$

$$f = 100\,564.48 \text{ N} = 100.56 \text{ kN}$$



$$\text{Torque} = f \times \frac{1}{2} (\text{diameter of wheel})$$

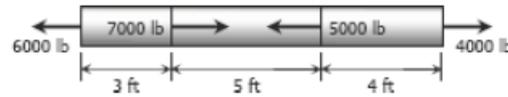
$$\text{Torque} = 100.56 \times 0.75025$$

$$\text{Torque} = 75.44 \text{ kN}\cdot\text{m}$$

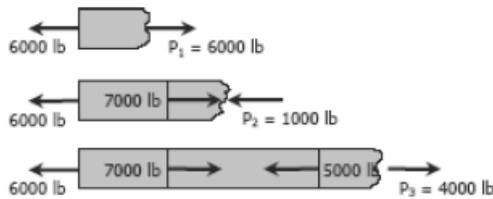
Problem 209

An aluminum bar having a cross-sectional area of 0.5 in^2 carries the axial loads applied at the positions shown in Fig. P-209. Compute the total change in length of the bar if $E = 10 \times 10^6 \text{ psi}$. Assume the bar is suitably braced to prevent lateral buckling.

Figure P-209 and P-210



Solution 209



$P_1 = 6000 \text{ lb tension}$
 $P_2 = 1000 \text{ lb compression}$
 $P_3 = 4000 \text{ lb tension}$

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

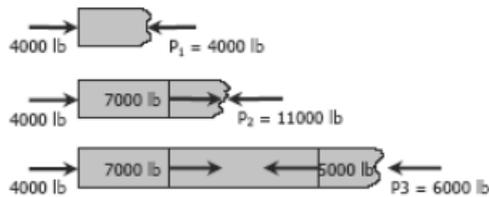
$$\delta = \frac{6000(3 \times 12)}{0.5(10 \times 10^6)} - \frac{1000(5 \times 12)}{0.5(10 \times 10^6)} + \frac{4000(4 \times 12)}{0.5(10 \times 10^6)}$$

$$\delta = 0.0696 \text{ in (lengthening)}$$

Problem 210

Solve Prob. 209 if the points of application of the 6000-lb and the 4000-lb forces are interchanged.

Solution 210



$P_1 = 4000 \text{ lb compression}$
 $P_2 = 11000 \text{ lb compression}$
 $P_3 = 6000 \text{ lb compression}$

$$\delta = \frac{PL}{AE}$$

$$\delta = -\delta_1 - \delta_2 - \delta_3$$

$$\delta = -\frac{4000(3 \times 12)}{0.5(10 \times 10^6)} - \frac{11000(5 \times 12)}{0.5(10 \times 10^6)} - \frac{6000(4 \times 12)}{0.5(10 \times 10^6)}$$

$$\delta = -0.19248 \text{ in} = 0.19248 \text{ in (shortening)}$$

Problem 211

A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. P-211. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $E_{br} = 83$ GPa.

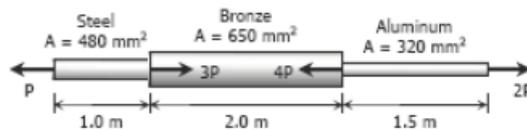


Figure P -211

Solution 211

Based on allowable stresses:

Steel:

$$P_{st} = \sigma_{st} A_{st}$$

$$P = 140(480) = 67\,200 \text{ N}$$

$$P = 67.2 \text{ kN}$$

Bronze:

$$P_{br} = \sigma_{br} A_{br}$$

$$2P = 120(650) = 78\,000$$

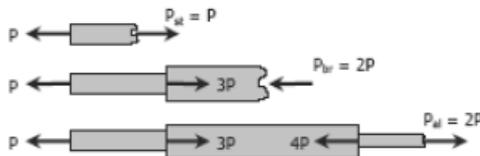
$$P = 39\,000 \text{ N} = 39 \text{ kN}$$

Aluminum:

$$P_{al} = \sigma_{al} A_{al}$$

$$2P = 80(320) = 25\,600 \text{ N}$$

$$P = 12\,800 \text{ N} = 12.8 \text{ kN}$$



Based on allowable deformation:

(steel and aluminum lengthens, bronze shortens)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$

$$3 = \frac{P(1000)}{480(200000)} - \frac{2P(2000)}{650(70000)} + \frac{2P(1500)}{320(83000)}$$

$$3 = \left(\frac{1}{96000} - \frac{1}{11375} + \frac{3}{26560} \right) P$$

$$P = 84\,610.99 \text{ N} = 84.61 \text{ kN}$$

Use the smallest value of P , $P = 12.8 \text{ kN}$

