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**DEPARTMENT OF ELECTRICAL ENGINEERING**

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# **ELECTRICAL MACHINES I**

- **Module-II: AC Transformers**
  - *Single phase transformers*
  - *Three-phase transformers*
  - *Auto-transformers*

## **Recommended textbooks**

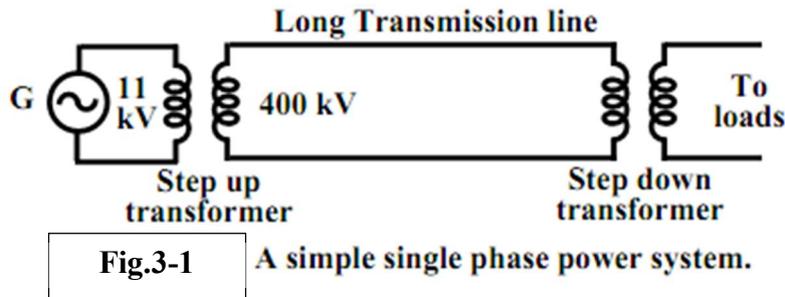
- *M.G. Say & E.O. Taylor, "Direct Current Machines", Pitman Pub.*
- *Hughes, "Electrical Technology", Prentice Hall*
- *B.L. Theraja, "A Textbook of Electrical Technology", Chand & Company LTD* .

# Operating Principles and Construction

## What is a Transformer?

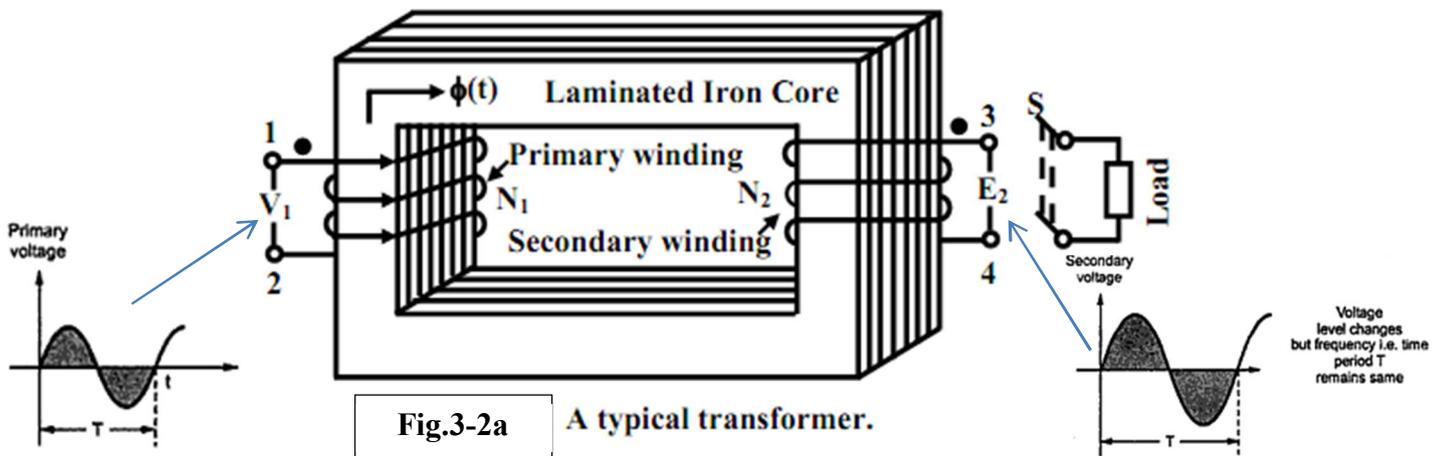
A **transformer** is a static piece of equipment used either for raising or lowering the voltage of an AC supply with a corresponding decrease or increase in current.

The use of transformers in transmission system is shown in the Figure below.



## Principle of Operation

A transformer in its simplest form will consist of a rectangular laminated magnetic structure on which two coils of different number of turns are wound as shown in Figure 3.2a.



The winding to which AC voltage is impressed is called **the primary** of the transformer and the winding across which the load is connected is called **the secondary** of the transformer.

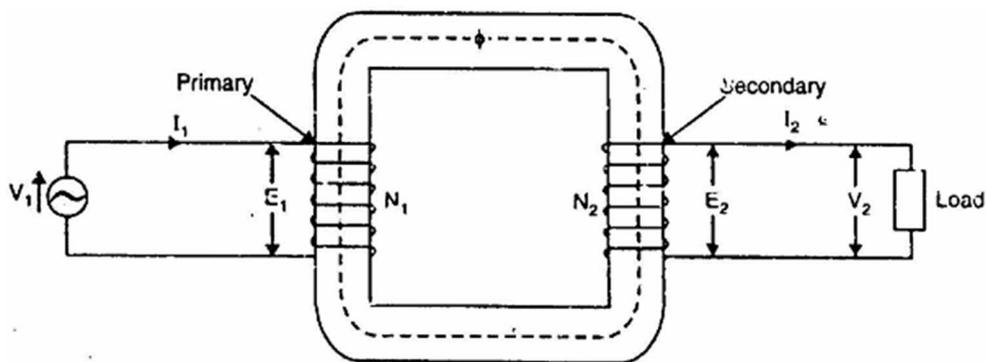


Fig.3-2b

Depending upon the number of turns of the primary ( $N_1$ ) and secondary ( $N_2$ ), an alternating emf ( $E_2$ ) is induced in the secondary. This induced emf ( $E_2$ ) in the secondary causes a secondary current  $I_2$ . Consequently, terminal voltage  $V_2$  will appear across the load. If  $V_2 > V_1$ , it is called a **step up-transformer**. On the other hand, if  $V_2 < V_1$ , it is called a **step-down transformer**.

When an alternating voltage  $V_1$  is applied to the primary, an alternating flux  $\Phi$  is set up in the core. This alternating flux links both the windings and induces emfs  $E_1$  and  $E_2$  in them according to **Faraday's laws of electromagnetic induction**. The emf  $E_1$  is termed as primary emf and emf  $E_2$  is termed as Secondary emf.

Clearly, 
$$E_1 = -N_1 \frac{d\phi}{dt}$$

and 
$$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of  $E_2$  and  $E_1$  depend upon the number of turns on the secondary and primary respectively. If  $N_2 > N_1$ , then  $E_2 > E_1$  (or  $V_2 > V_1$ ) and we get a step-up transformer. On the other hand, if  $N_2 < N_1$ , then  $E_2 < E_1$  (or  $V_2 < V_1$ ) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f.  $E_2$  will cause a current  $I_2$  to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

***The following points may be noted carefully:***

- (i) The transformer action is based on the laws of ***electromagnetic induction***.
- (ii) There is no electrical connection between the primary and secondary.
- (iii) There is no change in frequency i.e., output power has the same frequency as the input power.

## **Can DC Supply be used for Transformers?**

***The DC supply cannot be used for the transformers.*** This is because the transformer works on the principle of ***mutual induction***, for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and transformer will not work

There can be saturation of the core due to which transformer draws very large current from the supply when connected to DC.

***Thus DC supply should not be connected to the transformers.***

## **Construction**

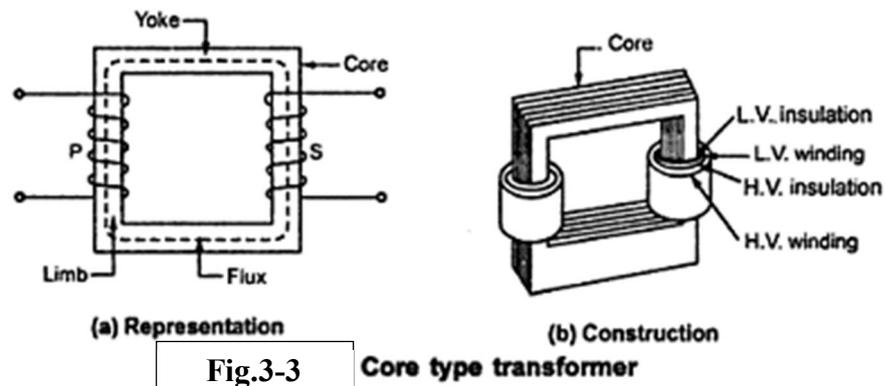
We usually design a power transformer so that it approaches the characteristics of an ideal transformer. To achieve this, following design features are incorporated:

- (i) The core is made of silicon steel which has low hysteresis loss and high permeability. Further, core is laminated in order to reduce eddy current loss. These features considerably reduce the iron losses and the no-load current.
- (ii) Instead of placing primary on one limb and secondary on the other, it is a usual practice to wind one-half of each winding on one limb. This ensures tight coupling between the two windings. Consequently, leakage flux is considerably reduced.

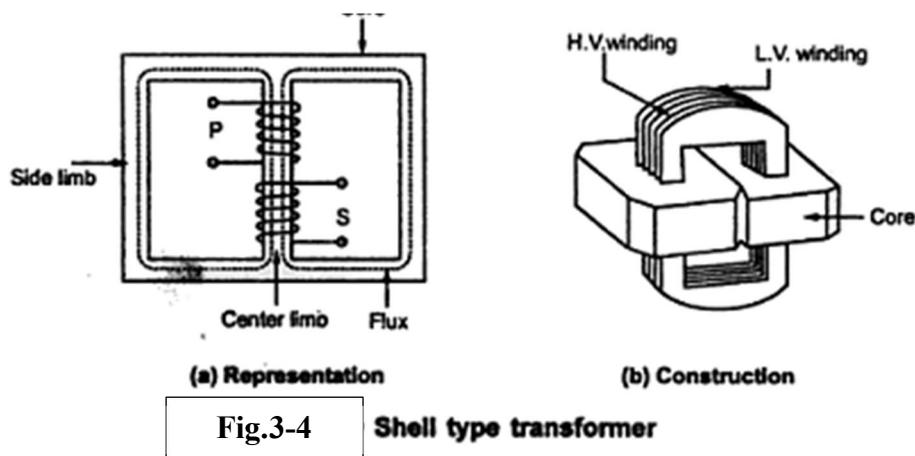
- (iii) The winding resistances are minimized to reduce Copper loss and resulting rise in temperature and to ensure high efficiency.

Transformers are of two types: (i) core-type transformer (see Fig.3-3) and (ii) shell-type transformer (see Fig.3-4).

**Core-Type Transformer:** In a core-type transformer, half of the primary winding and half of the secondary winding are placed round each limb to reduce the leakage flux.



**Shell-Type Transformer:** This method of construction involves the use of a double magnetic circuit. Both the windings are placed round the central limb to ensure a low-reluctance flux path.



## Comparison of Core and Shell Type Transformers

Core Type	Shell Type
The winding encircles the core.	The core encircles most part of the winding
It has single magnetic circuit	It has double magnetic circuit
The core has two limbs	The core has three limbs
The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
The winding are uniformly distributed on two limbs hence natural cooling is effective	The natural cooling does not exist as the windings are surrounded by the core.
Preferred for low voltage transformers.	Preferred for high voltage transformers.

### Cooling of Transformers

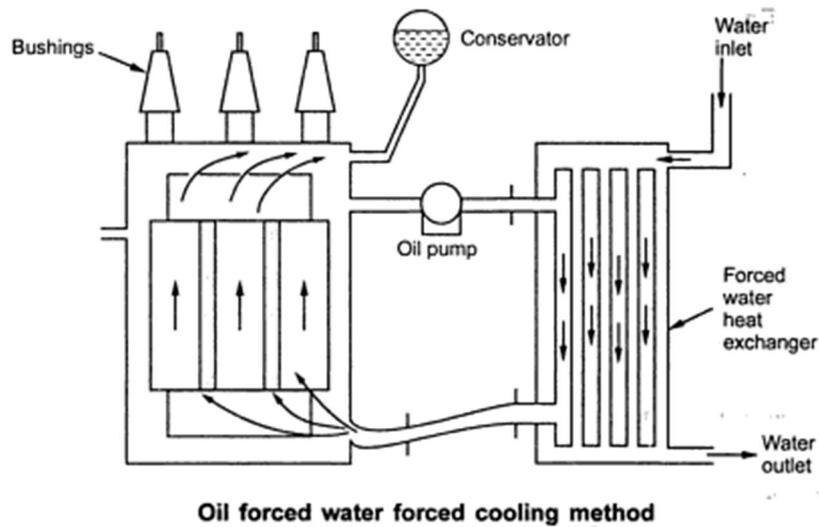
When transformer supplies a load, two types of losses occur inside the transformer. The iron losses occur in the core while copper losses occur in the windings. The power lost due to these losses appears in the form of heat. This heat increases the temperature of the transformer. *To keep the temperature rise of the transformer within limits, a suitable coolant and cooling method is necessary.*

The various cooling methods are designated with depended upon:

A: cooling medium used and B: type of circulation employed.

The various coolant used such as Air, Gas, Mineral oil, and water.

One of cooling method system is shown in figure below which is called ***Oil Forced Water Forced cooling system***;



## **EMF Equation of a Transformer**

Consider that an alternating voltage  $V_1$  of frequency  $f$  is applied to the primary as shown in Fig. 3-2b. The sinusoidal flux  $\Phi$  produced by the primary can be represented as:

$$\phi = \phi_m \sin \omega t$$

The instantaneous e.m.f.  $e_1$  induced in the primary is

$$\begin{aligned} e_1 &= -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt} (\phi_m \sin \omega t) \\ &= -\omega N_1 \phi_m \cos \omega t = -2\pi f N_1 \phi_m \cos \omega t \\ &= 2\pi f N_1 \phi_m \sin(\omega t - 90^\circ) \end{aligned} \quad (i)$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value  $E$  of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

or  $E_1 = 4.44 f N_1 \phi_m$

Similarly  $E_2 = 4.44 f N_2 \phi_m$

In an ideal transformer,  $E_1 = V_1$  and  $E_2 = V_2$ .

*Note.* It is clear from exp. (i) above that e.m.f.  $E_1$  induced in the primary lags behind the flux  $\phi$  by  $90^\circ$ . Likewise, e.m.f.  $E_2$  induced in the secondary lags behind flux  $\phi$  by  $90^\circ$ .

## Voltage Transformation Ratio (K)

From the above equations of induced emf, we have,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The constant  $K$  is called voltage transformation ratio. Thus if  $K = 5$  (i.e.

$N_2/N_1 = 5$ ), then  $E_2 = 5 E_1$ .

## Concept of Ideal Transformer

A transformer is said to be ideal if it satisfies following properties:

- i) It has no losses.
- ii) Its windings have zero resistance.
- iii) Leakage flux is zero i.e. 100 % flux produced by primary links with the secondary.
- iv) Permeability of core is so high that negligible current is required to establish the flux in it.

### **NOTE:**

For an ideal transformer, the primary applied voltage  $V_1$  is same as the primary induced emf  $E_1$  as there are no voltage drops.

For ideal transformer:

(i)  $E_1 = V_1$  and  $E_2 = V_2$  as there is no voltage drop in the windings.

$$\therefore \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(ii) there are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$V_1 I_1 = V_2 I_2$$

$$\text{or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of voltage transformation ratio. This simply means that if we raise the voltage, there is a corresponding decrease of current.

## Volt-Ampere Rating

Transformer rating is specified as the product of voltage and current and called *VA rating*.

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

The full load primary and secondary currents which indicate the safe maximum values of currents which transformer windings can carry can be given as:

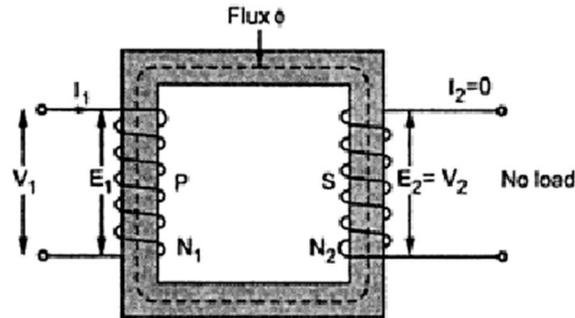
$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \dots \text{ (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

## Ideal Transformer on No Load

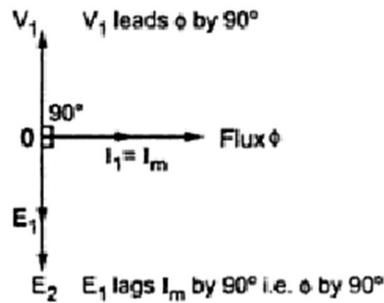
Consider an ideal transformer in Fig. 3-5. For no load  $I_2 = 0$ .  $I_1$  is just necessary to produce flux in the core, which is called *magnetising* current denoted as  $I_m$ .  $I_m$  is very small and lags  $V_1$  by  $90^\circ$  as the winding is purely inductive.

According to Lenz's law, the induced e.m.f opposes the cause producing it which is supply voltage  $V_1$ . Hence  $E_1$  and  $E_2$  are in antiphase with  $V_1$  but equal in magnitude and  $E_1$  and  $E_2$  are in phase.



**Fig.3-5**

This can be illustrated in the phasor diagram as shown below:

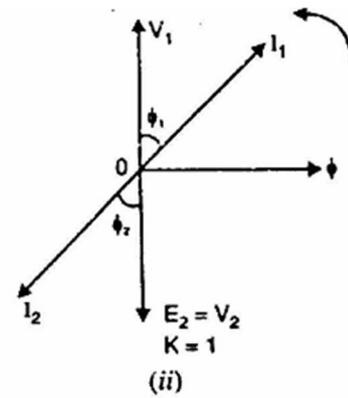
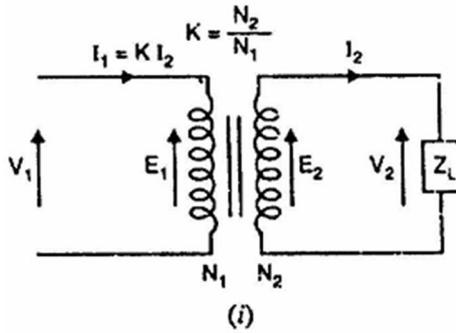


**Phasor diagram for ideal transformer on no load**

### **Ideal Transformer on Load**

Let us connect a load  $Z_L$  across the secondary of an ideal transformer as shown in Figure below:

The secondary emf  $E_2$  will cause a current  $I_2$  to flow through the load:



$$I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$$

The angle at which  $I_2$  leads or lags  $V_2$  (or  $E_2$ ) depends upon the resistance and reactance of the load. In the present case, we have considered inductive load so that current  $I_2$  lags behind  $V_2$  (or  $E_2$ ) by  $\phi_2$ .

The secondary current  $I_2$  sets up an m.m.f.  $N_2 I_2$  which produces a flux in the opposite direction to the flux  $\phi$  originally set up in the primary by the magnetizing current. This will change the flux in the core from the original value. However, the flux in the core should not change from the original value.

Thus when a transformer is loaded and carries a secondary current  $I_2$ , then a current  $I_1$ , ( $= K I_2$ ) must flow in the primary to maintain the m.m.f. balance. In other words, the primary must draw enough current to neutralize the demagnetizing effect of secondary current so that mutual flux  $\phi$  remains constant. Thus as the secondary current increases, the primary current  $I_1$  ( $= K I_2$ ) increases in unison and keeps the mutual flux  $\phi$  constant. The power input, therefore, automatically increases with the output. For example if  $K = 2$  and  $I_2 = 2\text{A}$ , then primary will draw a current  $I_1 = K I_2 = 2 \times 2 = 4\text{A}$ . If secondary current is increased to  $4\text{A}$ , then primary current will become  $I_1 = K I_2 = 2 \times 4 = 8\text{A}$ .

The Phasor diagram for the ideal transformer on load is shown in Figure (ii) above.

The secondary current  $I_2$  lags behind  $V_2$  (or  $E_2$ ) by  $\Phi_2$ . It causes a primary current  $I_1 = K I_2 = I_2$  (for  $K=1$ ) which is in antiphase with it.

(i)  $\phi_1 = \phi_2$

or  $\cos \phi_1 = \cos \phi_2$

Thus, power factor on the primary side is equal to the power factor on the secondary side.

(ii) Since there are no losses in an ideal transformer, input primary power is equal to the secondary output power i.e.,

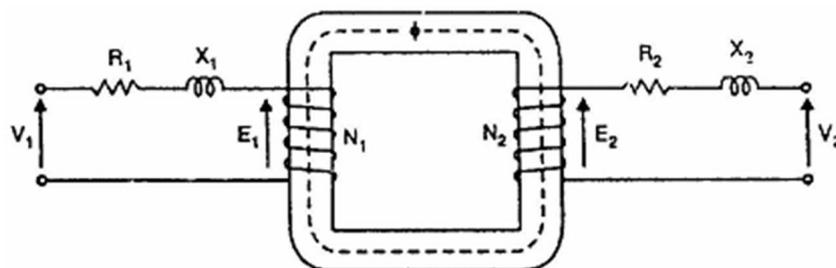
$$V_1 I_1 \cos \phi_1 = V_2 I_2 \cos \phi_2$$

## ✓ Practical Transformer

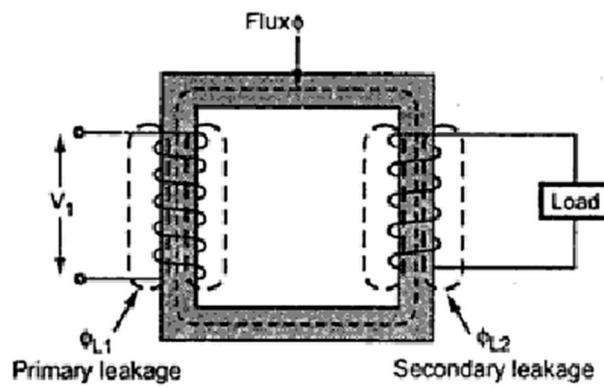
A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) **iron losses** (ii) **winding resistances** and (iii) **magnetic leakage**, giving rise to leakage reactance.

(i) **Iron losses.** Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it.

(ii) **Winding resistances.** Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance  $R_1$  and secondary resistance  $R_2$  act in series with the respective windings as shown below:

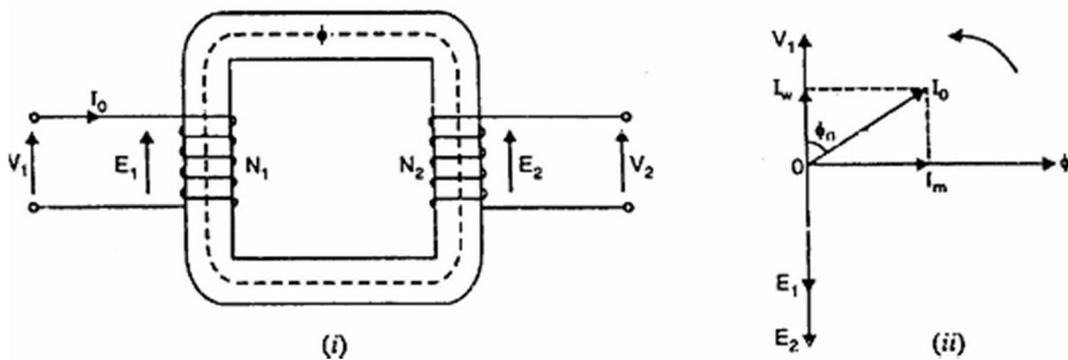


(iii) **Leakage reactance.** Both primary and secondary currents produce flux. The flux  $\Phi$  which links both the windings is the useful flux. However, primary current would produce some flux  $\Phi$  which would not link the secondary winding and is called **mutual flux** (for more information review Lecture Note 2) (See Fig. below).



## Practical Transformer on No Load

Consider the figure below:



The primary will draw a small current  $I_0$  to supply (i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current  $I_0$  is not  $90^\circ$  behind the applied voltage  $V_1$  but lags it by an angle  $\Phi_0 < 90^\circ$  as shown in the phasor diagram.

The no-load primary current  $I_0$  can be resolved into two rectangular components:

- (i) The component  $I_w$  in phase with the applied voltage  $V_1$ . This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_w = I_0 \cos \phi_0$$

ii)

The component  $I_m$  lagging behind  $V_1$  by  $90^\circ$  and is known as magnetizing component. It is this component which produces the mutual flux  $\phi$  in the core.

$$I_m = I_0 \sin \phi_0$$

Clearly,  $I_0$  is phasor sum of  $I_m$  and  $I_w$ ,

$$\therefore I_0 = \sqrt{I_m^2 + I_w^2}$$

No load p.f.,  $\cos \phi_0 = \frac{I_w}{I_0}$

It is emphasized here that no load primary copper loss (i.e.  $I_0^2 R_1$ ) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

No load input power,  $W_0 = \text{Iron loss}$

*Note.* At no load, there is no current in the secondary so that  $V_2 = E_2$ . On the primary side, the drops in  $R_1$  and  $X_1$ , due to  $I_0$  are also very small because of the smallness of  $I_0$ . Hence, we can say that at no load,  $V_1 = E_1$ .

## Practical Transformer on Load

We shall consider two cases (i) when such a transformer is assumed to have no winding resistance and leakage flux (ii) when the transformer has winding resistance and leakage flux.

### No winding resistance and leakage flux

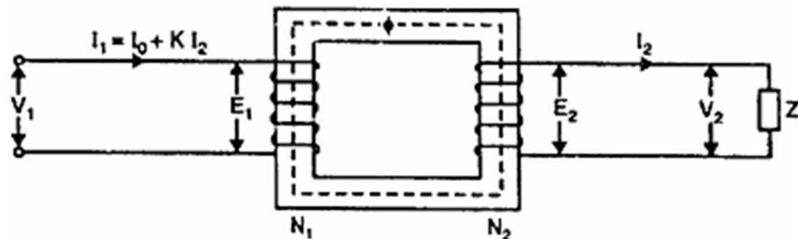


Fig. above shows a practical transformer with the assumption that resistances and leakage reactances of the windings are negligible. With this assumption,  $V_2 = E_2$  and  $V_1 = E_1$ .

Let us take the usual case of inductive load which causes the  $I_2$  to lag  $V_2$  by  $\Phi_2$ . The total primary current  $I_1$  must meet two requirements:

- It must supply the no-load current  $I_0$  to meet the iron losses in the transformer and to provide flux in the core.
- It must supply a current  $I'_2$  to counteract the demagnetizing effect of secondary current  $I_2$ . The magnitude of  $I'_2$  will be such that:

$$N_1 I'_2 = N_2 I_2$$

or 
$$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

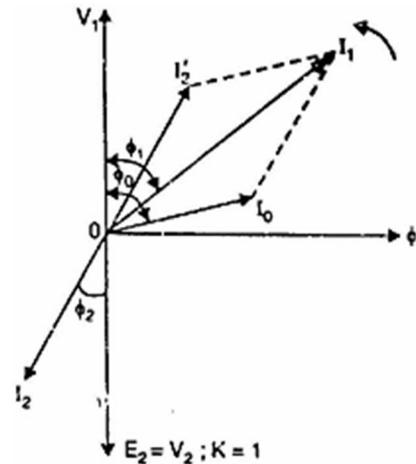
The total primary current  $I_1$  is the phasor sum of  $I'_2$  and  $I_0$  i.e.,

$$I_1 = I'_2 + I_0$$

where  $I'_2 = -K I_2$

Note that  $I'_2$  is  $180^\circ$  out of phase with  $I_2$ .

**Phasor Diagram:** Both  $E_1$  and  $E_2$  lag behind the mutual flux  $\phi$  by  $90^\circ$ . The current  $I'_2$  represents the primary current to neutralize the demagnetizing effect of secondary current  $I_2$ . Now  $I'_2 = K I_2$  and is antiphase with  $I_2$ .  $I_0$  is the no-load current of the transformer. The phasor sum of  $I'_2$  and  $I_0$  gives the total primary current  $I_1$ . Note that in drawing the phasor diagram, the value of  $K$  is assumed to be unity so that primary phasors are equal to secondary phasors.



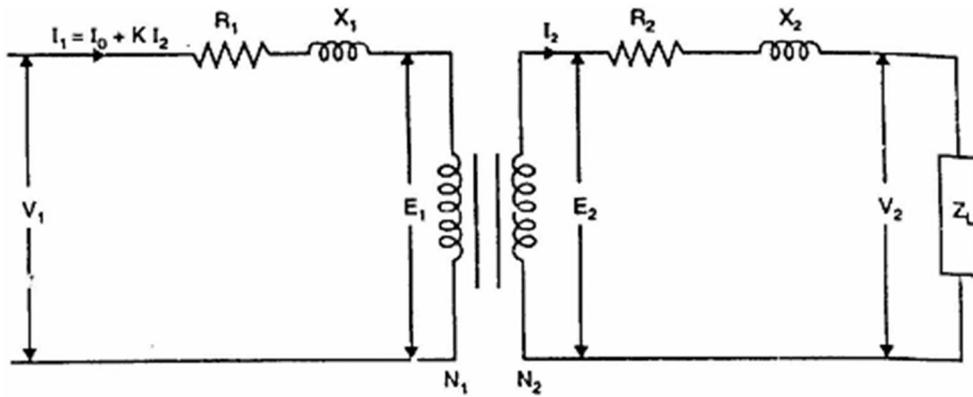
### Transformer with resistance and leakage reactance

The total primary current  $I_1$  must meet **two requirements**:

- It must supply the no-load current  $I_0$  to meet the iron losses in the transformer and to provide flux in the core.
- It must supply a current  $I'_2$  to counteract the demagnetizing effect of secondary current  $I_2$ . The magnitude of  $I'_2$  will be such that:

$$N_1 I'_2 = N_2 I_2$$

or 
$$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$



The total primary current  $I_1$  will be the phasor sum of  $I'_2$  and  $I_0$  i.e.,

$$I_1 = I'_2 + I_0 \quad \text{where} \quad I'_2 = -K I_2$$

$$V_1 = -E_1 + I_1(R_1 + jX_1) \quad \text{where} \quad I_1 = I_0 + (-K I_2)$$

$$= -E_1 + I_1 Z_1$$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$

$$= E_2 - I_2 Z_2$$

### Phasor Diagram:

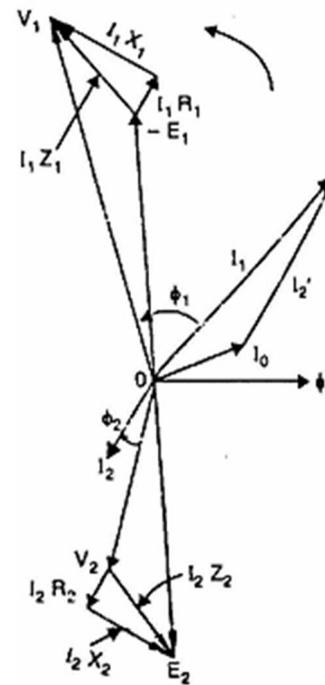
Note that counter emf that opposes the applied voltage  $V_1$  is  $-E_1$ . Therefore, if we add  $I_1 R_1$  (in phase with  $I_1$ ) and  $I_1 X_1$  ( $90^\circ$  ahead of  $I_1$ ) to  $-E_1$ , we get the applied primary voltage  $V_1$ . The phasor  $E_2$  represents the induced emf in the secondary by the mutual flux. The secondary terminal voltage  $V_2$  will be what is left over after subtracting  $I_2 R_2$  and  $I_2 X_2$  from  $E_2$ .

$$\text{Load power factor} = \cos \phi_2$$

$$\text{Primary power factor} = \cos \phi_1$$

$$\text{Input power to transformer, } P_1 = V_1 I_1 \cos \phi_1$$

$$\text{Output power of transformer, } P_2 = V_2 I_2 \cos \phi_2$$

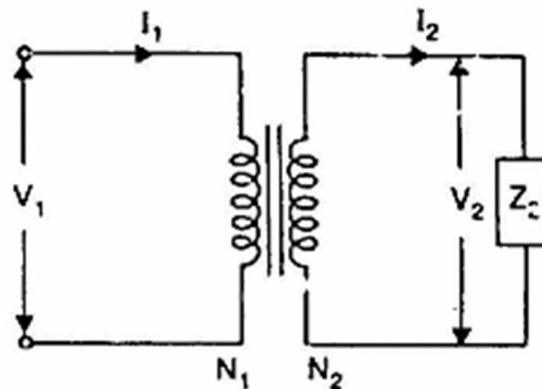


## Modelling and Equivalent Circuits of Single Phase Transformers

The term equivalent circuit of a transformer means the combination of fixed and variable resistances and reactances, which exactly simulates performance and working of the transformer.

### Impedance Ratio

Consider a transformer having impedance  $Z_2$  in the secondary as shown in the figure below:



$$Z_2 = \frac{V_2}{I_2}$$

$$Z_1 = \frac{V_1}{I_1}$$

$$\therefore \frac{Z_2}{Z_1} = \left( \frac{V_2}{V_1} \right) \times \left( \frac{I_1}{I_2} \right)$$

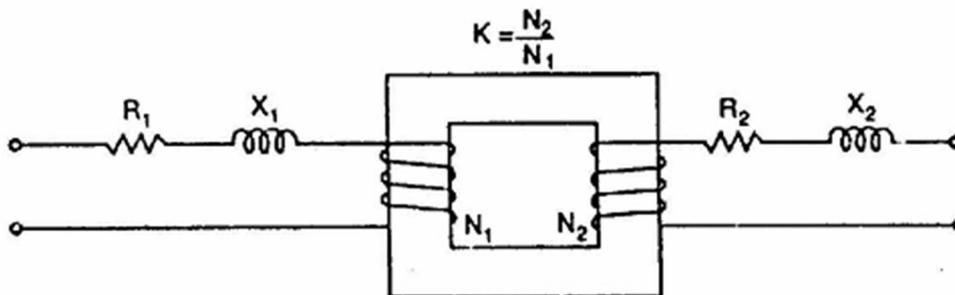
or

$$\boxed{\frac{Z_2}{Z_1} = K^2}$$

## Shifting Impedances in a Transformer

### NOTE:

Consider the following figure:



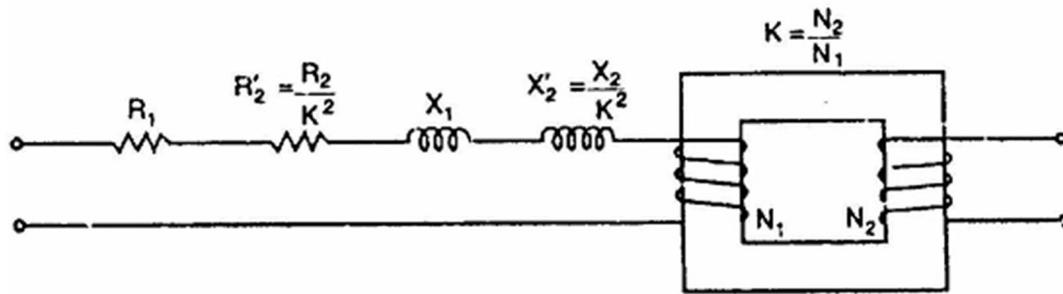
We can transfer the parameters from one winding to the other. Thus:

- ▶ A resistance  $R_1$  in the primary becomes  $K^2 R_1$  *when transferred to the secondary*.
- ▶ A resistance  $R_2$  in the secondary becomes  $R_2/K^2$  **when transferred to the primary**.
- ▶ A reactance  $X_1$  in the primary becomes  $K^2 X_1$  **when transferred to the secondary**.
- ▶ A reactance  $X_2$  in the secondary becomes  $X_2/K^2$  **when transferred to the primary**.

### NOTE:

- ▶ When transferring resistance or reactance from primary to secondary, **multiply it by  $K^2$**
- ▶ When transferring resistance or reactance from secondary to primary, **divide it by  $K^2$**
- ▶ When transferring voltage or current from one winding to the other, **only  $K$  is used**.

## A- Referred to primary



- *Equivalent resistance of transformer referred to primary*

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}$$

- *Equivalent reactance of transformer referred to primary*

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2}$$

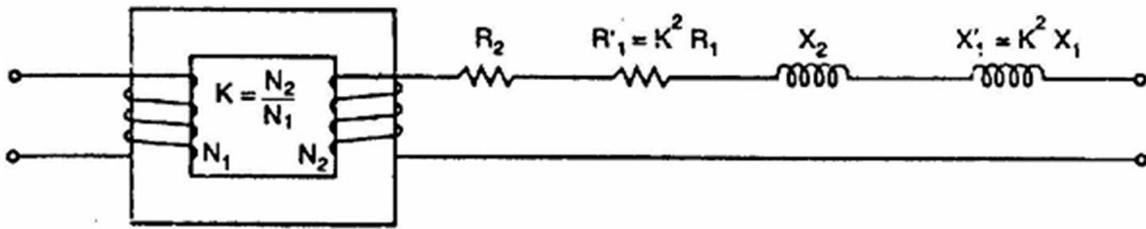
- *Equivalent impedance of transformer referred to primary*

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

- *The value of primary current  $I_1$*

$$I_1 = KI_2$$

## B- Referred to secondary



- *Equivalent resistance of transformer referred to secondary*

$$R_{02} = R_2 + R'_1 = R_2 + K^2 R_1$$

- *Equivalent reactance of transformer referred to secondary*

$$X_{02} = X_2 + X'_1 = X_2 + K^2 X_1$$

- *Equivalent impedance of transformer referred to secondary*

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

- *The value of secondary voltage referred to primary*

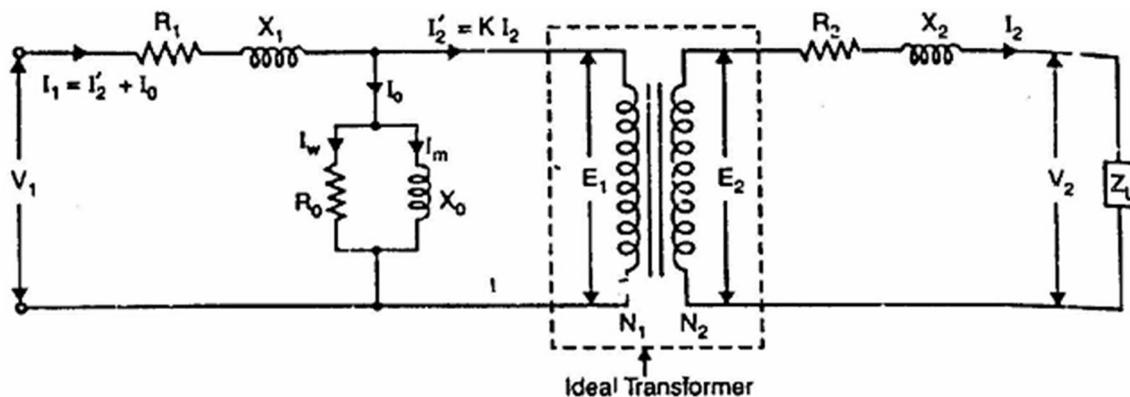
$$V_2 = K V_1$$

## What the Importance of Shifting Impedances?

If we shift all the impedances from one winding to the other, the transformer is eliminated and we get an equivalent electrical circuit. *Various voltages and currents can be readily obtained by solving this electrical circuit.*

## Exact Equivalent Circuit of a Loaded Transformer

The equivalent circuit for the transformer can be represented as shown in the figure.



Where:

$R_1$ : primary winding resistance

$R_2$ : secondary winding resistance

$X_1$ : leakage reactance of primary winding

$X_2$ : leakage reactance of the secondary winding

$R_0$ : represents the core losses (hysteresis and eddy current losses)

$X_0$ : represents magnetising reactance of the core

$I_m$ : magnetizing current (to create magnetic flux in the core)

$I_w$ : active current (required to supply the core losses)

$I_0$  = no load primary current

- **NOTE<sup>1</sup>**: Parallel circuit  $R_0 - X_0$  is the no-load equivalent circuit of the transformer or called exciting circuit.
- **NOTE<sup>2</sup>**: *The equivalent circuit has created two normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:*

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I'_2}{I_2}$$

- **NOTE<sup>3</sup>**: If  $Z_L$  is the external load across the secondary circuit, voltage  $E_2$  induced in the secondary by mutual flux will produce a secondary current  $I_2$ , hence:

$$V_2 = E_2 - I_2(R_2 + jX_2) = E_2 - I_2Z_2$$

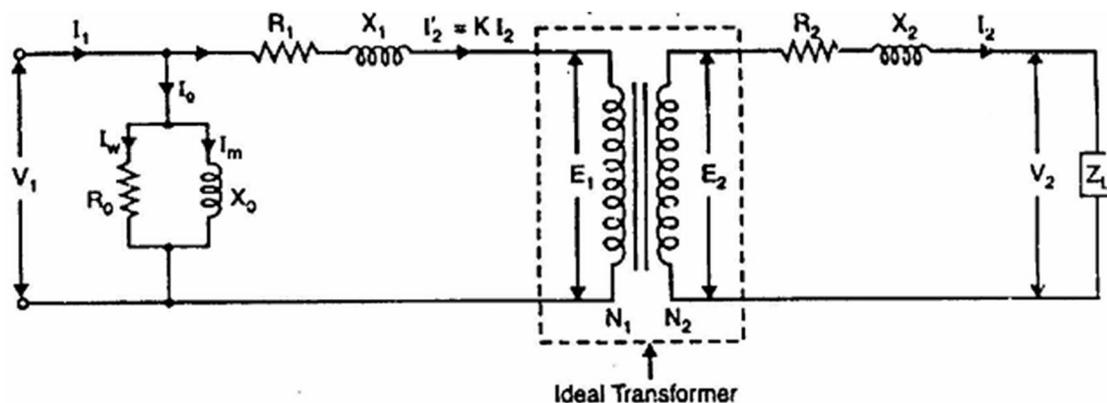
Similarly supply voltage can be given as

$$V_1 = -E_1 + I_1(R_1 + jX_1) = -E_1 + I_1Z_1$$

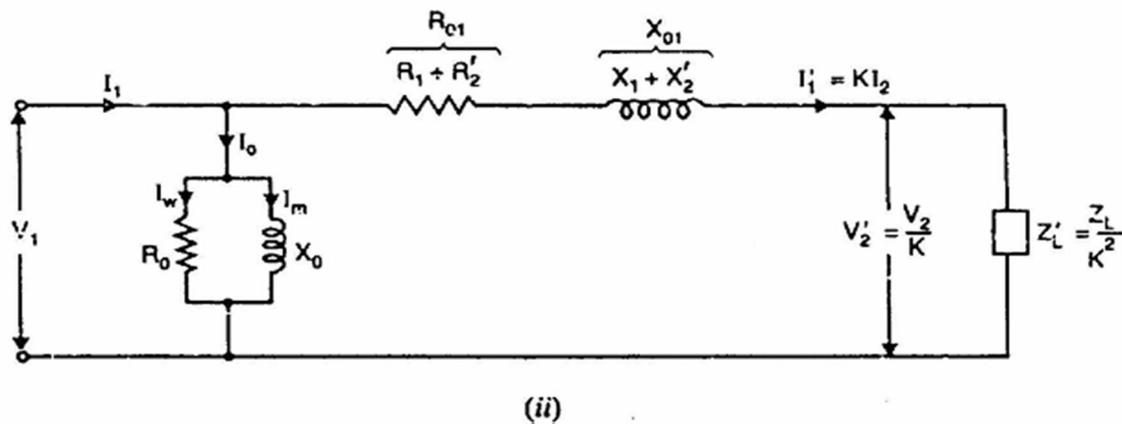
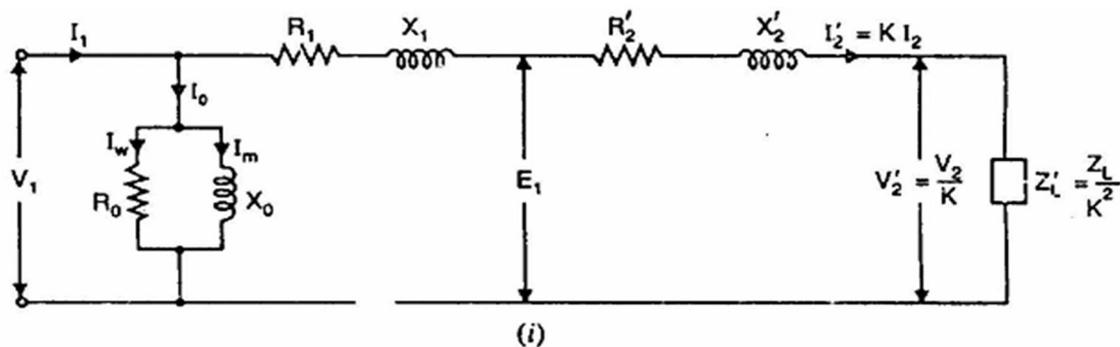
- **NOTE<sup>5</sup>**: When the transformer is loaded to carry the secondary current  $I_2$ , the primary current consists of two components:
  - ▶  $I_0$  to provide magnetizing current and the current required to supply the core losses.
  - ▶ primary current  $I'_2$  ( $= K I_2$ ) required to supply the load connected to the secondary

## Simplified Equivalent Circuit of a Loaded Transformer

If  $I_0$  of a transformer is small as compared to the rated primary current  $I_1$ , voltage drops in  $R_1$  and  $X_1$  due to  $I_0$  are negligible. Hence, the exact equivalent circuit can be simplified by transferring the shunt circuit  $R_0 - X_0$  to the input terminals as shown below:



If all the secondary quantities are referred to the primary, we can get the *simplified equivalent circuit of the transformer referred to the primary* as shown below



From the above circuits:

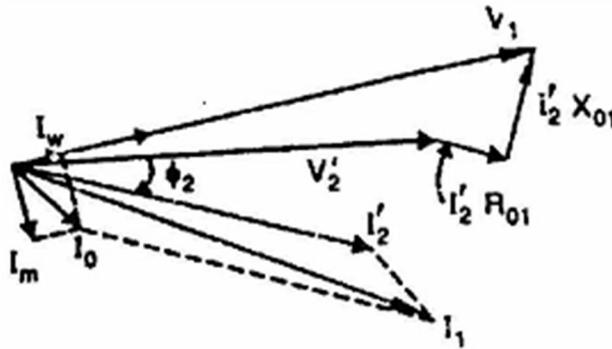
$$R'_2 = \frac{R_2}{K^2}; \quad X'_2 = \frac{X_2}{K^2}; \quad Z'_L = \frac{Z_L}{K^2}; \quad V'_2 = \frac{V_2}{K}; \quad I'_2 = K I_2$$

$$Z_{01} = R_{01} + j X_{01}$$

where

$$R_{01} = R_1 + R'_2; \quad X_{01} = X_1 + X'_2$$

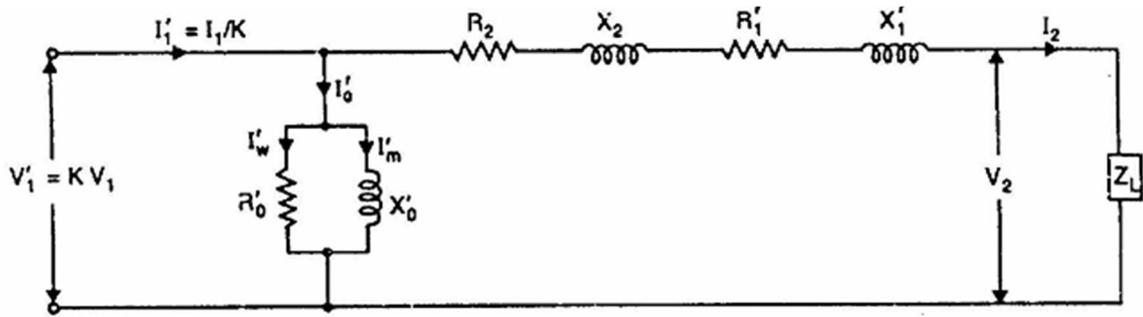
Hence the phasor diagram can be obtained as:



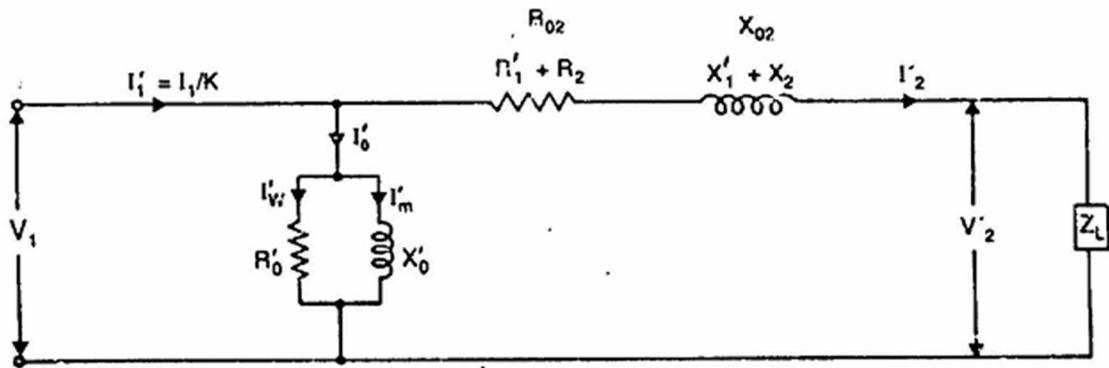
Based on the above phasor diagram we can notice the following:

- ▶ The referred value of load voltage  $V'_2$  is chosen as the reference phasor.
- ▶  $I'_2$  is lagging  $V'_2$  by phase angle  $\phi_2$ .
- ▶  $I'_2 R_{01}$  is in phase with  $I'_2$  and the voltage drop  $I'_2 X_{01}$ , leads  $I'_2$  by  $90^\circ$ .
- ▶  $I_W$  is in phase with  $V_1$  while  $I_m$  lags behind  $V_1$  by  $90^\circ$ .
- ▶ The phasor sum of  $I_W$  and  $I_m$  is  $I_0$ .
- ▶ The phasor sum of  $I_0$  and  $I'_2$  is the input current  $I_1$ .

If all the primary quantities are referred to the secondary, we can get the *simplified equivalent circuit of the transformer referred to the secondary* as shown below



(i)



(ii)

From the above circuit:

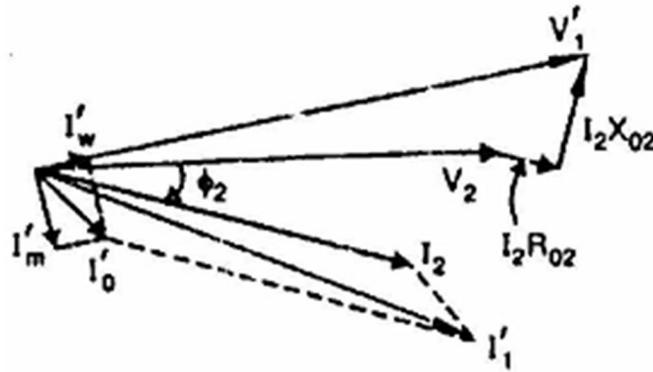
$$\boxed{R'_1 = K^2 R_1; \quad X'_1 = K^2 X_1; \quad V'_2 = K V_1; \quad I'_1 = \frac{I_1}{K}}$$

$$\boxed{Z_{02} = R_{02} + j X_{02}}$$

Where

$$\boxed{R_{02} = R_2 + R'_1; \quad X_{02} = X_2 + X'_1}$$

Hence the phasor diagram can be obtained as:

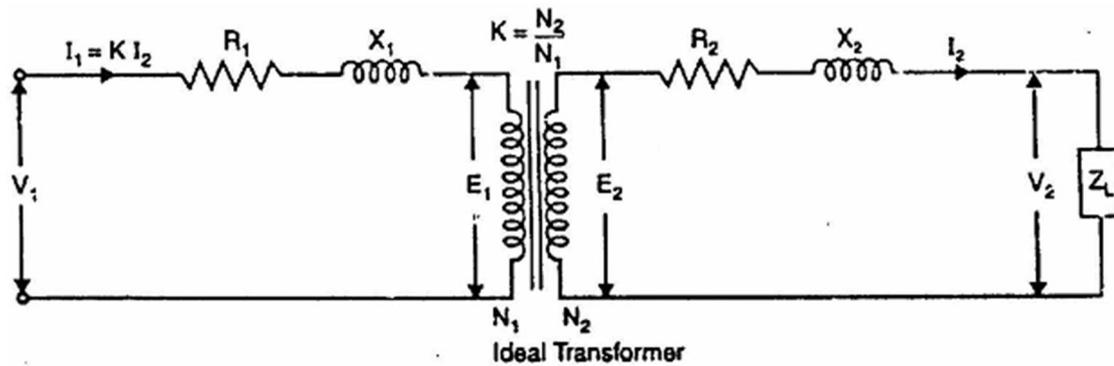


Based on the above phasor diagram we can notice the following:

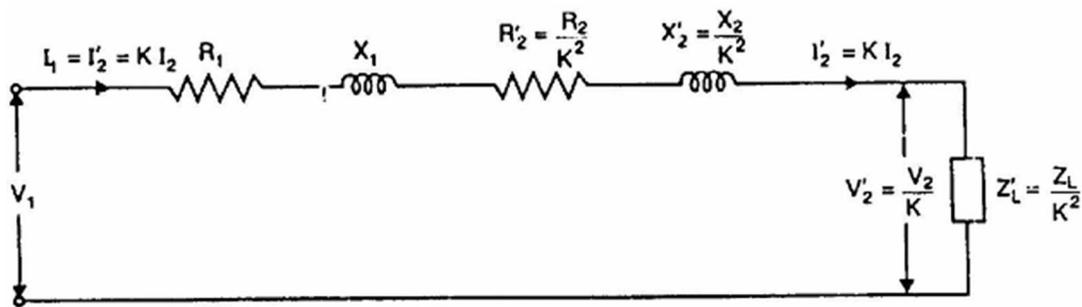
- ▶ *The referred value of load voltage  $V_2$  is chosen as the reference phasor.*
- ▶  *$I_2$  is lagging  $V_2$  by phase angle  $\phi_2$ .*
- ▶  *$I_2 R_{02}$  is in phase with  $I_2$  and the voltage drop  $I_2 X_{02}$ , leads  $I_2$  by  $90^\circ$ .*
- ▶  *$I_w'$  is in phase with  $V_1'$  while  $I_m'$  lags behind  $V_1'$  by  $90^\circ$ .*
- ▶ *The phasor sum of  $I_w'$  and  $I_m'$  is  $I_0'$ .*
- ▶ *The phasor sum of  $I_0'$  and  $I_2$  is the input current  $I_1'$ .*

## Approximate Equivalent Circuit of a Loaded Transformer

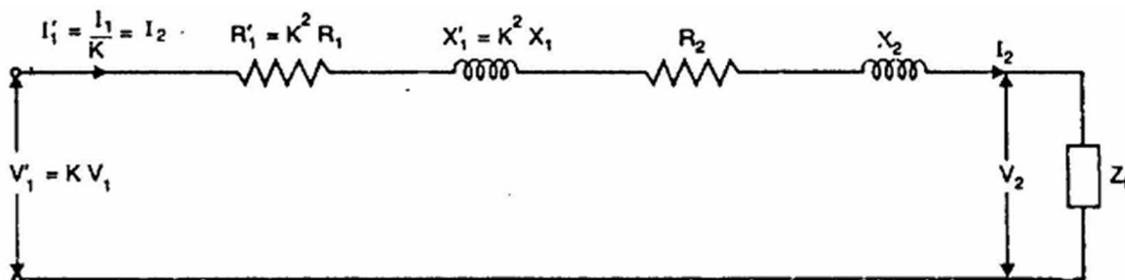
The no-load current  $I_0$  in a transformer is only 1-3% of the rated primary current and *may be neglected without any serious error*. The transformer can then be shown as in the figure below:



If all the secondary quantities are referred to the primary, we can get the *approximate equivalent circuit of the transformer referred to the primary* as shown below



If all the primary quantities are referred to the secondary, we can get the *approximate equivalent circuit of the transformer referred to the secondary* as shown below





Approximate drop in secondary voltage

$$= AN = AD + DN$$

$$= AD + BL \quad (\because BL = DN)$$

Approximate drop in secondary voltage is

$$= I_2 R_{02} \cos \phi_2 + I_2 X_{02} \sin \phi_2$$

For a load having a leading p.f.  $\cos \phi_2$ , we have,

$$\text{Approximate voltage drop} = I_2 R_{02} \cos \phi_2 - I_2 X_{02} \sin \phi_2$$

**Note:** If the circuit is referred to primary, then it can be easily established that

$$\text{Approximate voltage drop} = I_1 R_{01} \cos \phi_2 \pm I_1 X_{01} \sin \phi_2$$

# Testing, Efficiency, and Voltage Regulation

## Voltage Regulation of Transformer

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage ( ${}_0V_2$ ) and the secondary voltage  $V_2$  on load expressed as percentage of no-load voltage i.e.

$$\text{voltage regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$$

where

${}_0V_2$  = No-load secondary voltage =  $K V_1$

$V_2$  = Secondary voltage on load

As shown in *Lecture 4*

$${}_0V_2 - V_2 = I_2 R_{02} \cos \phi_2 \pm I_2 X_{02} \sin \phi_2$$

The +ve sign is for lagging p.f. and -ve sign for leading p.f.

**NOTE:** It may be noted that % voltage regulation of the transformer will be the same whether primary or secondary side is considered.

## Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core or Iron losses
2. Copper losses

**NOTE:** The above losses appear in the form of heat and produce (i) *an increase in temperature* and (ii) a *drop in efficiency*.

## A- Core or Iron losses ( $P_i$ )

These consist of *hysteresis and eddy current losses* and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test (see next sections).

$$\text{Hysteresis loss, } = k_h f B_m^{1.6} \text{ watts/m}^3$$

$$\text{Eddy current loss, } = k_e f^2 B_m^2 t^2 \text{ watts/m}^3$$

Both hysteresis and eddy current losses depend upon

- ▶ Maximum flux density  $B_m$  in the core and
- ▶ Supply frequency  $f$ .

**NOTE:** Since transformers are connected to constant-frequency, constant voltage supply, both  $f$  and  $B_m$  are constant. Hence, *core or iron losses are practically the same at all loads*. Hence,

$$\begin{aligned} \text{Iron or Core losses, } P_i &= \text{Hysteresis loss} + \text{Eddy current loss} \\ &= \text{Constant losses} \end{aligned}$$

**NOTE:** The hysteresis loss can be minimized *by using steel of high silicon content* whereas eddy current loss can be reduced *by using core of thin laminations*.

## B- Copper losses ( $P_C$ )

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test

Total copper Cu losses:

$$P_C = I_1^2 R_1 + I_2^2 R_2$$
$$= I_1^2 R_{01} \text{ or } I_2^2 R_{02}$$

Hence, total losses in a transformer are:

$$\text{Total losses in a transformer} = P_1 + P_C$$
$$= \text{Constant losses} + \text{Variable losses}$$

## Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as **the ratio of output power (in watts or kW) to input power (watts or kW) i.e.**

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

In practice, open-circuit and short-circuit tests are carried out to find the efficiency,

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

**NOTE:** The losses can be determined by transformer tests.

## Condition for Maximum Efficiency

$$\text{Output power} = V_2 I_2 \cos \phi_2$$

If  $R_{02}$  is the total resistance of the transformer referred to secondary, then,

$$\text{Total Cu loss, } P_C = I_2^2 R_{02}$$

$$\text{Total losses} = P_i + P_C$$

$$\begin{aligned} \text{Transformer } \eta &= \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{02}} \\ &= \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + P_i / I_2 + I_2 R_{02}} \end{aligned}$$

For a load of given pf, efficiency depends upon load current  $I_2$ . Hence, the efficiency to be maximum *the denominator should be minimum* i.e.

$$\frac{d}{dI_2} (V_2 \cos \phi_2 + P_i / I_2 + I_2 R_{02}) = 0$$

$$\text{or} \quad 0 - \frac{P_i}{I_2^2} + R_{02} = 0$$

$$\text{or} \quad P_i = I_2^2 R_{02}$$

i.e., Iron losses = Copper losses

**Hence efficiency of a transformer will be maximum when copper losses are equal to constant or iron losses.**

From above, the load current  $I_2$  corresponding to maximum efficiency is:

$$I_2 = \sqrt{\frac{P_i}{R_{02}}}$$

**NOTE:** In a transformer, iron losses are constant whereas copper losses are variable. In order to obtain **maximum efficiency**, the load current should be such that total Cu losses become equal to iron losses.

## Output kVA corresponding to Maximum Efficiency

Let  $P_C$  = Copper losses at full-load kVA

$P_i$  = Iron losses

$x$  = Fraction of full-load kVA at which efficiency is maximum

Total Cu losses =  $x^2 \times P_C$

for maximum efficiency  $x^2 \times P_C = P_i$

or 
$$x = \sqrt{\frac{P_i}{P_C}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

⇒ Output kVA corresponding to maximum efficiency:

$$\text{Full - load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

**NOTE:** The value of kVA at which the efficiency is maximum, is independent of pf of the load.

## All-Day Efficiency

All-day efficiency is of special importance for those transformers whose primaries are never open-circuited but the secondaries carry little or no load much of the time during the day.

The ratio of output in kWh to the input in kWh of a transformer over a 24-hour period is known as all-day efficiency i.e.:

$$\eta_{\text{all-day}} = \frac{\text{kWh output in 24 hours}}{\text{kWh input in 24 hours}}$$

**NOTE:** Efficiency of a transformer means commercial efficiency unless stated otherwise.

## Transformer Tests

The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests:

- (i) open-circuit test and (ii) short-circuit test

### Open-Circuit Test

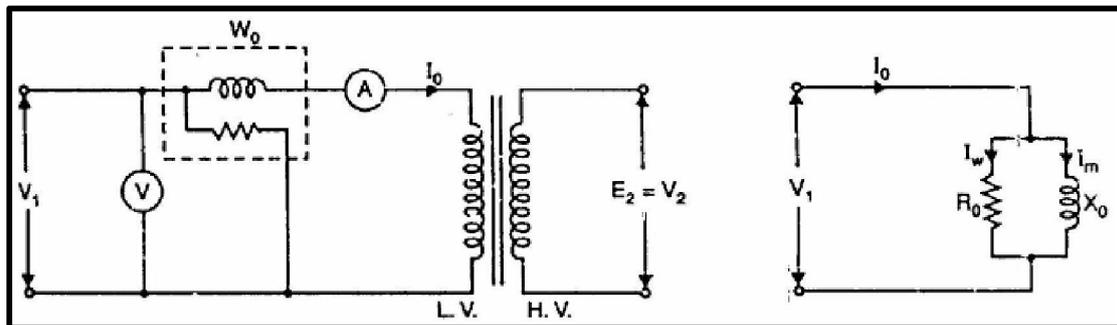
This test is conducted to determine:

- ▶ The iron losses (or core losses) and
- ▶ Parameters  $R_0$  and  $X_0$  of the transformer.

In this test (see Figure below), the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open-circuited.

As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core.

Cu losses in the primary under no-load condition are negligible as compared with iron losses.



For the figure above:

- Iron losses,  $P_i = \text{Wattmeter reading} = W_0$
- No load current = Ammeter reading =  $I_0$
- Applied voltage = Voltmeter reading =  $V_1$
- Input power,  $W_0 = V_1 I_0 \cos\phi_0$

⇒

$$\text{No-load p.f., } \cos \phi_0 = \frac{W_0}{V_1 I_0}$$

$$I_W = I_0 \cos \phi_0; \quad I_m = I_0 \sin \phi_0$$

$$R_0 = \frac{V_1}{I_W} \quad \text{and} \quad X_0 = \frac{V_1}{I_m}$$

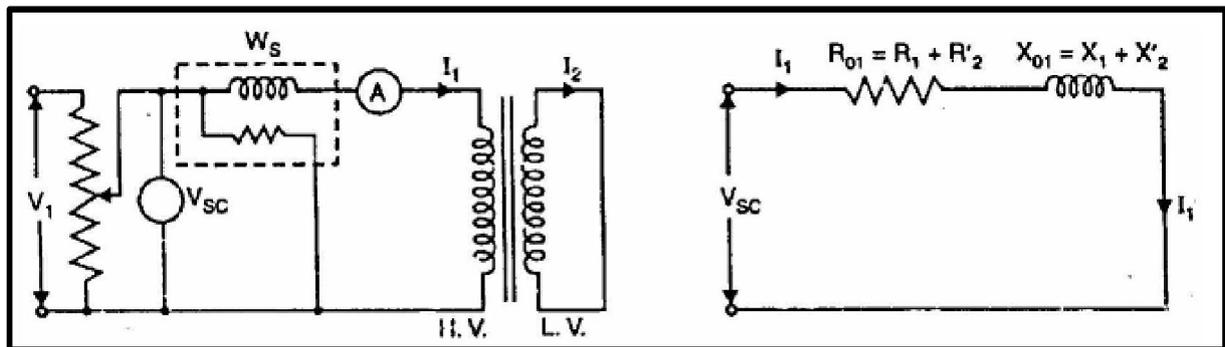
## Short-Circuit or Impedance Test

This test is conducted to determine:

- ▶ Full-load copper losses of the transformer and
- ▶  $R_{01}$  (or  $R_{02}$ ),  $X_{01}$  (or  $X_{02}$ ).

In this test (see Figure below), the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary.

The low input voltage is gradually raised till at voltage  $V_{SC}$ , full-load current  $I_1$  flows in the primary. Then  $I_2$  in the secondary also has full-load value since  $I_1/I_2 = N_2/N_1$ . Under such conditions, the copper loss in the windings is the same as that on full load.



For the figure above:

- Full load Cu loss,  $P_C = \text{Wattmeter reading} = W_s$
- Applied voltage = Voltmeter reading =  $V_{SC}$
- F.L. primary current = Ammeter reading =  $I_1$

⇒

$$P_C = I_1^2 R_1 + I_1^2 R'_2 = I_1^2 R_{01}$$

$$R_{01} = \frac{P_C}{I_1^2}$$

where  $R_{01}$  is the total resistance of transformer referred to primary

Total impedance referred to primary,

$$Z_{01} = \frac{V_{SC}}{I_1}$$

Total leakage reactance referred to primary,

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Short-circuit pf

$$\cos \phi_2 = \frac{P_C}{V_{SC} I_1}$$

## Efficiency from Transformer Tests

The full-load efficiency of the transformer at any pf can be obtained as:

$$\text{F.L. efficiency, } \eta_{\text{F.L.}} = \frac{\text{Full - load VA} \times \text{p.f.}}{(\text{Full - load VA} \times \text{p.f.}) + P_i + P_C}$$

where:  $P_i$  = Iron loss can be obtained from open-circuit test  
 $P_c$  = Copper loss can be obtained from short-circuit test  
F.L. = Full Load

Also the efficiency for any load,

$$\begin{aligned} \text{Corresponding total losses} &= P_i + x^2 P_C \\ \text{Corresponding } \eta_x &= \frac{(xx \text{ Full - load VA}) \times \text{p.f.}}{(xx \text{ Full - load VA} \times \text{p.f.}) + P_i + x^2 P_C} \end{aligned}$$

where  $xx$  = Fraction of full-load

**NOTE:** Iron loss remains the same at all loads.