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10. Air-Inertia load Distribution

10.1. Span-wise air load distribution:

This subject concerns both the aerodynamicist and the stress analyst. The aerodynamicist is usually concerned with properties, which affect the performance, stability and control of the airplane.

The stress analyst is concerned with the load distribution which will represent the most sever conditions for various parts of the internal structure of the airplane.

Exact equations for span-wise lift distribution which can be found in many aerodynamic books, can be solved for many wing planform. Analytical and numerical methods to solve these equations are available but the calculation is not simple.

Approximation methods to find span-wise lift distribution are simpler and available. The most popular methods are:

- Schrenk method.
- Diederich method.
- Fourier series method.

10.2. Schrenk method:

A simple approximated method to find solution for span-wise lift distribution which has been proposed by Dr. Ing Oster Schrenk and has been accepted by the Civil Aeronautics Administration (CAA) as a satisfactory method for civil a/c.

Schrenk method relies on the fact that the lift distribution does not differ much from elliptical platform shape if:

- The wing is upswept.
- The wing has no aerodynamic twist, i.e. zero lift lines for all wing sections lie in the same plane (constant airfoil section). Lift is:

$$L = 0.5\rho v^2 S C_L = qSC_L \qquad \dots 10.1$$

$$L = qb\bar{C}C_L \qquad \dots 10.2$$

Lift per unit span length is:

$$L/b = q\bar{C}C_L \qquad \dots 10.3$$

Where ($\bar{C}$) is mean chord for each unit span. Since dynamic pressure ($q = 0.5\rho v^2$) is constant, then lift distribution for unit span length depends on (CC_L) only, i.e. per unit span length:

$$L/b = const * CC_L \qquad \dots 10.4$$

For unit lift coefficient ($C_L = 1$), load per unit span length, L/b, depends on chord distribution CC_L, but the distribution CC_L is not actual wing chord distribution. The over bar is omitted since $\bar{C} = C$ for each unit span. For steady level flight at $C_L = 1$:

$$L = W = qS \qquad \dots 10.5a$$

$$\therefore L/q = W/S \qquad \dots 10.5b$$

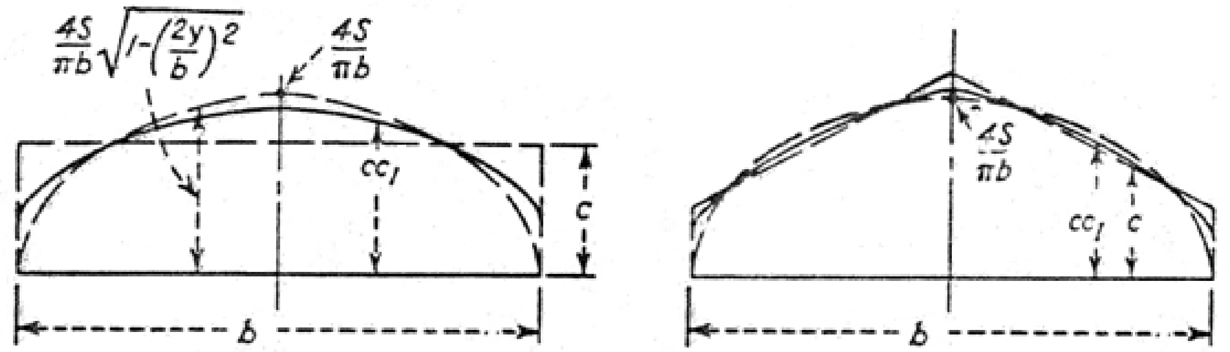
W/S is wing loading and is assumed constant.

Schrenk method proposed that the lift distribution per unit span length is the mean value of actual wing chord distribution and an elliptical wing chord distribution that has the same area (S) and the same span (b), see figure (10.1).

The lift distribution of elliptical platform wing is elliptical distribution. It obeys the wing elliptical chord distribution (like British spitfire of word II war). The elliptical chord distribution is:

$$C_{ellip} = C_{root}\sqrt{1-\eta^2} = \frac{4S}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2} \qquad \ldots 10.6a$$

Lift distribution per unit span for a wing at ($C_L = 1$) is:



(a) Rectangular planform ***(b) Trapezoidal planform***

Figure 10.1: Schrenk span-wise lift distribution

$$Sch.\,dist. = \frac{1}{2} * (actual\ wing\ chord\ distribution + elliptical\ wing\ chord\ distribution)$$

$$(C)_{Schrenk\ distribution} = \frac{1}{2}\left[(C)_{actual\ wing} + (C)_{elliptical\ wing}\right] \qquad \ldots 10.6b$$

For each section at a distance y from aircraft center line, i.e. root chord, the Schrenk distribution is:

$$(C)_{Sch.\ dist.} = \frac{1}{2}\left[(C)_{wing} + \frac{4S}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2}\right] \qquad \ldots 10.7$$

C_{wing} is the wing chordal distribution. If the value of lift coefficient is unity ($C_L \neq 1$) then just multiply $(C)_{Sch.\ dist.}$ by the actual value of wing lift coefficient. The Schrenk distribution is in (m). The local lift distribution in(N/m) is:

$$w = L/b = W/S * CC_L \qquad \ldots 10.8$$

And lift magnitude at each sectional span length is:

$$\ell = W/S * CC_L * \Delta y \qquad \ldots 10.9$$

Schrenk distribution is actual lift coefficient distribution so the local lift coefficient distribution is computed as:

$$C_{\ell,local} = CC_L/C_{wing} \qquad \ldots 10.10$$

It is very important that the local lift coefficient at any section should never exceed the maximum airfoil lift coefficient for this section, from NACA data sheet, else this section may stalled if $C_{\ell,local} > C_{\ell,max}$, see figure (10.2).

Wing geometrical or aerodynamical twist is used to prevent stall and a recalculation for the distribution is necessary.

Figure (10.2) denotes the effluence of taper ratio on span-wise lift distribution and figure (10.3) shows local lift curve distribution.

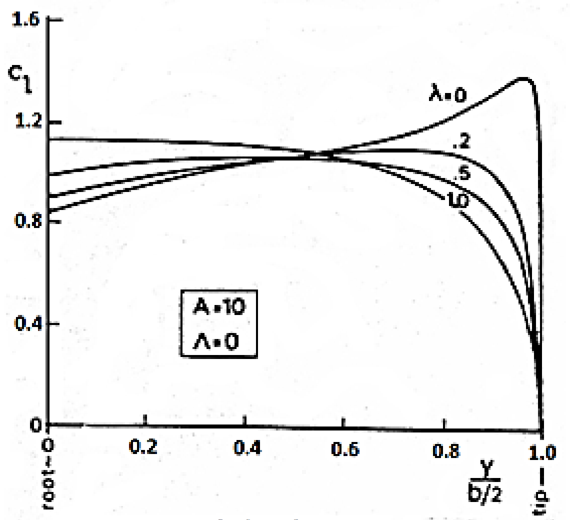
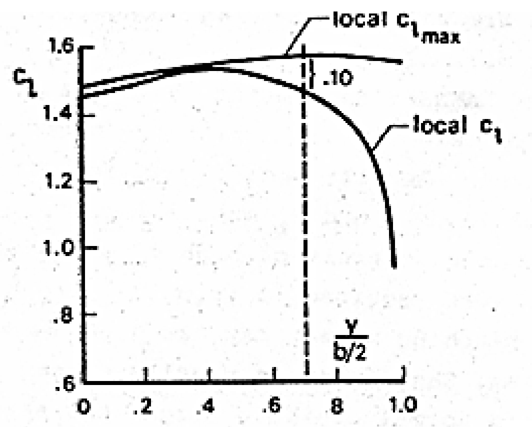


Figure 10.2: Lift distribution at C_L = 1 for straight wing with various taper ratios



Figuer 10.3: Calculated spanwise lift distribution at high angle of attack where flap is up.

The result should be tabulated in a table as below with at leas teen sections.

I.	y, m	$\frac{2y}{b}$	$\frac{4S}{\pi b}\sqrt{1-(2y/b)^2}$, m	C_{wing}, m	CC_L, m	$C_{\ell,local}, m$	$w,\ N/m$	ℓ, N
					$0.5(3+4)$	$5 \div 4$	5*W/S	$7 * \Delta y$
	1	2	3	4	5	6	7	8
1.	0	0						
2.	.	.						
3.	.	.						
10.	. b/2	. 1						

C_{wing} : Wing chord at any section (m).
C_L : Wing lift coefficient.
C_ℓ : Local lift coefficient at each section.
b : Wing span (m).
S : Wing area (m^2).
W/S : Wing loading (N/m^2).
η : Non-dimensional parameter.
w : Air load distribution (N/m)
ℓ : =0.5 $* (w_1 + w_2)\Delta y$, Local lift (N).

Column (5) gives Schrenk distribution while column (7) gives Schrenk air-load distribution. Column (8) gives local air-load value.

This lift distribution is obviously inaccurate at the wing tips, see figure (10.1). A well designed wing will have a rounded tip where approximate method gives closer results to actual distribution than for square tip. Analytical and numerical methods also need empirical corrections are often applied.

For wings with aerodynamic twist, the distribution is evaluated in two parts.

1. Basic lift distribution is obtained for the angle of attack where entire wing has no lift. Where some outboard sections have negative lift and some inboard section have positive lift.
2. Addition lift distribution is evaluated by assuming that the wing has lift but no twist. And the distribution can be evaluated by Schrenk method.

For each section basic and addition lift are added together to give actual section lift. The details are left for the student who is interest. Diederich method seems simpler and more general.

10.3. S.F & B.M distribution:

Schrenk distribution is also used to evaluated shear force and bending moment distribution span wise. Evaluation of S.F & B.M values at each wing section has a great importance in wing structure analysis. S.F & B.M values are calculated along span-wise increment (Δy) as:

$$S.F = \int_0^{b/2} w.dy = \sum_{i=1}^{i=n} \left(\frac{w_1 + w_2}{2}\right) * \Delta y \qquad \text{... } 10.11$$

$$B.M = \int_0^{b/2} S.F.dy = \iint_0^{b/2} w.dy = \sum_{i=1}^{i=n} \left(\frac{S.F_1 + S.F_2}{2}\right) * \Delta y \qquad \text{... } 10.12$$

The result should be tabulated as:

I	y	Load intensity w	Interval Δy	Shear increment $\left(\frac{w_1+w_2}{2}\right)\Delta y = SF$	Shear force $\sum SF$	Shear increment $\frac{(SF_1+SF_2)}{2} = BM$	Bending moment = $\sum BM$
	m	N/m	m	N	N	$N.m$	$N.m$
	1	2	3	4	5	6	7
1.							
2.							
. . .							
	b/2						

Example:

Find spanwise air-load, shear force and bending .moment distributions over a straight taper wing for an aircraft has the following data:

$Aircraft\ weight$: $12750\ kg$; $\quad Wing\ loading$: $270.127\ kg/m^2$
$Aspect\ ratio$: 10.0 ; $\quad Taper\ ratio$: 0.6
$Load\ factor$: 1.0; $\quad$ If ($C_{\ell,max} = 1.1$) is there any stalled section.

Solution:

$S = W/(W/S) = 12750/270.127 = 47.2\ m^2$

$b = \sqrt{AR * S} = \sqrt{10.0 * 47.2} = 21.73\ m$

$\bar{C} = b/AR = 21.73/10 = 2.173\ m$

For straight taper wing (trapezoidal),

$\bar{C} = (C_{root} + C_{tip})/2$

$\bar{C} = C_{root}(1 + \lambda)/2$

$C_{root} = 2\bar{C}/(1 + \lambda) = 2 * 2.173/(1 + 0.6) = 2.716\ m$

$C_{tip} = \lambda C_{root} = 0.6 * 2.716\ m$

I.	y	$2y/b$	$\frac{4S}{\pi b}\sqrt{1-(2y/b)^2}$	C_{wing}	CC_L	$C_{L,local}$	w
	m		m	m	m	m	N/m
					$0.5(3+4)$	$5 \div 4$	$5 * W/S$
	1	2	3	4	5	6	7
1.	0	0	2.763	2.716	2.741	1.009	740.419
2.	1	0.092	2.753	2.616	2.685	1.026	725.292
3.	2	0.184	2.718	2.516	2.617	1.040	706.923
4.	3	0.276	2.658	2.416	2.537	1.050	685.313
5.	4	0.368	2.571	2.316	2.444	1.055	660.191
6.	5	0.460	2.455	2.216	2.336	1.054	631.017
7.	6	0.552	2.306	2.116	2.211	1.049	597.251
8.	7	0.644	2.112	2.016	2.066	1.025	558.083
9.	8	0.736	1.872	1.916	1.894	0.989	511.621
10.	9	0.828	1.550	1.816	1.683	0.927	454.624
11.	10	0.920	1.084	1.716	1.400	0.816	378.178
12	10.863	1.000	0.000	1.63	0.815	0.500	220.154

It seems that section 4 and 5, where ($C_{\ell,local} = 1.055\ and\ 1.054$) respectively, are closer to be stalled. In order to maintain safety margin from stall, an airfoil section with ($C_{\ell,max} > 1.1$) is recommended, but it is acceptable and the distribution of ($C_{\ell,max}$) is flat. The characteristics of trapezoidal wing are better than those for rectangular wing since the former is closer to the elliptic wing.

I	y	Load intensity w	Interval Δy	Shear increment $\left(\frac{w_1 + w_2}{2}\right)\Delta y = SF$	Shear force $\sum SF$	Shear increment $\frac{(SF_1 + SF_2)}{2} = BM$	Bending moment = $\sum BM$
	m	N/m	m	N	N	N.m	N.m
	1	2	3	4	5	6	7
1.	0	740.419			6388.781		30767.804
			1	732.856		6022.353	
2.	1	725.292			5655.925		24745.451
			1	716.108		5297.871	
3.	2	706.923			4939.817		19447.580
			1	696.118		4591.758	
4.	3	685.313			4243.699		14855.822
			1	672.752		3907.323	
5.	4	660.191			3570.947		10948.499
			1	645.604		3248.145	
6.	5	631.017			2925.343		07700.354
			1	614.134		2618.276	
7.	6	597.251			2311.209		05082.078
			1	577.667		2022.376	
8.	7	558.083			1733.542		03059.702
			1	534.852		1466.116	
9.	8	511.621			1198.690		01593.586
			1	483.123		957.129	
10.	9	454.624			0715.567		00636.457
			1	416.401		0507.367	
11.	10	378.178			0299.166		00129.090
			0.863	299.166		0129.090	
12.	10.863	220.154			0000.000		00000.000

To check the validity of solution, compare the value of shear force at wing root with weigh of the a/c which must equal half the weight.

Check:

$Half\ weight\ =\ 0.5(12750)\ =\ 6375\ kg.$

$Shear\ force\ at\ root\ = 6388.781\ kg$

$Error\ =\ (6375 - 6388.781)/\ 6375 =\ -0.216\%$

$Half\ weight\ * \bar{\bar{y}}\ =\ 0.5(12750) * 4.979 = 31741.125 kg.m$

$Bending\ moment\ at\ root = 30767.804$

$Error\ =\ (31741.125 - 30767.804)/31741.125 =\ +3.07\%$

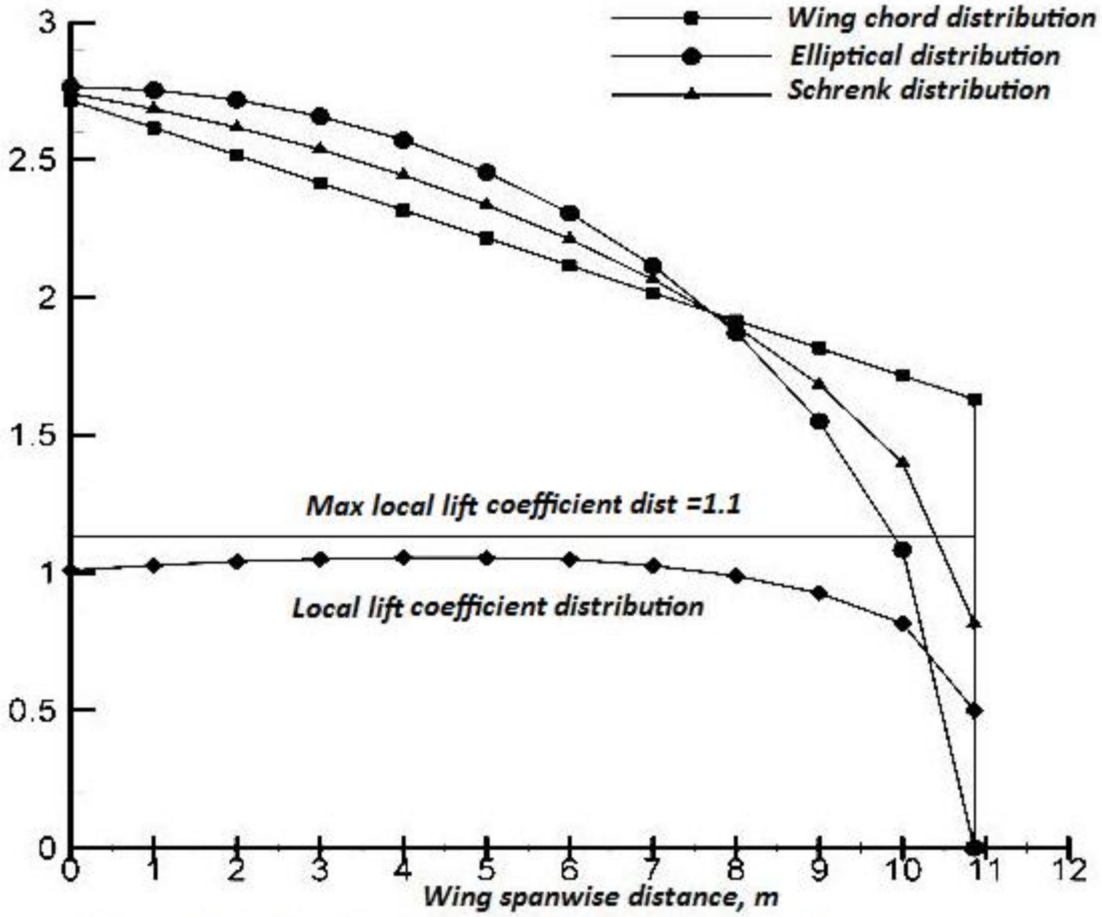


Figure 10.4: Wing chord, elliptical, Schrenk and local lift curve distribution

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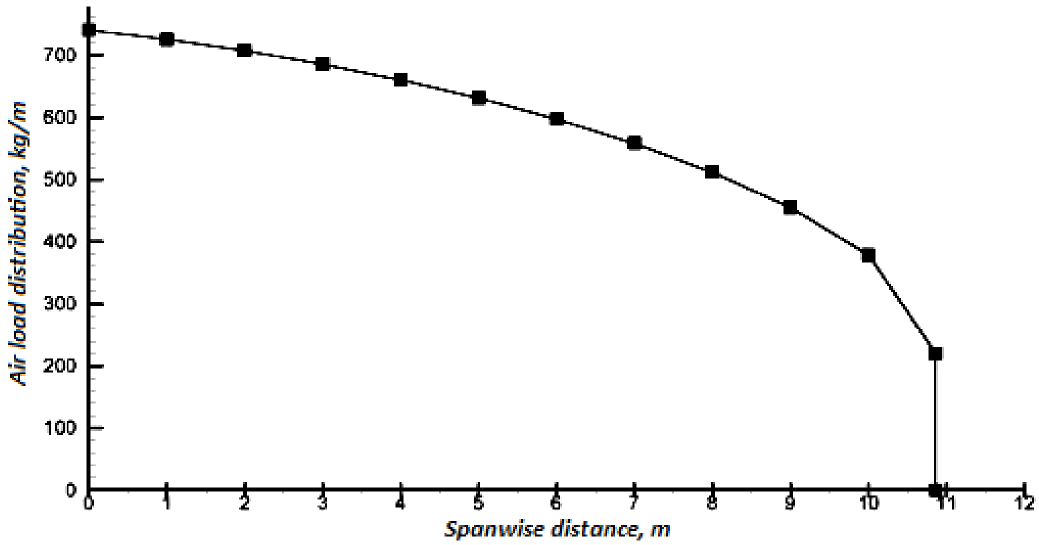
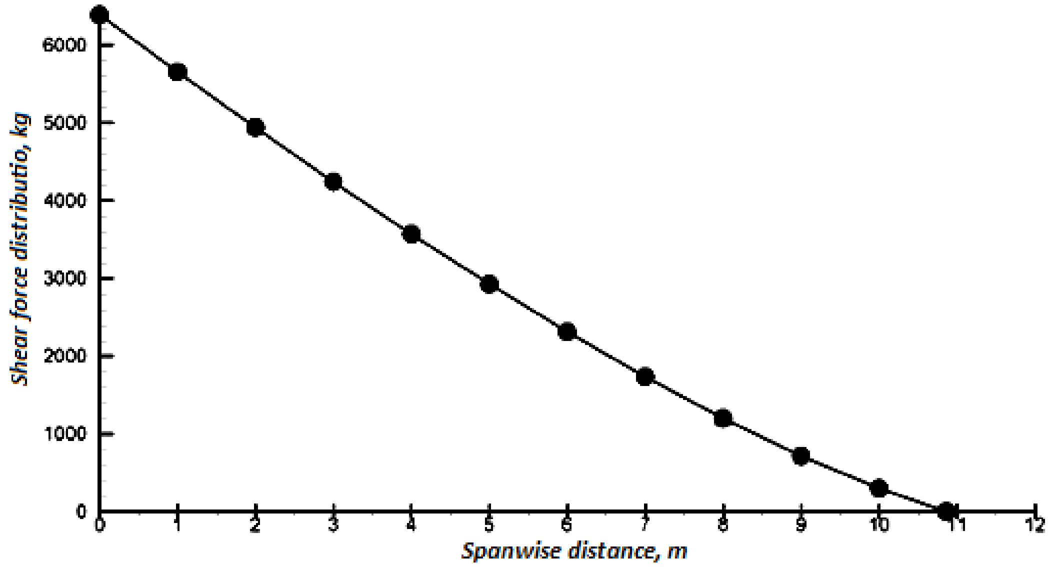




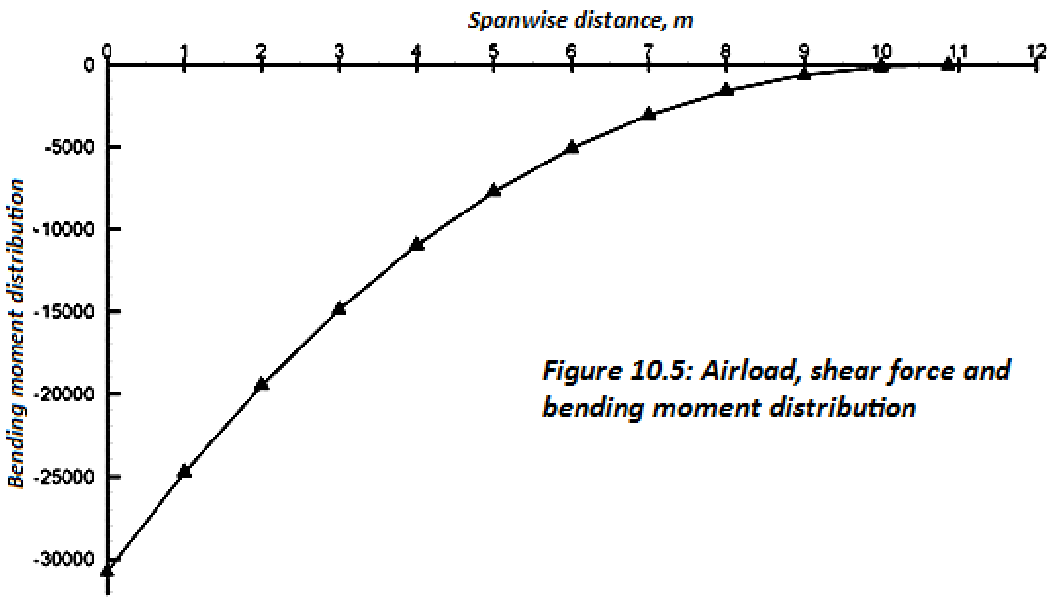


Figure 10.5: Airload, shear force and bending moment distribution

10.4. Diederich method:

Diederich method is a modification to Anderson method. The lift may be divided into additional lift (L_a) and basic lift (L_b), then:

$$C_\ell = C_{\ell a} + C_{\ell b}$$

In terms of non-dimensional parameter (L_a & L_b) used by Anderson R .F. (NACA report 572, 1936)

$$C_\ell \frac{C}{\bar{C}} = L_a C_L + \frac{\mathcal{E}_t a_o}{E} L_b \qquad \text{... 10.13}$$

$$L_a = \frac{C_{\ell a} C}{C_L \bar{C}} \qquad \text{... 10.14}$$

$$L_b = \frac{C_{\ell b} C}{\bar{C}} \frac{E}{\mathcal{E}_t a_o} \qquad \text{... 10.15}$$

$$E = 1 + \frac{2\lambda}{A(1+\lambda)} \qquad \text{... 10.16}$$

Anderson presents tables for (L_a) and (L_b) for straight –taper wings with linear twist in incompressible flow which can be inserted in equation (18) to evaluated (C_ℓ).

Diederich F. W. (NACA TR 2751, 1952) proposed the following semi-empirical method, which yields a satisfactory result for pre-design purpose. It is valid for wing with arbitrary platform and lift distribution, provided that the quarter chord line of a wing half is approximately straight. This method can be used for straight and swept wings in incompressible or compressible and sub-critical flow.

a- Additional lift distribution:

$$L_a = C_1 \frac{C}{\bar{C}} + C_2 \frac{4}{\pi}\sqrt{1-\eta^2} + C_3 f \qquad \text{... 10.17}$$

Coefficients (C_1, C_2 and C_3) are evaluated from figure (10-6). Lift distribution function (f) is evaluated from figure (10-7). For straight wings ($\Lambda_{0.25} = 0.0$) the function (f) is elliptical:

$$f = \frac{4}{\pi}\sqrt{1-\eta^2} \qquad \text{... 10.18}$$

And the equation for (L_a) is simplified to:

$$L_a = C_1 \frac{C}{\bar{C}} + (C_2 + C_3)\frac{4}{\pi}\sqrt{1-\eta^2} \qquad \text{... 10.19}$$

If ($C_1 = (C_2 + C_3)$), the distribution becomes Schrenk distribution.

b- Basic lift distribution: it is evaluated from:

$$L_b = \beta E \left[L_a C_4 cos\Lambda_\beta \left(\frac{\mathcal{E}}{\mathcal{E}_t} + \alpha_{o,1} \right) \right] \qquad \text{... 10.20}$$

$\mathcal{E}$: Aerodynamic twist. For linear twist ($\mathcal{E} = \eta \mathcal{E}_t$).

$\mathcal{E}_t$: Aerodynamic twist for tip section.

Λ_β: Corrected sweep angle,$(= \Lambda_\beta/\beta)$.

η : Non-dimensional spanwise station, $(= 2y/b)$.

β : Prandtl's compressibility correction, $(= \sqrt{1 - M^2})$.

C_4: Factor evaluated from figure (10.8).

$\alpha_{o,1}$: Factor equal to the local aerodynamic twist at the spanwise station for which $(C_{\ell b} = 0)$.

$$\alpha_{o,1} = -\int_0^1 \frac{\mathcal{E}}{\mathcal{E}_t} L_a d\eta \qquad \text{... 10.21}$$

For elliptical Additional lift distribution (L_a) and linear twist distribution

$$\alpha_{o,1} = 4/3\pi \qquad \text{... 10.22}$$

For straight-taper unswept wings with linear twist distribution :

$$\alpha_{o,1} = -\left(C_1 \frac{1 + 2\lambda}{3(1 + \lambda)} + (C_2 + C_3)\frac{4}{3\pi}\right) \qquad \text{... 10.23}$$

The factor (C_4) is evaluated from figure (10.6).

10.5. Inertia Loads

The maximum load on any part of the airplane structure is at the stage where airplane is accelerated. The loads produced by landing impaction, maneuvering or encountering gust in flight case are always greater than steady state or equilibrium conditions. Therefore various loading factors should be considered.

During stress analysis different inertia loads for different airplane parts should be

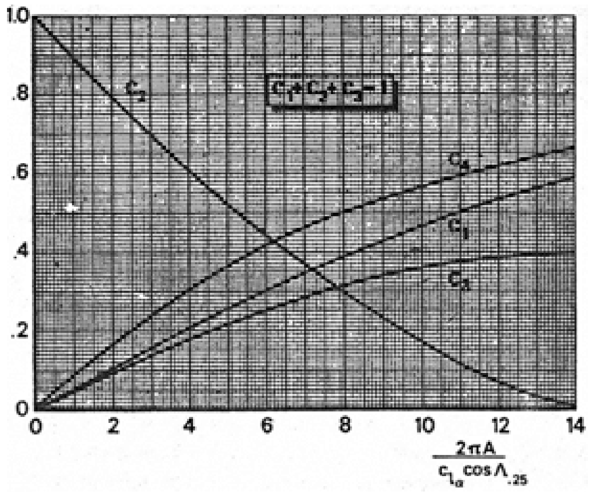


Figure 10.6: Factors in Diederich's method

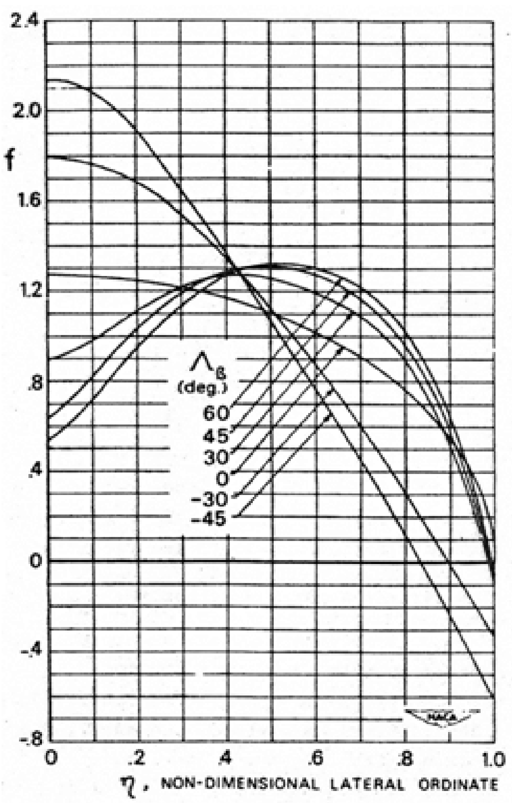


Figure 10.7: The lift distributionfunction f

considered. Since the sever conditions occurs at wing due to many different dynamic loads during flying, our attention will be focused on wing group comp.

The wing, from structural analysis point of view, can be regarded as a simple cantilever beam supported at root and free to deflect at tip.

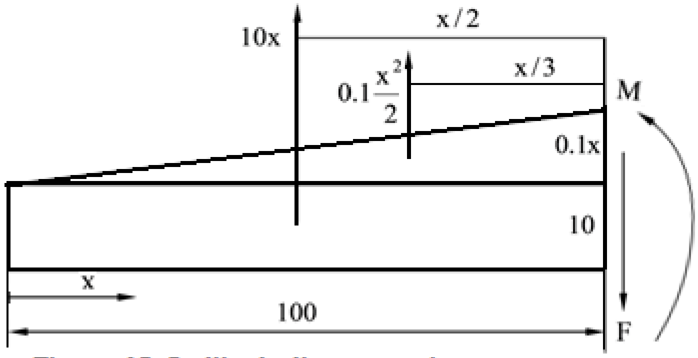


Figure 10.9: Illustrative example

Loads at wing are due to:

- Wing structural weight distribution.
- Fuel weight distribution.
- Concentrated loads due to power unit.
- Concentrated loads due to undercarriage.
- Other loads due to different parts accommodated in the wing.

S.F. and B.M. diagrams for inertia loads are evaluated by many methods that the student studied these methods in 2nd year within subject strength of material, such as:

- By considering forces to the left of each section.
- By integration of equations defining loads and shear curves.
- By obtaining areas under curves geometrically.

Example: Find S.F. and B.M. diagram for the beam shown.

Solution:

Since load intensity increases linearly from $(10\, kg/cm)$ at $(x = 0)$ to $(20\, kg/cm)$ at $(x = 100)$, then load at any section is:

$w = 10 + 0.1x$

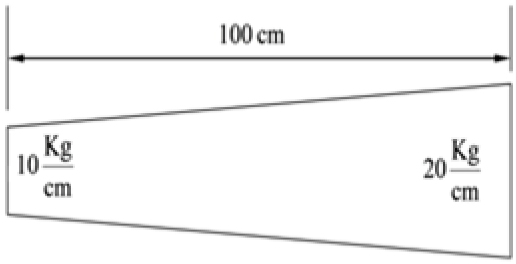


Figure 10.8: Illustrative example

a) By direct integration of load distribution (w).

$$S.F. = \int w dx = \int (10 + 0.1x) dx = 10x + 0.1\frac{x^2}{2}$$

$$B.M. = \int \left(10x + 0.1\frac{x^2}{2}\right) dx = 10\frac{x^2}{2} + 0.1\frac{x^3}{6}$$

b) By considering forces to the left of each section. Divide the load distribution into two regions rectangular and triangular, then:

$$S.F = 10x + 0.1\frac{x^2}{2}$$

$$B.M = 10x * \frac{x}{2} + 0.1\frac{x^2}{2} * \frac{x}{3} = 10\frac{x^2}{2} + 0.1\frac{x^3}{6}$$

c) by dividing load distribution into strips, and then:

Area of strip 1 =S.F. at x1
Area of strip 1+2 =S.F. at x2
Area of strip 1+2+3 =S.F. at x3
Area of strip 1+2+3+4 =S.F. at x4

And so on for all sections. After that S.F. diagram is drawn and also is divided into many strips to find B.M. diagram. Although the method lengthy, it is quite beneficial for irregular distribution.

(S.F.) at any section is equal to the area under load curve positioned to the left of the section. (For the above example)

$$S.F. = rectangular\ area\ (10x) + \ triangular\ area(10x * \frac{x}{2})$$

(B.M.) at any section is equal to the area under shear force curve positioned to the left of the section.

$$B.M. = \ triangular\ area\ (10x * \frac{x}{2}) + \ parabola\ area\ (0.1\frac{x^2}{2} * \frac{x}{3})$$

Note: Area of parabola = (maximum ordinate * third of the base)

10.6. Wing group load distribution

For precise calculation of fuel tank volume it is necessary to account for the actual section shape of the wing structural layout. But a first guess a total fuel volume tank is needed.

❖ Fuel tank:

- Fuel tank cross-section area at any section of chord C and thickness ratio t/c approximately is:

$$S_{f,tank} = \frac{t}{c} * \frac{C}{2} * \frac{C}{2}$$

- Volume of fuel tank:

 - Truncated pyramid

$$V_{f,t} = \frac{\ell}{3}\left(S_1 + S_2 + \sqrt{S_1 S_2}\right)$$

*Obelisk

$$V_{f,t} = \frac{\ell}{3}\left(S_1 + S_2 + \frac{a_1 b_2 + a_2 b_1}{2}\right)$$

- Weight of fuel tank which is filled with fuel:

$$W_{f,t} = V_{f,t} * \rho_{fuel}$$

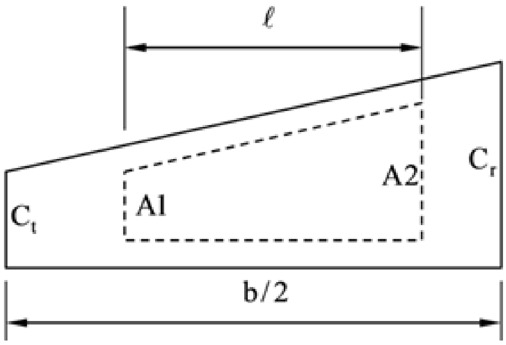


Figure 10.10: Fuel tank layout

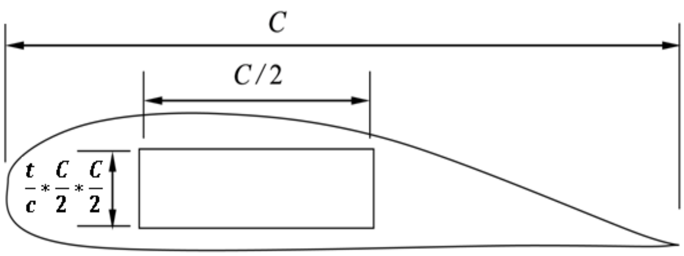


Figure 10.11: Wing section area & fuel tank cross section area approximately

❖ Wing:

- Wing cross section area:

Very crud, roughly is $S_{w,sec} = \frac{t}{c} * C * \frac{C}{2}$, In order to estimate area of the airfoil section at root or tip or other sections, Simpson's rule with graphical paper or computer aided design software are recommended.

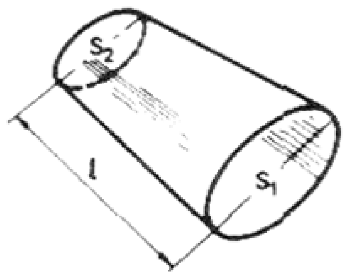
Volume of wing shape:

$$V_{wing} = 0.54\frac{S^2}{b}\left(\frac{t}{c}\right)_{root}\frac{1 + \lambda\sqrt{\tau} + \lambda\tau}{(1+\lambda)^2}$$

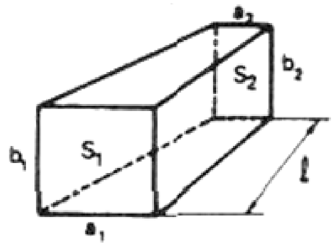
S = wing Gross area.
b = Wingspan.
$(t/c)_r$ = Thickness/chord ratio at wing root.
λ = Taper ratio.
τ = Ratio $(t/c)_{tip}/(t/c)_{root}$

- Weight of wing structure, W_w, was evaluated early.

We assume that the wing is a cantilever where wing structure weight and fuel weight are distributed linearly and homogenously. As we calculated ends cross section area, the load distribution is evaluated as flow.



Truncated pyramid



Obelisk

Figure 10.12: fuel tanks geometry for wings

$$\frac{w_1}{w_1} = \frac{A_1}{A_2}$$

$$w_1 = w_2 \frac{A_1}{A_2}$$

w_1: Weight of cross section (1) per unit length (N/m).
w_2: Weight of cross section (2) per unit length (N/m).
A_1: Area of cross section (1) per unit length (m^2).
A_2: Area of cross section (2) per unit length (m^2).

Fuel load distribution for half wing is:

$$0.5(w_1 + w_2) * \ell = 0.5W_{feul}$$

$$0.5(w_1 + w_2) * \ell = 0.5W_{wing}$$

Notes:

$$Area_{Simpson} = \frac{e}{3}\left[f_O + 4\left(f_1 + f_2 + f_3 + ..etc\right) + 2\left(f_2 + f_4 + f_6 + ..etc\right) + f_n\right]$$

Where (n) is number of sub-divisions, and must be even, ($e = (chord/n)$.

$$\frac{S_{root}}{S_{tip}} \approx \frac{C_{root} * (t/c)_{root}}{C_{tip} * (t/c)_{tip}}$$

For maximum S.F. and B.M., all weight should be multiply by maximum load factor.
It is better to use (N) for force and $(N.m)$ for moment.

❖ Contribution of concentrated loads:

The contribution of concentrated loads such as engine weight and undercarriage weight should be also considered during S.F. and B.M calculation.

10.6. Total shear force and bending moment.

The final S.F. and B.M that affect the wing is the summation of air load and inertia load contribution.

$$S.F.)_{total} = S.F.)_{air} + S.F.)_{inertia}$$

$$B.M.)_{total} = B.M.)_{air} + B.M.)_{inertia}$$

If sections taken for air and inertia load distribution are the same then the summation is simple, else use S.F. diagram and B.M. diagram for air and inertia load distribution to find the values at the same sections.

10.7. Fuselage group load distribution

Load distribution includes the mass of fuselage and any parts attached to it. Air loads are so small, except for fighters with integrated fuselage, compared with inertia loads.

The load distribution is integrated from front and rear airplane edges to amid point which usually lies on (y-axis) is passing through (1/4) the chord, usually aerodynamic mean chord. Figure 10.14 shows a typical fuselage S.F. and B.M. diagram

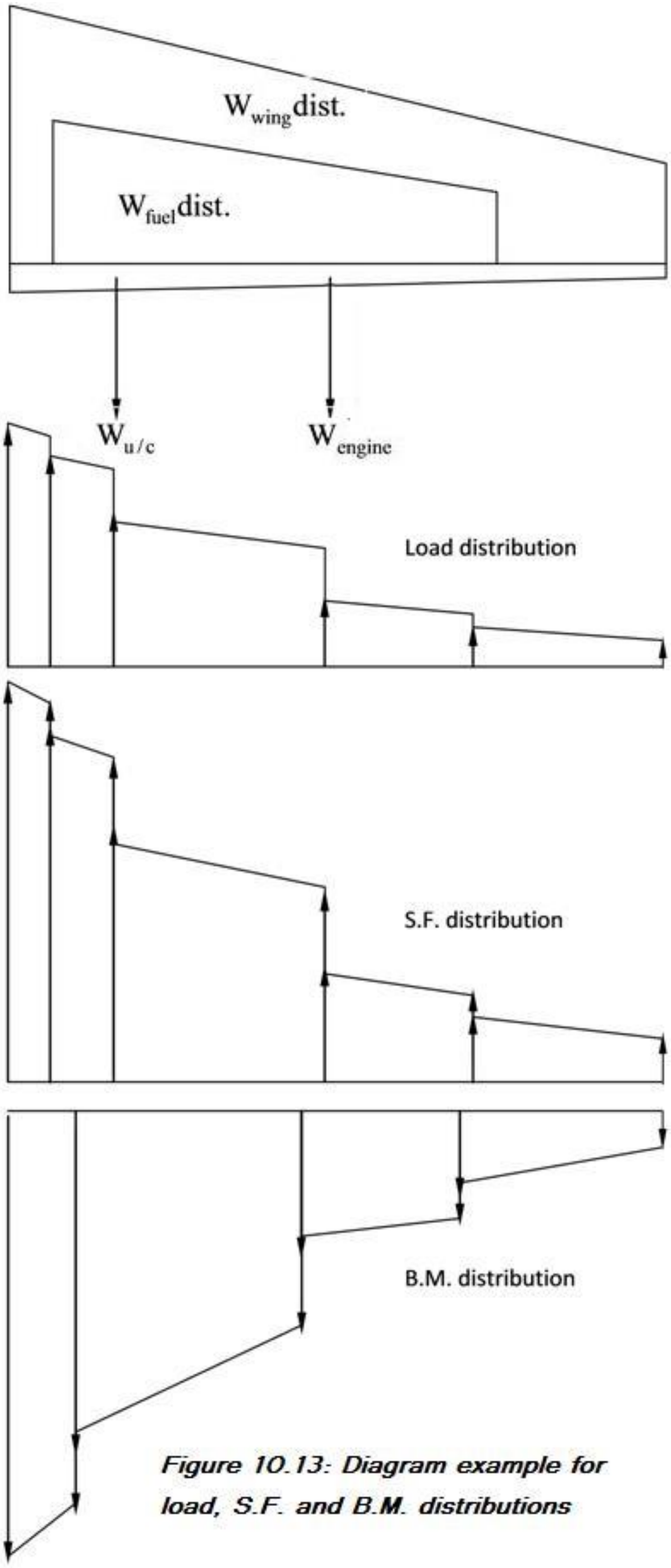


Figure 10.13: Diagram example for load, S.F. and B.M. distributions

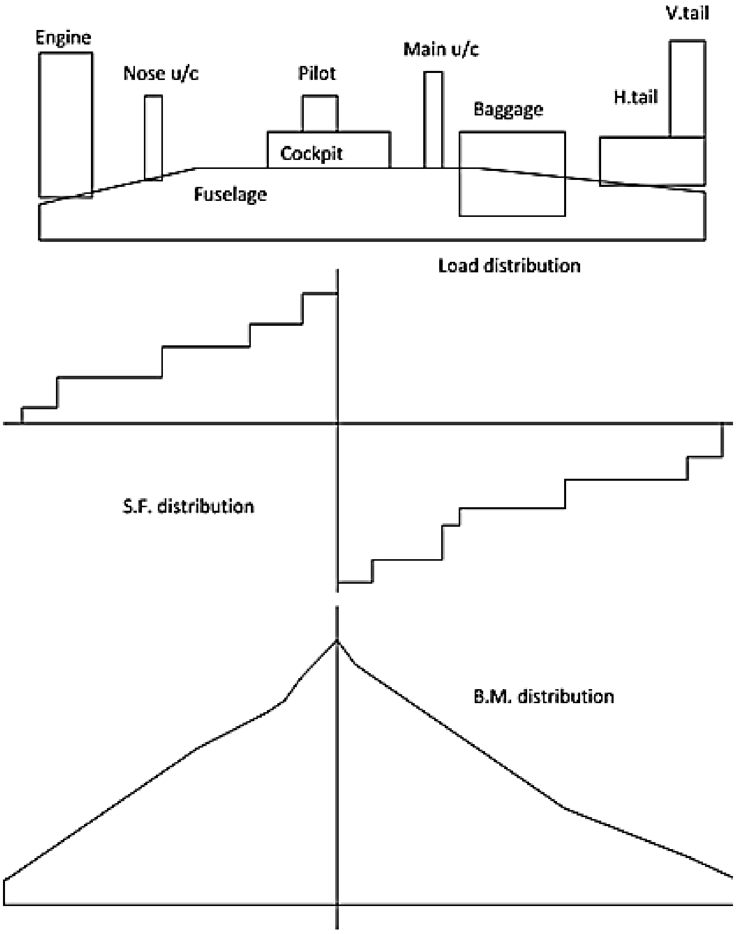


Figure 10.14: typical S.F. and B.M. diagram for fuselage group.

11. Gust and Flight Envelope

11.1. Flight envelope:

The various loading conditions are plotted against aircraft speed. For a particular aircraft to indicate the flight performance limits. This inter relationship diagram is often referred to as flight envelope or (v-n) diagram. To ensure general minimum standards of strength and safety, airworthiness regulations lay down several factors which the primary structure of the aircraft must satisfy. These factors are:

- Limited load: the maximum load that the a/c is expected to experience in normal operation. It also called "applied load".
- Proof load: the maximum load that a/c structure can withstand without distortion. The proof factor is (1.0 to 1.25).
- Ultimate load: the maximum design load which should be taken into account for various uncertainties. For civil aircraft applications, the factor of safety equals 1.5.

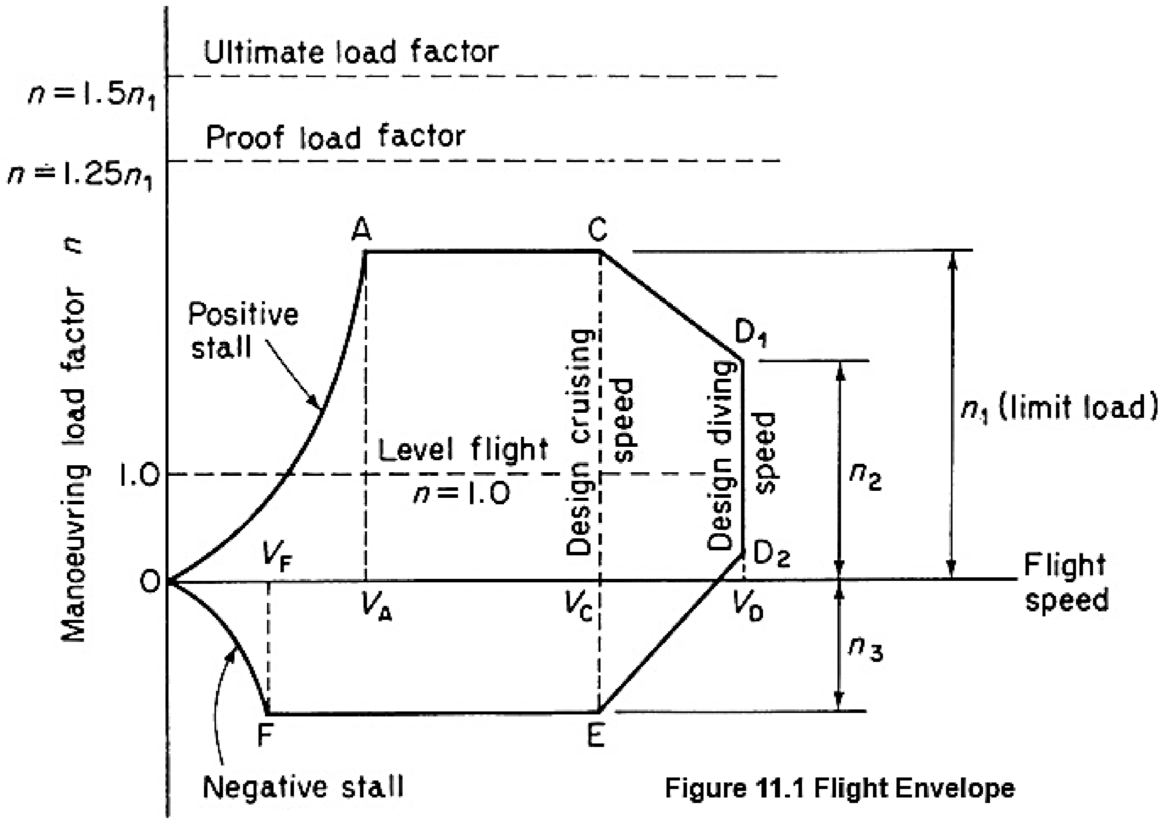


Figure 11.1 Flight Envelope

- Line OA: Limiting condition by stalling characteristics for positive value of $C_{L,max}$.
- Point A: Maximum (n) for highest angle of attack, positive value of $C_{L,max}$.
- Line AC: Maximum load factor (n) for which a/c is designed.
- Point C: Maximum (n) for lowest angle of attack, positive value of $C_{L,max}$
- Point F: Maximum (n) for highest angle of attack, negative value of $C_{L,max}$.
- Line OF: Limiting condition by stalling characteristics for negative value of $C_{L,max}$.

- Line FE: Maximum load factor (n) for negative maneuvers.
- Point E: Maximum (n) for lowest angle of attack, negative value of $C_{L,max}$.

The velocities v_C & v_E are design cruise speed and v_{D_1} & v_{D_2} are maximum diving speed. The envelope $(OACD_1D_2EFO)$ is called flight envelope or $(V-n)$ diagram for particular a/c at steady flight. Using true air speed (V_t) makes $(V-n)$ diagram to be drawn for a range of altitudes from sea level to the operation ceiling of the a/c, while using equivalent air speed (v_{eq}) makes the $(V-n)$ diagram universal.

According to European Aviation Safety Agency (EASA) Requirement, Joint Aviation Authority (JAA) requirement (JAR 25), Federal Aviation Administration(FAA) requirement (FAR 23 and FAR 25) and the International Aviation Organization (ICAO) requirement the $V-n$ diagram has the following details:

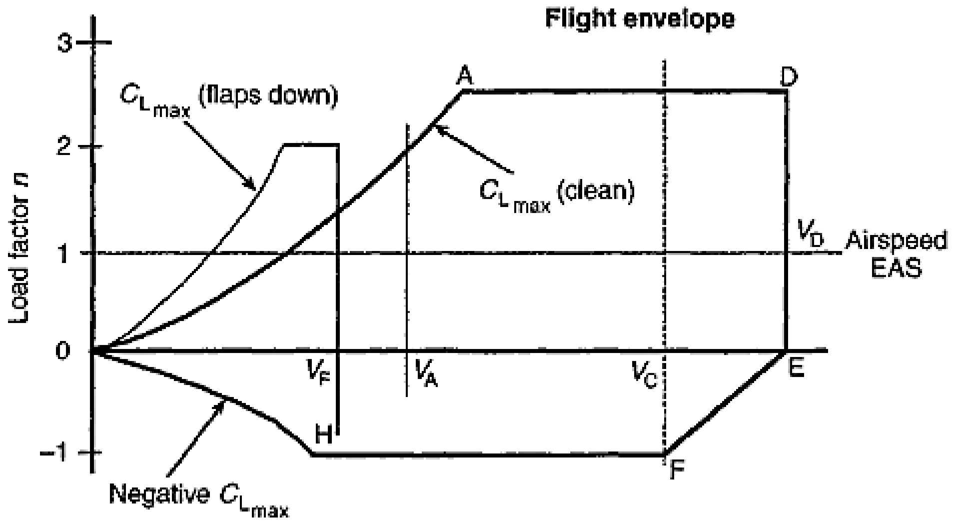


Figure 11.2 Flight Envelope

$$n = \frac{L}{W} = \frac{\rho V_t^2 C_L}{2(W/S)} = \frac{\rho_o V_{eq}^2 C_L}{2(W/S)} \qquad \dots 1$$

$$v_{eq} = \sqrt{\rho/\rho_o}\, V_t$$

Load factors are laid down by airworthiness authorities. BCAR imposed the values in table (11.1).

Load factor	Normal	Semi-aerobatics	aerobatics
n_1	$2.1+\dfrac{24000}{W+10000}$	4.5	6.0
n_2	$0.75n_1$ $but > 2.0$	3.5	4.5
n_3	-1.0	1.8	3.0

Table 11.1: Load factors according to BCAR

According to (FAR) part 25 for normal a/c:-

$$n^+ \geq 2.1 + \frac{24000}{W+10000} \quad but \quad 2.5 \leq n^+ \leq 3.8$$

$$-1.0 \leq n^- \leq -0.4\, n^+$$

According to (FAR) part 25 for transport a/c:-

$$n^+ \geq 2.1 + \frac{24000}{W+10000} \quad but \quad 3.0 \leq n^+ \leq 4.4$$

$$-1.0 \leq n^- \leq -2.0$$

Category	$n+$	$n-$
Normal	2.5 to 3.8	-1.0 to -1.5
Utility	4.4	-1.8
Aerobatic	6.0	-3.0
Home built	5.0.	-2.0
Transport	3.0 to 4.0	-1.0 to -2.0
Strategic bomber	3.0	-1.0
Tactical bomber	4.0	-2.0
Fighter	6.5 to 9.0	-3.0 to -6.0

Table 11.2: Typical load factor, FAR

Note: Weight of a/c should be in pound ($1\ kg\ =\ 2.202\ lb.$). Table (11.2) gives typical load factors according to FAR 25.

11.2. Drawing of flight envelope:

As the value of load factor (n) at each envelope corner is evaluated, the value of corresponding velocity is now computed.

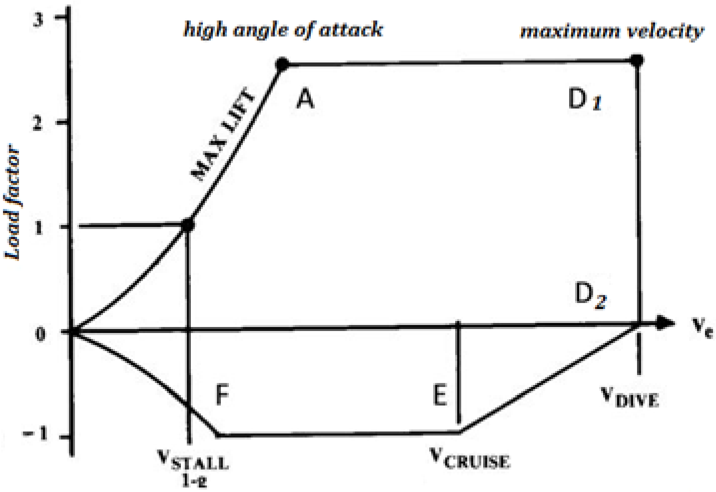


Figure 11.3: Flight envelope according to FAR

$L = 0.5\rho_o V_{eq}^2 S C_L$

$n = \frac{L}{W}$

$nW = 0.5\rho_o V_{eq}^2 S C_L$

$$V_{eq} = \sqrt{\frac{2n(W/S)}{\rho_o C_L}} \qquad \ldots 2$$

The stall velocity at $n = 1$, the subscript (eq) is omitted forth going for simplicity, is:

$$V_{stall} = \sqrt{\frac{2(W/S)}{\rho_o C_{L,max}}}$$

$$\therefore v_{eq} = v_{stall}\sqrt{n} \qquad \ldots 3$$

❖ Line OA is line for maximum and $+ve$ lift coefficient.

In order to draw line OA take five points starting from ($n = 0.0,\ 0.5,\ 1.0, \ldots n_1$) and substitute in equation (3). At ($n = 1.0$, $V_{eq} = V_{stall}$) the velocity at each point is:

$V_{OA} = V_{stall}\sqrt{n}$

❖ Line OF : The same procedure where ($n = 0.0,\ 0.25,\ 0.5, \ldots n_3$).

$V_{OF} = v_{stall}\sqrt{n}$

❖ Line AC: at this line ($n = n_1$) and the velocity of point C is cruise speed and it is evaluated according to $BCAR$ valid also for FAR):

$v_C = V_{cruise} = k\sqrt{W/S}$

$k = 38\ if\ \ W/S\ \ in\ \ lbf/ft^2\ \ then\ \ v\ in\ mph$

$k = 7.7\ if\ \ W/S\ \ in\ \ kg/m^2\ \ then\ \ v\ in\ \ m/s$

❖ Line CD: at D ($n = n_2$) and the velocity of point D is dive speed and it is evaluated as:

According to $BCAR$:

$V_D = 1.4 v_{cruise} \quad in\ \ m/s$

According to FAR:

$V_D = 1.25 v_{cruise} \quad in\ \ m/s$

$1\ knot = 1.1508\ \ m/h = 1.852\ \ km/h. \qquad 1\ mph\ \ = \ 2.237\ \ m/s$

11.3. Load factor application:-

In section (11-1) we determined load factor (n). It is necessary to relate this load factor to given type of maneuver. Two cases arise, the first involve a steady pull out from dive and the second is a correctly banked turn.

11.3.1. Load factor at steady pull out:-

Let us suppose that the aircraft has just begun its pull out from a dive so that it is describing a curved flight path but is not yet at its lowest point. The loads acting on the a/c at this stage of maneuver are shown in figure (11.3), where (R) is the radius of curvature of the flight path. (V^2/R) is the centripetal acceleration towards the center of curvature of the flight path.

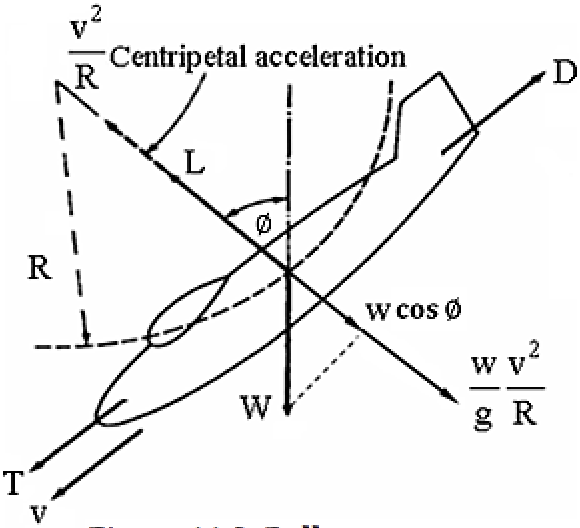


Figure 11.3: Pull out maneuver

$$L = \frac{W}{g}.\frac{V^2}{R} + W\cos\theta \qquad ...4$$

$$n = \frac{L}{W} = \frac{W}{g}.\frac{V^2}{R} + W\cos\theta \qquad ...5$$

For critical condition which is the lowest point of the pull out,$(dn/d\theta = 0)$ gives $\theta = 0.0$.

$$n = \frac{W}{g}.\frac{V^2}{R} + W \qquad ...6$$

It is quite possible for severe pull out, small (R), the a/c is over stressed by load exceeding ultimate loads. At high speed, (R) must be kept large and control surfaces movement must be limited by stops or any sufficient means. At low speed, pull out may stall the a/c so stall warning device are used as safety precautions, especially at low altitude, since for modern high speed a/c a stall can be disastrous.

11.3.2. Load factor at correctly banked turn:-

In this maneuver the a/c flies in a horizontal turn with no sideslip at constant speed. If the radius of the turn is(R) and the angle of bank is $(\emptyset)$, then the forces acting on the a/c re those shown in figure (11.4).

$$L = \sin\emptyset = \frac{W}{g}.\frac{V^2}{R} \qquad ...7$$

$$L = \cos\emptyset = W \qquad ...8$$

$$n = \frac{L}{W} = \sec\emptyset \qquad ...9$$

$$\tan\emptyset = \frac{V^2}{gR} \qquad ...10$$

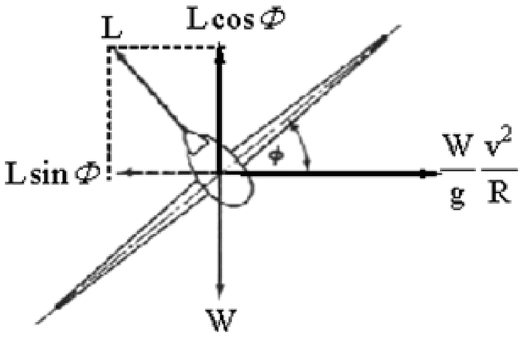


Figure 11.4: Correctly banked turn

For horizontal flight turn, the tighter the turn, i.e. (R) is reduced, the greater the angle of bank $(\emptyset)$ should be. If $(\emptyset)$ is increased load factor (n) will increased also. Aerodynamic theory shows that, for a limiting value of (n), the minimum time taken to turn through a given angle at a valve of engine thrust occurs when the lift coefficient (C_L) is a maximum, that is with the a/c on the point of stalling.

11.4. Gust envelope:-

A gust which is an ascending air current, may hit an a/c during level flight at still air. As the a/c enter the gust which has a vertical velocity. The angle of attack will increase by $(\Delta\alpha)$.

There are several examples of gust profiles and for each profile there is a method of analysis. Figure (11.5) gives an example for gust profile.

Early airworthiness requirements specified instantaneous application of gust velocity (U), as a sharp-edge gust.

To evaluate the corresponding gust loads the designer may either calculate the complete motion of a/c during the disturbance or replace the graded (ramp) gust by an equivalent sharp-edge gust.

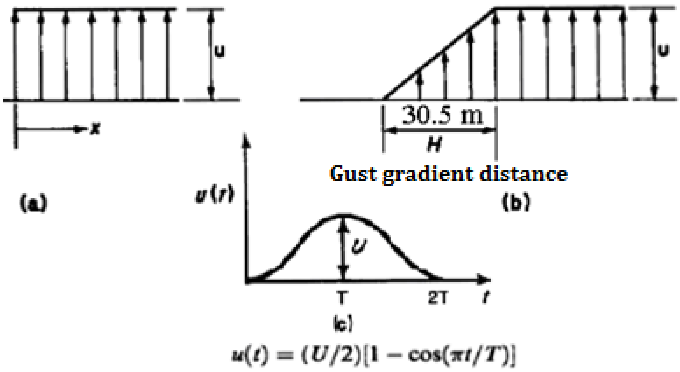


Figure 11.5: a) sharp-edged gust; b) graded gust; c) 1-cosine gust

11.4.1. Equivalent "Sharp Edge" Gust:-

The simplifying assumptions introduced in the determination of gust loads resulting from the 'sharp-edged' gust, have been discussed in the earlier part of this section. In Fig. 11.6 the aircraft is flying at a speed (V) with wing incidence α_o in still air. After entering the gust of upward velocity (u), the incidence increases by an amount $(\Delta\alpha)$:

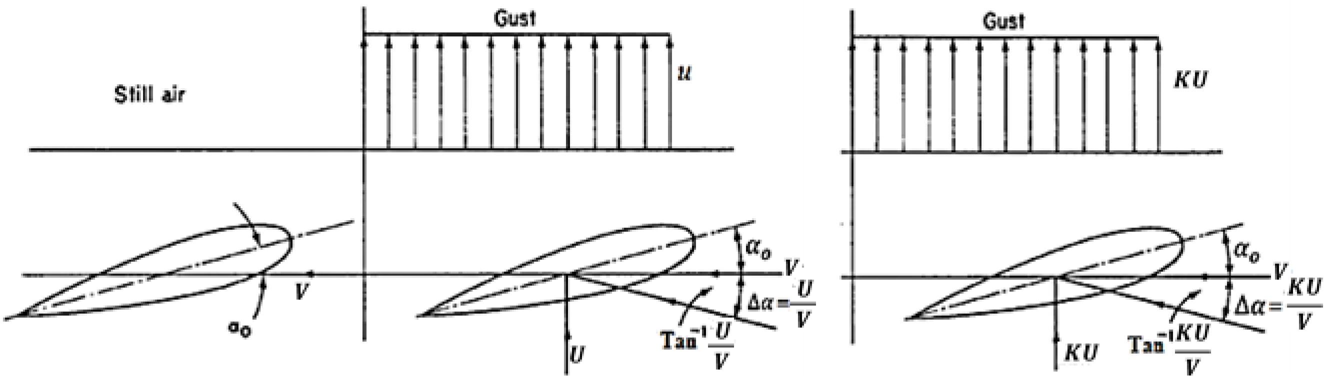


Figure 11.6: Increase in wing angle of attack due to a sharp-edged gust and graded gust

---

$\Delta\alpha = tan^{-1}\left(\frac{U}{V}\right) = \frac{U}{V}$

$\Delta\alpha \ll \alpha_o$

(U) is usually small compared with (V). This is accompanied by an increase in aircraft speed to $V = \sqrt{U^2 + V^2} \approx V$ where $\Delta\alpha \ll \alpha_o$ and $KU \ll V$

The 'graded' gust of Fig. 11.5(b) may be converted to an equivalent 'sharp-edged' gust by multiplying the maximum velocity in the gust by a *gust alleviation factor*, K. Thus:

$$\Delta\alpha = tan^{-1}\left(\frac{KU}{V}\right) = \frac{KU}{V} \qquad \ldots 11$$

$V = \sqrt{(KU)^2 + V^2} \approx V$

The increase in wing lift AL is then given by:

$$\Delta C_L = a_1\Delta\alpha = \frac{a_1KU}{V} \qquad ..12$$

$$\Delta \mathrm{L} = 0.5\rho_o V^2 S\Delta C_L = 0.5\rho_o V^2 S\frac{a_1KU}{V}$$

$$\Delta \mathrm{L} = 0.5\rho_o VSa_1KU \qquad \ldots 13$$

Neglecting the change of lift on the tail plane as a first approximation, the gust load factor Δn is:

$$\Delta n = \frac{\Delta L}{W} = \frac{0.5\rho_o VSa_1KU}{W}$$

$$\Delta n = \frac{0.5\rho_o Va_1KU}{W/S} \qquad \ldots 14$$

$$K = \frac{0.88 + \mu}{5.3 + \mu} \qquad \ldots 15$$

$$\mu = \frac{2(W/S)}{g\rho_o\bar{\bar{C}}a_1} \qquad \ldots 16$$

V: Aircraft equivalent air speed in (m/s^2).

U: Gust speed in (m/s^2).

a_1: Wing lift curve slope

K: Gust alleviation (effectiveness) factor.

λ: Aerodynamic mass ratio.

W/S: Wing loading in (N/m^2).

α_o: Angle of attack in (rad).

$\Delta\alpha$: Increment of α_o due upward gust velocity.

$U = 20 \;\; m/s \;\; for\, points\, A\, \&\, F$

$U = 15.25 \;\; m/s \;\; for\, points\, C\, \&\, E$

$U = 7.50 \;\; m/s \;\; for\, points\, D_1\, \&\, D_2$

When a/c in level flight the load factor is unity before striking the gust ($n = 1$), then:

For up gust:

$n_g = 1 + \Delta n$

$$n_g = 1 + \frac{0.5\rho_o V a_1 KU}{W/S} \qquad \dots 17$$

For down gust:

$\mathrm{n}_g = 1 - \Delta n$

$$\mathrm{n}_g = 1 - \frac{0.5\rho_o V a_1 KU}{W/S} \qquad \dots 18$$

For tail plane contribution, the change in tail plane angle of attack ($\Delta\alpha_t$) is not equal to the change in wing angle of attack ($\Delta\alpha_w$), due to downwash effect at tail.

$$\Delta L_t = 0.5\rho_o V^2 S_t \Delta C_{L,t} = 0.50.5\rho_o V^2 S_t a_t \Delta\alpha \qquad \dots 19$$

$$\Delta L = \Delta L_w + \Delta L_t \qquad \dots 20$$

Neglecting any change in a/c velocity (V) at wing and tail.

$$\Delta \mathrm{n}_g = \frac{\Delta L}{W} = \frac{\Delta L_w + \Delta L_t}{W} = \frac{\rho_o VKU}{2W}(S_w a_w + S_t a_t)$$

$$\Delta \mathrm{n}_g = \frac{\rho_o VKU}{2\, W/S_w}\left(a_w + \frac{S_t}{S_w} a_t\right) \qquad \dots 21$$

$$\mathrm{n}_g = 1 \mp \Delta n \qquad \dots 22$$

For first simple approximation, take a/c overall lift coefficient, which can be evaluated from wind tunnel tests, to replace for ($\Delta L = \Delta L_w + \Delta L_t$). For no further information the tail term may be neglected for the sack of simplicity

a_w: Wing lift-angle of attack curve slope ($1/rad$).

a_t: Horizontal tail lift-angle of attack curve slope ($1/rad$).

S_w: Wing area (m^2).

S_t: Horizontal tail area (m^2).

Lift – angle of attack slope should is taken from earlier work in wing design chapter, but a rough approximation is:-

$$a \approx \frac{a_o}{1 + \frac{a_o}{\pi AR}} \approx \frac{a_o}{1 + \frac{2}{AR}}$$

The value of (n_g) is evaluated from equation (17 & 18) or equation (22) at each envelope corner and a diagram like the one in figure (11-7) should be drawn.

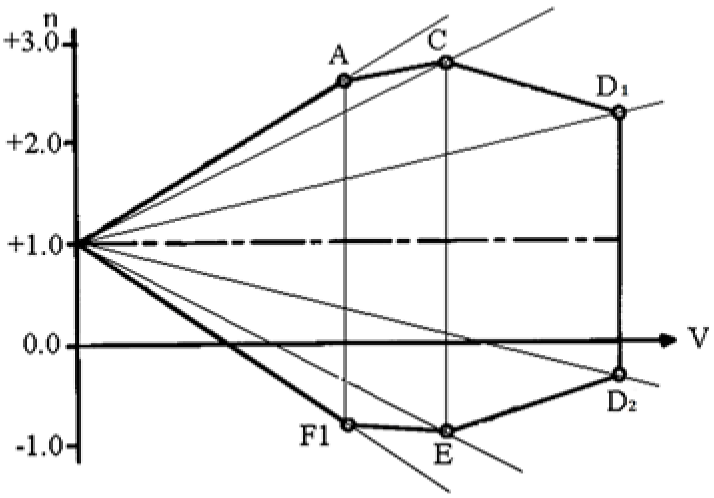


Figure 11.7: Typical gust envelope

11.5. Flight-Gust Envelope:-

I	point	V	U	n_f	n_g	$n_{largest}$	Remarks
1	$\acute{A}$						Stall
2	$\acute{C}$						Cruise
3	$\acute{D}_1$						Dive
4	$\acute{D}_2$						Dive
5	$\acute{E}$						Cruise
6	$\acute{F}$						Stall

Table 11.3: typical flight and gust load factor arrangement

The final result of calculations of load factors for flight and gust conditions should be tabulated and a flight-gust envelope for the largest values of n at each point should be tabulated as in table (11.3) and be drawn as in figure (11.8).

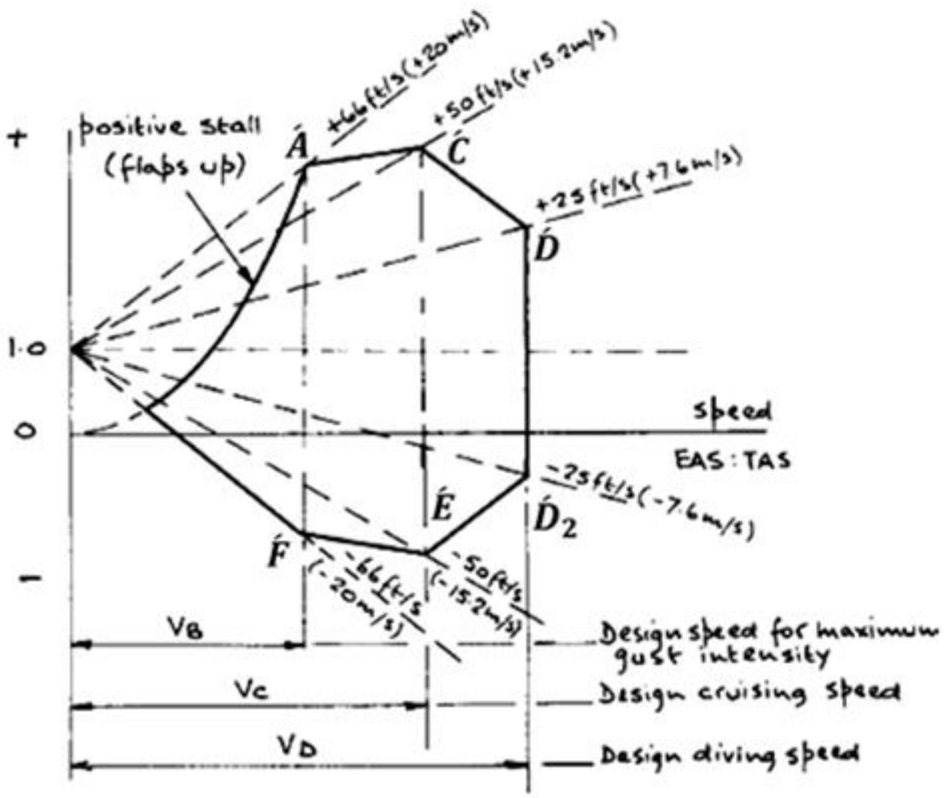


Figure 11.8: Typical Flight-Gust Envelope

chapter 12. Wing Lift, L_W and Tail Lift, L_T at Symmetric Maneuvering Conditions.

12.1. Introduction:

The subject is related to the evaluation of wing lift and tail lift at all corners points of flight-gust envelope which represents different at symmetric maneuvering (flight) Conditions. At each envelope point, load factor and velocity are defined and they for certainly will affect the magnitude and direction of air load on wing and tail.

The attitude of aircraft whether it is in steady level flight or not also affects the magnitude and direction of the wing and tail air load. Where, for angular acceleration or deceleration, the aircraft pitching moment inertia, B, has a great influence in the calculations.

Lastly, wing pitching of moment, $M_{a.c.}$, which is a wing character has also the same influence. This type of pitching moment should be transfer from aerodynamic center to aircraft center of gravity and the new pitching moment, M_o, should be used.

As the location of the wing and tail has been decided early, then all necessary dimensions have been calculated. Since aircraft center of gravity was estimated early, then the distance from (c.g.) for each aircraft component to aircraft c.g. is known. So these air loads are calculated as follow:

12.2. Moments of inertia:

According to class II method (a more detailed method) described by Dr. Jan Roskam, the aircraft pitching moment of inertia is evaluated as follow:

$$I_{yy} = \sum_{1}^{n} I_{yy,i} + \sum_{1}^{n} m_i \left[\left(x_i - x_{c.g.}\right)^2 + \left(z_i - z_{c.g.}\right)^2 \right] \qquad 12.1$$

The first term in Eqn. (12.1) represents the moment (or product) of inertia of component i about its own center of gravity.

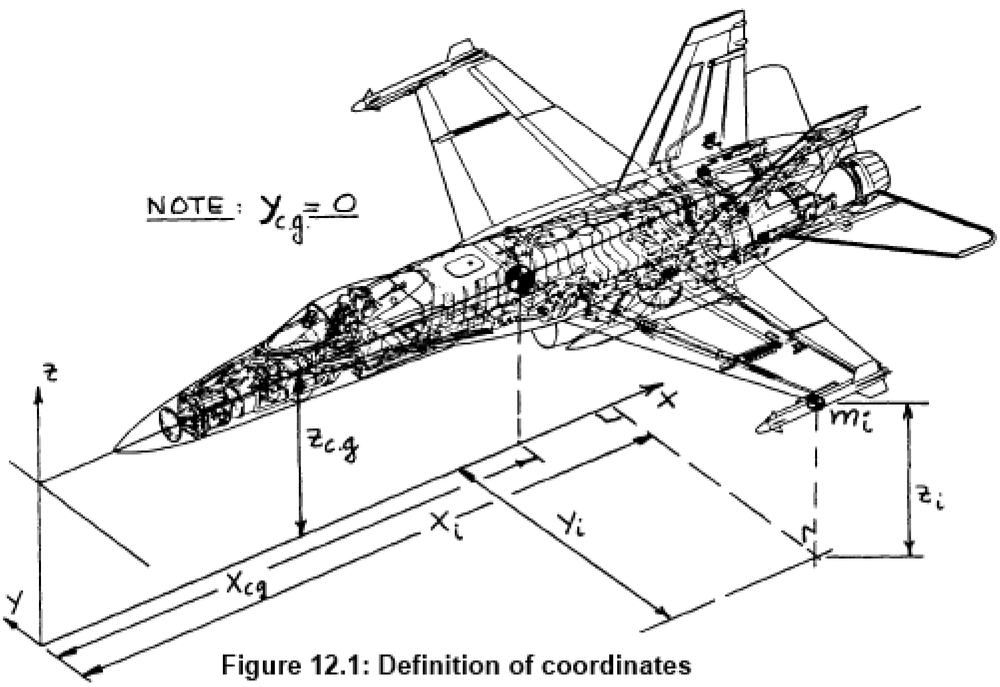


Figure 12.1: Definition of coordinates

Moments (and products) of inertia of airplane components about their own center of gravity can be computed in a relatively straightforward manner by assuming uniform mass distributions for structural components. An example of the latter would be the airplane fuel system. Major fuel system components such as pumps, bladders and the like can be considered to be concentrated masses distributed around the fuel system c.g. Equations (12.1) is simplified to compute the moments of inertia of the fuel system about its own e.g.

$$I_{yy}=\sum_{1}^{n} m_i\left[\left(x_i-x_{c.g.}\right)^2+\left(z_i-z_{c.g.}\right)^2\right] \qquad 12.2a$$

For rolling moment of inertia, I_{xx}:

$$I_{xx}=\sum_{1}^{n} m_i\left[\left(y_i-y_{c.g.}\right)^2+\left(z_i-z_{c.g.}\right)^2\right] \qquad 12.2b$$

For yawing moment of inertia, I_{zz}:

$$I_{zz}=\sum_{1}^{n} m_i\left[\left(y_i-y_{c.g.}\right)^2+\left(x_i-x_{c.g.}\right)^2\right] \qquad 12.2c$$

And the cross moment of inertia, I_{xy}, I_{xz} and I_{yz} are evaluated as:

$$I_{xy}=\sum_{1}^{n} m_i\left[\left(y_i-y_{c.g.}\right)\left(x_i-x_{c.g.}\right)\right] \qquad 12.3a$$

$$I_{yz}=\sum_{1}^{n} m_i\left[\left(y_i-y_{c.g.}\right)\left(z_i-z_{c.g.}\right)\right] \qquad 12.3b$$

$$I_{zx}=\sum_{1}^{n} m_i\left[\left(z_i-z_{c.g.}\right)\left(x_i-x_{c.g.}\right)\right] \qquad 12.3c$$

For symmetric airplane the $\left(y_i-y_{c.g.}\right)\approx zero$ then $I_{xy}=0$ and $I_{yz}=0$. And also the term I_{zx} is not important at preliminary design stage. Taking into consideration that the term $\left(x_i-x_{c.g.}\right)^2 \gg \left(z_i-z_{c.g.}\right)^2$ so then the system of equation is reduced to:

$$I_{yy}\approx I_{zz}=\sum_{1}^{n} m_i\left(x_i-x_{c.g.}\right)^2 \gg I_{xx} \qquad 12.3a$$

$$I_{xx}=\sum_{1}^{n} m_i\left(z_i-z_{c.g.}\right)^2 \qquad 12.3b$$

According to class I method (a less detailed method) described by Dr. Jan Roskam, the moments of inertia of the airplane in terms of the radius of gyration for the airplane are then found from the following equations:

$$I_{xx} = mR_x^2 = m.b^2.\bar{R}_x^2/4 \qquad 12.4a$$

$$I_{yy} = mR_y^2 = m.L^2.\bar{R}_y^2/4 \qquad 12.4b$$

$$I_{zz} = mR_z^2 = m.Z^2.\bar{R}_z^2/4 \qquad 12.4c$$

Where $\bar{R}_x^2$, $\bar{R}_y^2$ and $\bar{R}_z^2$ are a non-dimensional radius of gyration and they are:

$\bar{R}_x = 2R_x/b$

$\bar{R}_y = 2R_y/L$

$\bar{R}_z = 2R_z/Z$

$Z = (b + L)/2$

Where b is airplane span and L is airplane length. Airplanes of the same mission orientation tend to have similar values for the non-dimensional radius of gyration. Tables 12.l through 12.l2 present numerical values for these non-dimensional radii of gyration for different types of airplanes.

The procedure for estimating inertias as follow

1) List the values of W_{to} or W_{emp}, b, L and the size Z for the airplane being designed.
2) Select values for the non-dimensional radii of gyration corresponding to W_{to} or W_{emp}.
3) Compute the airplane moments of inertia from Eqs 12.4.
4) Compare the estimated inertias with the data of Figures 12.2 through 12.4. If the comparison is poor make adjustments.

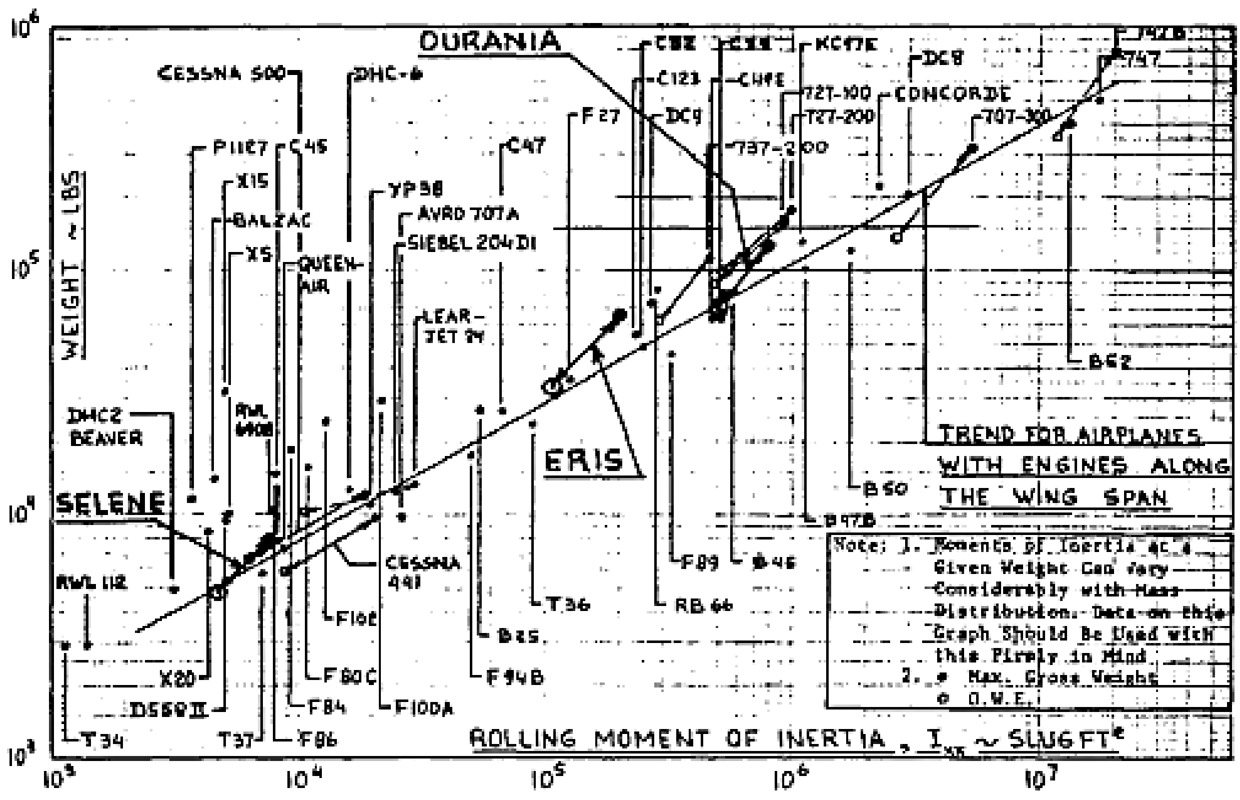


Figure 12.2:correlation of rolling moment of inertial with weight

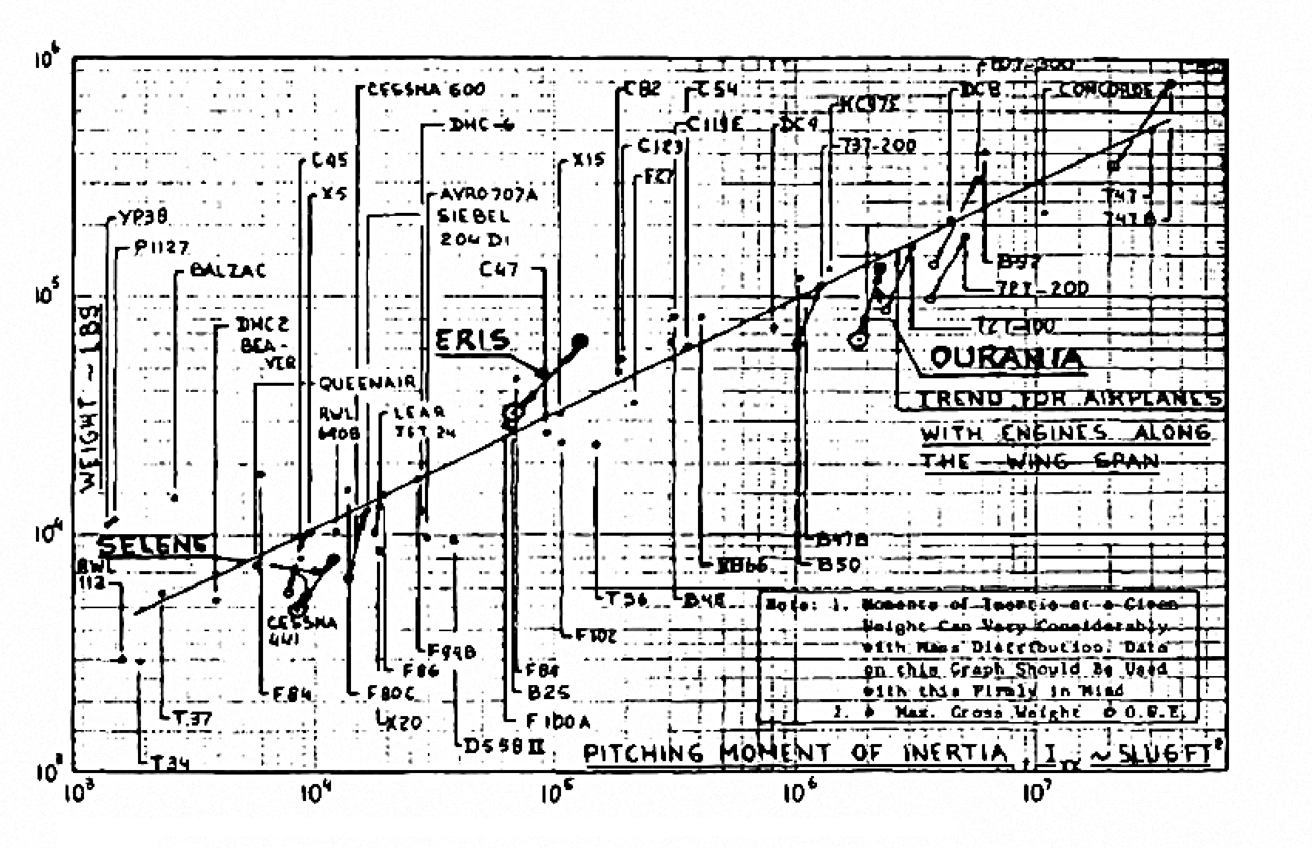


Figure 12.3: correlation of pitching moment of inertia with wight

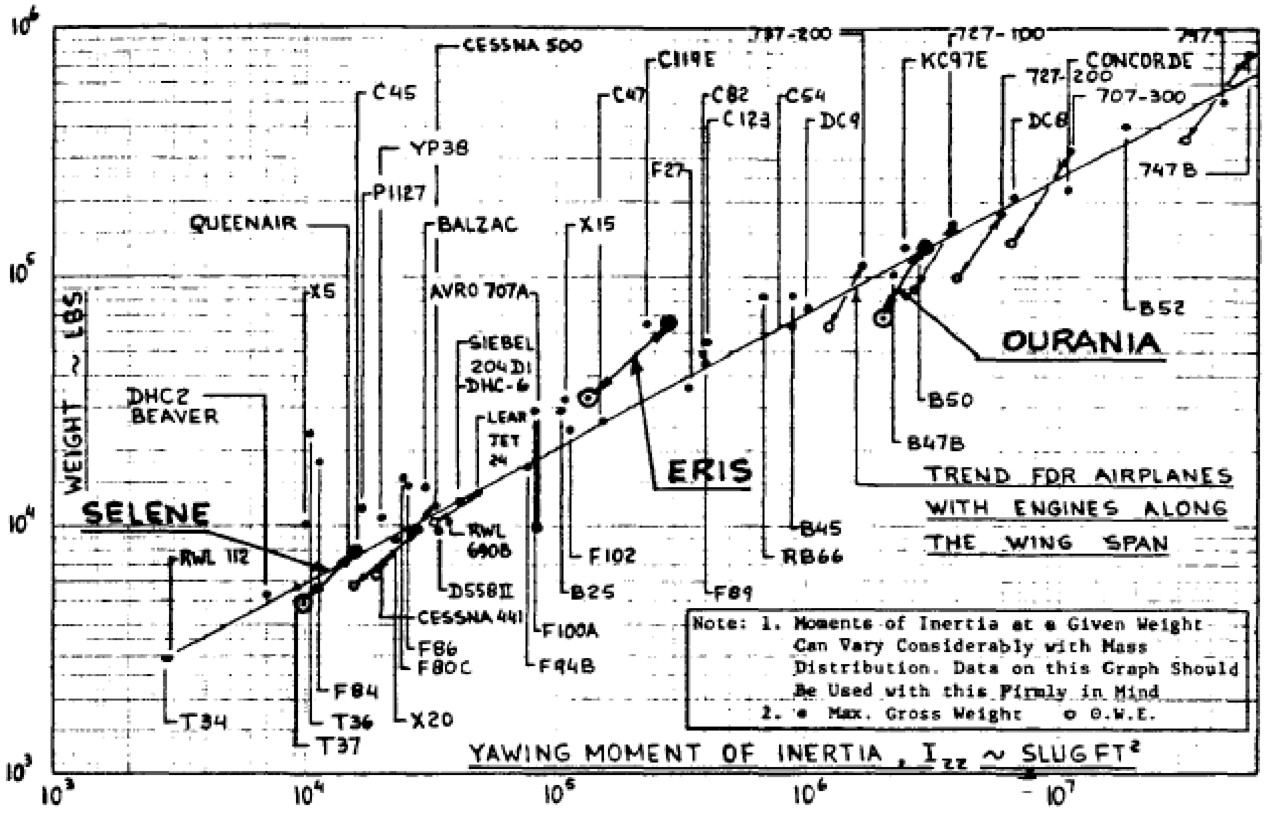


Figure 13.4: correlation of yawing moment of inertia with weight

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Table B1 Non-dimensional Radii of Gyration for Homebuilt Propeller Driven Airplanes

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition

At the time of printing, no data were available for this type airplane

Table B2 Non-dimensional Radii of Gyration for Single Engine Propeller Driven Airplanes

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Beech N-35*	3,125	32.8	25.1	29.0	0.248	0.338	0.393	1 in fusel.
Cessna 150M**	1,127	33.5	21.5	27.5	0.254	0.405	0.418	1 in fusel.
Cessna 172M**	1,477	36.2	26.5	31.4	0.242	0.386	0.403	1 in fusel.
Cessna 177A**	1,761	35.6	27.0	31.3	0.212	0.362	0.394	1 in fusel.
Cessna R182**	1,885	36.2	28.0	32.1	0.342	0.397	0.393	1 in fusel.
Cessna 210K***	2,700	36.8	28.3	32.6	0.222	0.356	0.379	1 in fusel.

*at W_{TO} **at W_{OE} ***atW_{OE} plus 25 percent fuel

Note: one pilot included in all data

Table B3 Non-dimensional Radii of Gyration for Twin Engine Propeller Driven Airplanes

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Beech 55	4,880	37.8	25.7	31.8	0.260	0.329	0.399	2 on wing
Beech 95	4,000	37.8	25.3	31.6	0.251	0.327	0.391	2 on wing
Beech D-50	6,500	45.9	31.5	38.7	0.240	0.313	0.384	2 on wing
Beech D18S	9,000	47.7	34.2	41.0	0.232	0.360	0.396	2 on wing
Cessna 402*	5,000	39.9	36.3	38.1	0.414	0.278	0.502	2 on wing
Cessna 402	6,200	39.9	36.3	38.1	0.373	0.269	0.461	2 on wing
Cessna 404*	4,851	46.7	39.5	43.1	0.324	0.318	0.446	2 on wing
Cessna 404	8,400	46.7	39.5	43.1	0.340	0.284	0.445	2 on wing
Cessna 441*	5,642	49.3	39.0	44.2	0.285	0.345	0.429	2 on wing
Cessna 441	9,925	49.3	39.0	44.2	0.256	0.212	0.336	2 on wing

*at W_E

Table B4 Non-dimensional Radii of Gyration for Agricultural Airplanes

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition

At the time of printing, no data were available for this type of airplane

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Table B5 Non-dimensional Radii of Gyration for Business Jets

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Morane/S 760	7,066	33.3	32.9	33.1	0.374	0.328	0.486	2 in W/F
Lockh. Jetstar	39,288	53.7	58.8	56.3	0.370	0.356	0.503	4 on fusel.
Cessna 500*	6,505	47.1	43.5	45.3	0.236	0.384	0.430	2 on fusel.
Cessna 500**	12,000	47.1	43.5	45.3	0.306	0.303	0.423	2 on fusel.
Cessna 550*	7,036	51.7	47.2	49.5	0.243	0.400	0.447	2 on fusel.
Cessna 550**	13,500	51.7	47.2	49.5	0.293	0.312	0.420	2 on fusel.

*at W_e **at W_{TO}

Table B6 Non-dimensional Radii of Gyration for Regional Turbopropeller Driven Airplanes

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Fokker F-27A	38,500	95.2	77.2	86.2	0.235	0.363	0.416	2 on wing
DHC6 Twin Otter	12,500	65.0	51.8	58.4	0.203	0.326	0.350	2 on wing

Table B7a Non-dimensional Radii of Gyration for Jet Transports

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Convair 880	185,000	120.0	124.2	122.1	0.320	0.342	0.465	4 on wing
Convair 880	191,500	120.0	124.2	122.1	0.322	0.339	0.464	4 on wing
Convair 990	240,000	120.0	134.8	127.4	0.335	0.338	0.473	4 on wing
Convair 990	245,000	120.0	134.8	127.4	0.305	0.334	0.472	4 on wing
Boeing 727-100	165,000	108.0	133.2	120.6	0.249	0.375	0.452	3 on fusel.
Boeing 727-100*	89,000	108.0	133.2	120.6	0.247	0.442	0.518	3 on fusel.
Boeing 727-200	180,000	108.0	153.2	130.6	0.248	0.394	0.502	3 on fusel.
Boeing 727-200*	100,000	108.0	153.2	130.6	0.240	0.451	0.550	3 on fusel.
Boeing 737-200	113,000	93.0	100.0	96.5	0.246	0.382	0.456	2 on wing
Boeing 737-200*	62,000	93.0	100.0	96.5	0.264	0.456	0.517	2 on wing
Boeing 747-100B	800,000	195.7	231.3	213.5	0.290	0.329	0.445	4 on wing
Boeing 747-100B*	350,000	195.7	231.3	213.5	0.332	0.380	0.508	4 on wing
McDD DC9-10	74,000	89.4	104.3	96.9	0.242	0.360	0.435	2 on fusel.
McDD DC8	210,000	142.4	150.5	146.5	0.301	0.349	0.434	4 on wing

*at W_{OE}

Table B7b Non-dimensional Radii of Gyration for Piston-Propeller Driven Transports

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Lockheed L-749A	107,000	123.0	95.2	109.1	0.300	0.298	0.426	4 on wing
Lockheed L-1049	120,000	123.0	113.6	118.3	0.316	0.336	0.448	4 on wing
Lockheed L-1649	146,500	150.0	116.2	133.1	0.371	0.278	0.473	4 on wing
Douglas DC-4	60,360	138.3	97.6	118.0	0.250	0.320	0.388	4 on wing
Douglas DC-6	97,200	117.5	100.5	109.0	0.322	0.324	0.456	4 on wing
Airspeed Ambass.	49,500	115.0	80.4	97.7	0.278	0.314	0.400	2 on wing
Martin 404	45,000	93.3	74.6	84.2	0.272	0.378	0.444	2 on wing
Convair T-240	41,800	91.7	74.7	83.2	0.286	0.351	0.443	2 on wing
Convair T-340	44,500	105.7	79.2	92.4	0.308	0.345	0.457	2 on wing
Beech Twin Quad	20,000	70.0	52.7	61.4	0.225	0.303	0.346	4 in wing

Table B7c Non-dimensional Radii of Gyration for Turbo-Propeller Driven Transports

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Bristol 175(k)*	103,000	130.0	110.0	120.0	0.317	0.356	0.455	4 on wing
Bristol 167(1)**	187,000	230.0	177.0	203.5	0.330	0.356	0.478	4 on wing
Lockh. Electra	116,000	99.0	104.7	101.9	0.394	0.341	0.497	4 on wing

*Britannia **Brabazon

Table B8 Non-dimensional Radii of Gyration for Military Trainers

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2 ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Cessna T-37A	6,300	38.4	30.0	34.2	0.220	0.142	0.245	2 in fusel.

Table B9a Non-dimensional Radii of Gyration for Fighters (Jet)

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
McD F2H-1	14,413	41.6	40.2	40.9	0.230	0.359	0.465	2 in W/F
McD F3H-2N	26,878	35.3	58.8	47.1	0.252	0.107	0.449	1 in fusel.
McD F-101A	36,969	39.7	67.4	53.6	0.209	0.329	0.428	2 in fusel.
VS Attacker	10,450	36.9	37.3	37.1	0.244	0.328	0.400	1 in fusel.
DH Vampire 20	10,891	40.0	30.1	35.1	0.286	0.318	0.409	1 in fusel.
Gl. Meteor II	11,100	43.0	41.4	42.2	0.286	0.330	0.404	2 in wing
Lockheed F-80A	11,940	38.9	34.3	36.6	0.286	0.356	0.444	1 in fusel.
Lockheed F-94B	13,650	37.5	40.1	38.8	0.284	0.396	0.488	1 in fusel.
Lockheed F-104G	20,900	21.9	54.8	38.4	0.224	0.392	0.563	1 in fusel.
NAA F-86A	13,900	37.1	37.5	37.3	0.266	0.346	0.400	1 in fusel.
NAA FJ-3	16,883	37.0	37.6	37.3	0.281	0.352	0.438	1 in fusel.
NAA F-100D	29,800	38.0	47.0	42.5	0.252	0.376	0.462	1 in fusel.
Vought XF8U-1	21,300	35.7	54.4	45.1	0.225	0.404	0.507	1 in fusel.
Vought F8U-3	30,600	40.0	58.9	49.5	0.225	0.375	0.467	1 in fusel.
GD XF-91	18,600	31.3	43.3	37.3	0.323	0.424	0.548	1 in fusel.
GD TF-102A	32,859	38.1	63.2	50.7	0.295	0.386	0.520	1 in fusel.
GD F-106B	36,834	38.3	70.7	54.5	0.247	0.379	0.516	1 in fusel.
Northrop F-89D	38,000	58.0	54.0	56.0	0.440	0.304	0.532	2 in fusel.
Republic RF-84F	19,000	33.6	47.5	40.6	0.310	0.308	0.432	1 in fusel.
Republic F-105D	34,058	35.0	64.4	49.7	0.231	0.425	0.567	1 in fusel.
Grumman F9F-8	16,744	34.5	41.9	38.2	0.248	0.374	0.454	1 in fusel.
Grumman XF10F-1	26,160	36.8	57.8	46.9	0.251	0.323	0.414	1 in fusel.
Grumman F11F-1	16,500	31.6	40.8	36.2	0.221	0.404	0.484	1 in fusel.

Table B9b Non-dimensional Radii of Gyration for Fighters (Propeller)

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Brewster Buffalo	5,066	35.0	26.0	30.5	0.208	0.358	0.374	1 in fusel.
Seversky P35	5,788	36.0	26.8	31.4	0.198	0.367	0.360	1 in fusel.
VS Spitfire-I	6,250	36.8	29.9	33.4	0.240	0.334	0.384	1 in fusel.
BP Defiant	6,410	39.4	35.0	37.2	0.234	0.360	0.404	1 in fusel.
Curtiss P36	6,825	37.3	31.7	34.5	0.172	0.356	0.370	1 in fusel.
Bell P39	7,533	34.0	30.0	32.0	0.276	0.340	0.425	1 in fusel.
Grumman F6F	10,560	42.8	33.5	38.2	0.242	0.346	0.404	1 in fusel.
Hawker Typhoon	11,017	41.5	31.7	36.6	0.277	0.300	0.394	1 in fusel.
Republic P47	12,500	40.7	36.0	38.4	0.296	0.322	0.428	1 in fusel.
Vought F4U	12,850	41.0	34.5	37.8	0.268	0.360	0.420	1 in fusel.
Bl.Firebr'd-III	13,660	49.8	38.2	44.0	0.250	0.300	0.397	1 in fusel.
Westland Welkin	18,340	70.0	42.0	56.0	0.270	0.304	0.408	2 in wing
Bristol Beauftr	22,635	57.8	42.5	50.2	0.330	0.299	0.447	2 in wing
Bristol Brigand	39,000	71.7	46.4	59.1	0.299	0.338	0.438	2 in wing

Table B10a Non-dimensional Radii of Gyration for Bombers (Piston-Propeller)

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Martin B-26	26,600	65.0	57.6	61.3	0.270	0.320	0.410	2 on wing
HP Halifax	55,000	99.0	71.6	85.3	0.346	0.306	0.395	4 on wing
Shorts Stirling	64,000	99.0	87.3	93.2	0.360	0.330	0.488	4 on wing
Boeing B-29	105,000	141.0	99.0	120.0	0.316	0.320	0.376	4 on wing
Boeing B-50	120,000	141.0	99.0	120.0	0.304	0.332	0.450	4 on wing
GD B-36	357,500	230.0	162.0	196.0	0.316	0.262	0.428	6 in wing

Table B10b Non-dimensional Radii of Gyration for Bombers (Jet)

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
Martin XB-51	53,785	53.0	81.0	67.0	0.194	0.404	0.498	3 in/on fus.
Martin B57A	48,554	64.0	66.0	65.0	0.312	0.278	0.412	2 in wing
Boeing XB-47	125,000	116.0	107.0	111.5	0.346	0.320	0.474	4 in wing
Boeing B-52A	390,000	185.0	156.5	170.8	0.346	0.306	0.466	8 on wing
Northrop RB-49A	213,500	172.0	53.0*	112.5	0.316	0.316	0.510	6 in wing
NAA B-45A	82,600	89.0	75.3	82.2	0.325	0.290	0.438	4 on wing
NAA B-45C	82,600	89.0	75.3	82.2	0.340	0.299	0.456	4 on wing

*Flying wing

Table B10c Non-dimensional Radii of Gyration for Military Patrol Airplanes (Propeller Driven)

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
<u>Piston-Propeller Driven</u>								
Lockheed P2V-4	67,500	100.0	81.6	90.8	0.368	0.300	0.484	2 on wing
Lockheed P2V-7*	67,500	100.0	91.7	95.9	0.372	0.266	0.462	4 on wing
Grumman S2F-3	26,147	69.7	43.5	56.6	0.240	0.347	0.387	2 on wing
Grumman W2F-1	41,549	80.6	56.3	68.4	0.235	0.366	0.387	2 on wing
<u>Turbo-propeller Driven</u>								
Lockheed P3V-1	127,200	99.7	116.8	108.3	0.357	0.249	0.421	4 on wing

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Table B10d Non-dimensional Radii of Gyration for Military Transports

(Propeller Driven)

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
<u>Piston-Propeller Driven</u>								
Douglas C-54	61,840	117.5	94.0	105.8	0.286	0.294	0.406	4 on wing
Fairchild C-119B in wing	64,000	109.3	88.5	98.9	0.287	0.282	0.390	2 0n wing
Boeing C-97	128,340	141.2	110.3	125.8	0.276	0.325	0.424	4 on wing
GD XC-99	265,000	230.0	182.5	206.3	0.276	0.346	0.432	6 on wing
<u>Turbo-propeller Driven</u>								
Lockheed C-130B	135,000	132.6	97.8	115.2	0.363	0.319	0.489	4 on wing
Lockheed C-130E	155,000	132.6	97.8	115.2	0.375	o.290	0.486	4 on wing

*Has two jet engines outboard of the piston engines

Table B11 Non-dimensional Radii of Gyration for Flying Boats

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	$\bar{R}_x$	$\bar{R}_y$	$\bar{R}_z$	Number of engines and disposition
XBTM-1	20,009	50.0	41.2	45.6	0.230	0.340	0.380	1 in fusel.
XP4M-1	80,000	114.0	84.0	99.0	0.248	0.320	0.414	2 on wing
VS Seagull	14,230	52.5	44.0	48.3	0.297	0.364	0.402	1 in WF

Table B12 Non-dimensional Radii of Gyration for Supersonic Cruise Airplanes

Airplane Type	GW lbs	Wing Span, b, ft	Total Length, L, ft	e = (b+L)/2, ft	R_x	R_y	R_z	Number of engines and disposition
NAA A3J-1	44,305	53.0	72.5	62.8	0.240	0.372	0.472	2 in fusel.

12.3. Symmetric maneuvering (flight) conditions:

For vertical equilibrium, from figure (12.5.):

$$L_w + L_t = nW \qquad \text{... 12.5}$$

Taking moment about airplane center of gravity in the plane of symmetry,(vertical plane divided the a/c into two symmetry halves):

$$L_w * a + M_o - L_w * \ell_t = \pm B\dot{q} \qquad \text{... 12.6}$$

$$M_o = 0.5\rho_o V^2 S\bar{C} C_{M_o} \qquad \text{... 12.7}$$

For level flight, there is no angular acceleration, i.e. ($\dot{q} = 0$) then eq. (12.2) becomes:

$$L_w * a + M_o - L_w * \ell_t = 0 \qquad \text{... 12.8}$$

$\dot{q}$: Angular acceleration, rate of change of angular velocity in ($radians/s^2$).

V: Aircraft velocity, ft/s or m/s.

B : Pitching moment of inertia, $lb.ft^2$ or $kg.m^2$

W: Airplane weight, lbs or N.

m_i: Mass of each part of the airplane, lb or kg.

X_i : Distance from each part c.g. to airplane c.g. ft or m.

C_{M_o}: Pitching moment coefficient about airplane c.g., which is evaluated during wind tunnel test.

If (C_{M_o}) is not available then take $(C_{M_{a.c.}})$ about airplane a.c. If the latter is not available also, then take $(c_{M_{a.c.}})$ from NACA data sheet for the airfoil you chosen which is a two-dimensional pitching momentum.

M_o: pitching moment about a/c c.g. it comes from transferring $M_{a.c}$ from aerodynamic center to aircraft center of gravity.

$C_{Ma.c.}$: pitching moment coefficient about a/c aerodynamic center.

$c_{M_{a.c.}}$: A two-dimensional, airfoil pitching moment about a.c.

For the analysis of symmetric maneuvering conditions are mainly consist of :

a) Maneuvering balanced conditions. Assuming the airplane to be in equilibrium with zero pitching acceleration, the maneuvering conditions (A) through (I) on the maneuvering envelope in figure 12.6 must be investigated.
b) Pitch maneuver conditions. For maximum or specified pitch control displacement must be investigated as in (a) with positive and negative pitching acceleration.

The positive and negative acceleration must be equal to at least (according to Part 25: Airworthiness Standards: Transport Category, Special FAR, subpart C)

$$\dot{q} = +\frac{39}{V}n(n-1.5) \qquad \dots 12.9a$$

$$\dot{q} = -\frac{26}{V}n(n-1.5) \qquad \dots 12.9b$$

n is the positive load factor at the speed under consideration, and V is the airplane equivalent speed in knots.

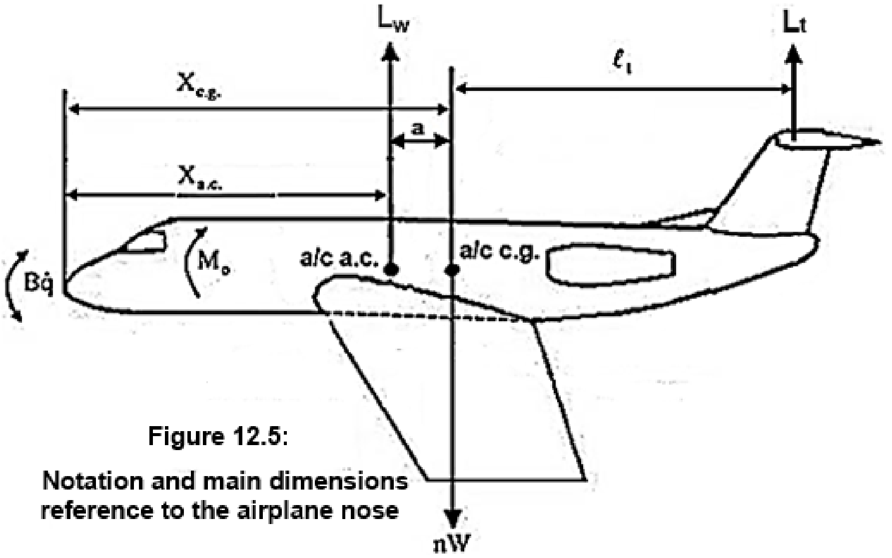


Figure 12.5:
Notation and main dimensions reference to the airplane nose

In order to evaluate Pitching moment of inertia, B, a similar to table (12.13.) is used, or from eq 12.4. The shown table is very simple. It includes main a/c parts only. The clockwise B about a/c c.g. is $+ve$, while the counter clockwise B about a/c c.g. is $-ve$. The actual table should include all a/c parts, payload and fuel for different loading cases.The values of (L_w) and (L_w) at each corner of flight-gust envelope, with and without angular acceleration, are now computed and should be tabulated as illustrated in table (12.14):

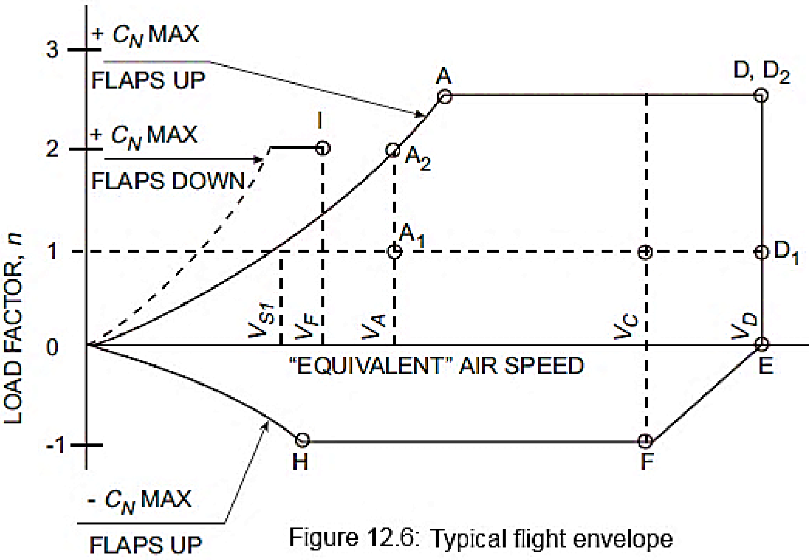


Figure 12.6: Typical flight envelope

	item	M kg	x_i m	Mx_i $Kg.m$	Mx_i^2 $Kg.m^2$	z_i m	Mz_i $Kg.m$	Mz_i^2 $Kg.m^2$
1.	*Wing*							
2.	*Fuselage*							
3.	*Tail*							
4.	*Undercarriage*							
5.	*Cockpet*							
6.	*Engines*							
7.	*Payload*							
8.	*Fuel Tanks*							
9.	*Passengers*							
10.	*…etc*							
				$\sum mx_i$	$\sum mx_i^2$		$\sum mz_i$	$\sum mz_i^2$

Table 12.13: Determination of airplane pitching moment of inertia

point	n	V_{eq}	M_o	L_t	L_w	L_t	L_w	L_t	L_w
				$\dot{q} = zero$		$+ve\ B\dot{q}$		$-ve\ B\dot{q}$	
1	2	3	4	5	6	7	8	9	10
A									
D_1									
D_2									
E									
F									

Table12.14 Arrangement results for calculation L_W & L_t at Different Flight Conditions

13. Drag Estimation

13.1. Introduction:

In this initial study the landing conditions are assumed similar to take off conditions. Detailed drag estimation is usually very elaborate exercise for which most a/c manufacturers have developed their own procedures.

Area drag method is very important. Whereby drag area for each principal part of the a/c is found, the aircraft characteristic drag area is found and then from which a/c zero lift drag ($C_{D,o}$) is found.

$$C_{D,o} = C_{D,pressure} + C_{D,skin\,friction} \quad ...13.1$$

Usually skin friction drag is dominant for slender streamed bodies, see table (13.1) for typical values. Aircraft drag coefficient is evaluated from the following formula:

$$C_D = C_{D,o} + C_{D,i} + C_{D,w} \quad ...13.2$$

$$C_{D,i} = \frac{1}{e.AR.\pi} C_L$$

Where

$C_{D,o}$: zero lift drag (profile drag).
$C_{D,w}$: Wave drag due to compressibility.
$C_{D,i}$: lift dependent drag (parasite drag).

	C_{D_o}	e
high-subsonic jet aircraft	.014 - .020	.75 - .85*
large turbopropeller aircraft	.018 - .024	.80 - .85
twin-engine piston aircraft	.022 - .028	.75 - .80
small single engine aircraft		
retractable gear	.020 - .030	.75 - .80
fixed gear	.025 - .040	.65 - .75
agricultural aircraft :		
- spray system removed	.060	.65 - .75
- spray system installed	.070 - .080	.65 - .75

Table 13.1: Profile drag values for different a/c types

13.2. Drag Area method:

Area drag method is an elementary method for preliminary design stage which is assumed satisfactory. Preliminary stage drag estimation may be accomplished by adding the individual area drag contribution of the various components of the a/c;

$$C_{D,o} = \frac{\sum C_{D,j}.S_j}{S_w} \quad ...13.3$$

The term ($C_{D,j}.S_j$) is called the drag area of each a/c component and it will be evaluated for each individual aircraft external part.

13.3. Zero lift drag at take-off stage:

A simplified area drag method based on graphics presented by “Royal Aeronautical Society” through “Engineering Science Data United Ltd” is presented here in some details. This procedure is valid also during other flight stages. Methods mentioned by in books such “Synthesis of Subsonic Airplane Design by Egbert Torenbeek” and “Airplane Design by Dr. Jan Roskam” are strongly recommended.

13.3.1. Wing:

i) Compute Reynolds number, $R_e = \rho v \bar{C}/\mu$.

ii) Assume transition point from laminar to turbulent flow, usually from wind tunnel tests. Let us say for example it occurs at $(20\%\bar{C})$.

iii) Find $(C_{D,o})$ for flat plate at zero incidence from sheet W 02.04.02.

iv) Find correction factor, λ, from sheets W 02.04.02 and W 02.04.03 to account for transition point and thickness chord ratio ,t/c, taking into consideration trailing edge angle,τ,.where λ is the ratio of profile drag coefficient of wing to that of corresponding flat plate at the same (Re and τ).

Sheet W 02.04.02 is used for (i) Conventional sections (RAF series and $NACA$ 4 digit series). (ii) Early experimentally low drag sections of the EQH and EC type. (iii) Low drag section of $NACA$ 6,5 series. Sheet W 02.04.03 is an extension of to cover low drag section of the $NACA$ 6A and similar series. Sheets W 02.04.01, W 02.04.02 and W 02.04.03 apply to incompressible condition only. Now:

$$C_{D,o} = C_{D,o} \times \lambda \times roughness\ correction \qquad \text{... 13.4}$$

For roughness (≤ 0.001) mm no correction is needed. For other surfaces:

Correction = 1.05 for metal.
= 1.10 for good paint.
= 1.50 for doped fabric.

v) Find Profile drag coefficient increment due to full span single slotted flap from sheet F02.01.06 having flap angle in degree,δ_f and flap chord in m, c_f.

$$C_{D,o_{full\ span\ flap}} = C_{D,o_{no\ flap}} + \Delta C_{D,o} \qquad \text{... 13.5}$$

vi) Find conversion factor, μ, for profile drag increment due to part span flap from sheet F02.01.07.

$$C_{D,o_{full\ span\ flap}} = C_{D,o_{no\ flap}} + \mu\Delta C_{D,o} \qquad \text{... 13.6}$$

The term $(\mu\Delta C_{D,o})$ is used during take off and landing stages only, and is ignored during cruising.

Example.

Find the conversion factor for the flap arrangement shown. It has a split flap of chord $0.2C$ at a flap angle of $\delta = 68°$, where $b_{f1} = 12.2\ m$, $b_{f2} = 7.32\ m$, $b_{f3} = 5.79\ m$, $b_{f4} = 1.52\ m$, Span, $b = 19.8\ m$, thickness chord ratio,$t/c = 0.18C$, Aspect ratio, $AR = 6$ and taper ratio, $\lambda = 0.5$.

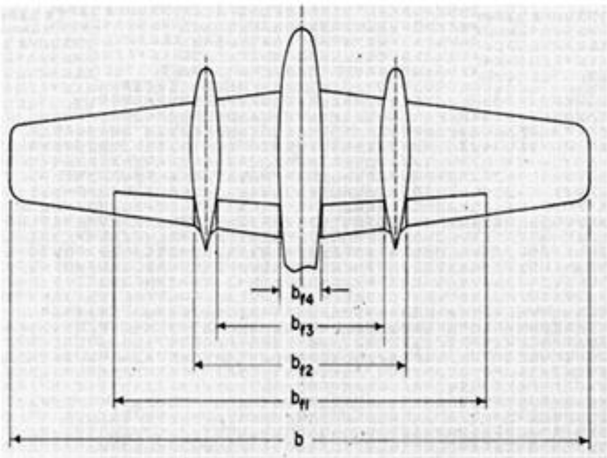


Figure 13.1: part span example

Solution:

From sheet $F02.01.07$.

$b_{f1}/b = 0.616 \rightarrow \mu_1 = 0.70$;$b_{f2}/b = 0.370 \rightarrow \mu_2 = 0.45$

$b_{f3}/b = 0.292 \rightarrow \mu_3 = 0.36$;$b_{f4}/b = 0.077 \rightarrow \mu_4 = 0.10$

$\mu = \mu_1 - \mu_2 + \mu_3 - \mu_4 = 0.70 - 0.45 + 0.36 - 0.10 = 0.51$

vii) Add allowable factor for flap gaps, hinges, linkages, etc. of value (1.10). the final wing profile drag coefficient is then:

$$C_{Do_{wing,final}} = 1.1 C_{D,o_{wing\ part\ span\ flap}} \quad \text{...13.7}$$

viii) Evaluate wing drag area:

$$wing\ drag\ area = C_{Do,wing} \times S_{wing} \quad \text{...13.8}$$

13.3.2. Empennage:

For horizontal tail: the procedure is the same as for the wing, use roughness factor correction (1.05) and allowable factor for flap gaps, hinges, linkages, etc. of value (1.5).

$$H.tail\ drag\ area = C_{Do,Htail,final} \times S_{H.tail} \quad \text{...13.9}$$

- For vertical tail: the procedure is the same as for the wing, use roughness factor correction (1.05) and allowable factor for flap gaps, hinges, linkages, etc. of value (1.32).

$$V.tail\ drag\ area = C_{D,o_{V.tail}} \times S_{V.tail} \quad \text{...13.10}$$

13.3.3. Fuselage:

i. Find diameter to length ratio (d_f/l_f) and Reynolds number $R_e = \rho v l_f/\mu$.

ii. Use sheet $B02.04.01$ for transition from laminar to turbulent flow at nose or sheet $B02.04.02$ for transition point at ($0.1 l_f$) or sheet $B02.04.03$ for transition point at ($0.2 l_f$), to evaluate ($C_{Do,fuselage}$).

iii. Use roughness factor correction (1.05) and allowable factor for protrusions, gaps, hinges, linkages (1.05).

$$C_{Do,fuselage,final} = 1.05 \times 1.05 C_{Do,fuselage} \quad \text{...13.11}$$

iv. Evaluate fuselage drag area:

$$fuselage\ drag\ area = C_{Do,fuselage,final} \times S_{fuselage} \quad \text{...13.12}$$

13.3.4. Cockpit: Take the following data:

$C_{Do,cockpit} = 0.05$ For smooth well rounded cockpit.

$= 0.20$ for angular or open cockpit.

$A_{cockpit,projected} \approx 0.2\ A_{fuselage,frontal}$

$$Cockpir\ drag\ area = C_{Do,cockpit} \times A_{cockpit,projected} \quad \text{...13.13}$$

13.3.5. Nacelle:

i. Find diameter to length ratio (d_N/l_N) and Reynolds number ($R_e = \rho v l_N/\mu$).

ii. Use sheet $B02.04.01$ or sheet $B02.04.02$ or sheet $B02.04.03$ to evaluate $C_{Do,Nacelle}$.

iii. Use roughness factor correction (1.10) and allowable factor for gaps, hinges, linkages, etc (1.05), and also nacelle sharp edges (1.2)

$$C_{Do,Nacelle,final} = C_{Do,Nacelle} \times (1.1)_{gaps} \times (1.05)_{rough} \times (1.2)_{sharp} \quad \ldots 13.14$$

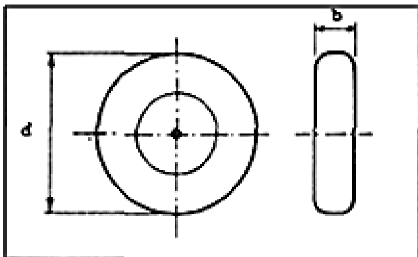
iv. Evaluate nacelle drag area:

$$Nacelle\ drag\ area = C_{Do,Nacelle,final} \times A_{Nacee,wetted} \times number\ of\ nacelles \quad \ldots 13.15$$

13.3.6. Undercarriage:

At takeoff and landing stages all undercarriages are extended down and they are contribute a great deal of drag to a/c. at cruise stage all u.c. are usually extracted and contribute nothing to a/c drag except for fixed type u.c. Table (13-2a) represents the drag coefficients of several types of wheels. In table (13-2b) drag can be appreciably reduced by various types of streamlined fairings.

Drag area of undercarriage is evaluated as flow:



d = tire diameter
b = tire width
$R = \frac{V\,d}{\nu}$, Reynolds number referred to tire diameter
$C_D = \frac{drag}{q_\infty b\,d}$

(a)

Wheel type	d inch	b inch	S (ft^2)	R$\times 10^{-6}$	BARE WHEEL C_D
8.50-10, low-pressure tire	25 5/16	8 1/2	1.494	1.60	.250
27 inch streamline tire	27	9 1/4	1.734	1.70	.176
25 x 11-4 extra-low-pressure tire	24 1/2	11 1/4	1.914	1.55	.226
30-5 disk wheel with 30-5 tire	29 3/4	5	1.033	1.87	.350
30-5 disk wheel with 32-6 tire	31 1/2	6 1/4	1.367	1.98	.310

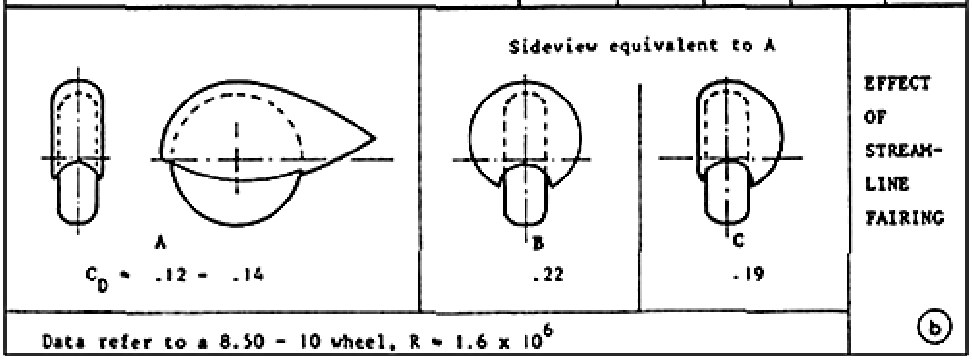


Table 13.2a: Typical profile drag coefficient of different u.c layout

❖ Main wheel:- let as assume that there are two bogies, each one has two wheels and two fairings, then:

$$(C_{Do}.S)_{u.c.} = 2(C_{Do} \times S)_{wheel} + 2(C_{Do} \times S)_{fairing} + 2(C_{Do} \times S)_{struts} \quad \ldots 13.16$$

Where C_{Do} from table 13.2b for each item and S is the frontal area

❖ Nose wheel: - the same as for main u.c. there are usually one assembly with two (one) wheels plus fairings and a leg.

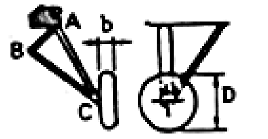
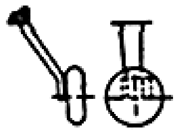
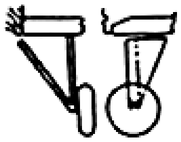
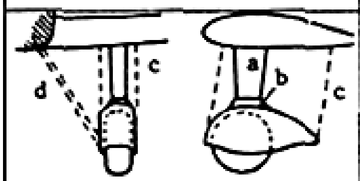
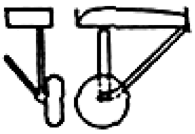
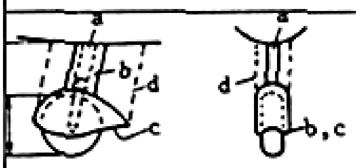
CONFIGURATION	REMARKS			C_{D_π}
 wheel type 8.5-10 **	no streamline members, no fairings			1.28
	with streamline members	junctions not faired		.56
		junctions A and B faired		.47
		junctions A, B and C faired		.43
		with wheel fairing type C (Fig. F-17)		.36
 streamline member	no fairing	27-inch streamline wheels		.23
			8.5-10 wheels	.29
	wheel fairing	type B	8.5-10 wheels	.27
		type C	8.5-10 wheels	.25
	no fairing	27-inch streamline wheels		.25
			8.5-10 wheels	.31
	wheel fairing type A		8.5-10 wheels	.23
	no fairing		8.5-10 wheels	.51
	wheel fairing type C		8.5-10 wheels	.34
 8.5-10 wheels	circular strut, no fairings			.05
	streamline strut	corners not faired	(a)	.26
		corners faired	(b)	.17
	trouser fairing	cantilever	(c)	.17
		with sidestay	(d)	.38
	8.5-10 wheels	not faired		.53
		with fairing c		.34
 NOSE GEAR	round strut with fork		(a)	.64
	faired strut with fork		(b)	.42
	faired strut, wheel faired		(c)	.15
	trouser fairing		(d)	.29
 TAILWHEEL	no fairing			.58
	with rear fairing			.49
	with forward fairing			.41
	completely faired			.27

Table 13.2b: Typical profile drag coefficient of different u.c layout

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13.4. Interference drag:-

The evaluation of interference drag is a bit complicated because on a/c many types of interference drag and for each one the influence on vortex (pressure) drag and profile (skin friction) drag should be study. There are the following interference drag types:

- Wing-fuselage.
- Nacelle-wing.
- Nacelle-fuselage.
- Tail-fuselage.
- Tail-wing.

The wing-fuselage interference drag is seemed to be the significant one. The following empirical relation can be used:

$$(drag\ area)_{interference} = \frac{S_w (t/c)_{root}}{12n} \qquad \ldots 13.17$$

$n = 20$ for low wing.

$n = 25$ for high wing.

So the aerodynamic efficiency for high wing is better than that for low wing, since the high wing produces lower drag than low wing..

13.5. Zero lift drag at cruise stage.

A similar procedure can be adopted to find drag equation at cruise stage where no flap contribution and no undercarriage contribution. A more detailed and a more accurate procedure is as the follow:

Drag at cruise = incompressible wing drag + compressible wing drag + extra to wing drag + lift dependent drag.

$$(C_{Do})_{cruise} = (C_{Do})_{\substack{wing \\ incompressible}} + (C_{Do})_{\substack{wing \\ compressible}} + (C_{Do})_{extra} + (C_{Di})_{\substack{lift \\ dependent}} \qquad \ldots 13.18$$

13.5.1. Incompressibility wing drag:- as explain previously or from the following empirical formula:

$$(C_{Do})_{\substack{wing \\ incompressible}} = \frac{1.15\{1.0 + 3.0(t/c)\cos^2\Psi\}}{(\log_{10} Re)^{21/8}} \times \frac{S_{net}}{S_{gross}} \qquad \ldots 13.19a$$

$$\tan\Psi = \tan\Lambda - \frac{(1-\lambda)\{1.6(\chi)_{R/c} - 0.28\}}{(1+\lambda)AR} \qquad \ldots 13.19b$$

$$\frac{S_{net}}{S_{gross}} = 1.0 - \frac{2.0 d_{fus} AR^{0.5} S_{gross}^{0.5} - d_{fus}^2 (1.0-\lambda)}{(1+\lambda)AR.S_{gross}} \qquad \ldots 13.19c$$

t/c	: Thickness/chord ratio.	;Ψ	:Aerodynamic sweep of wing.
Re	: Reynolds number.	;$\chi_{R/c}$	:Roof position of chord wise pressure distribution.
AR	: Aspect ratio	;S_{net}	: Net wing area.
S_{gross}	: Gross wing area.	;Λ	: sweep angle at ¼ chord.
λ	: Taper ratio.	;d_{fus}	: Fuselage maximum diameter.

13.5.2. Compressibility wing drag:-

Compressibility effect on drag are generally ignored at Mach number below (0.5), see figure (13-2). The following increment may be assumed:

$(C_{Do})_{\substack{wing \\ compressible}} = 0.002$ for high speed cruise condition.

$(C_{Do})_{\substack{wing \\ compressible}} = 0.005$ for long range cruise condition.

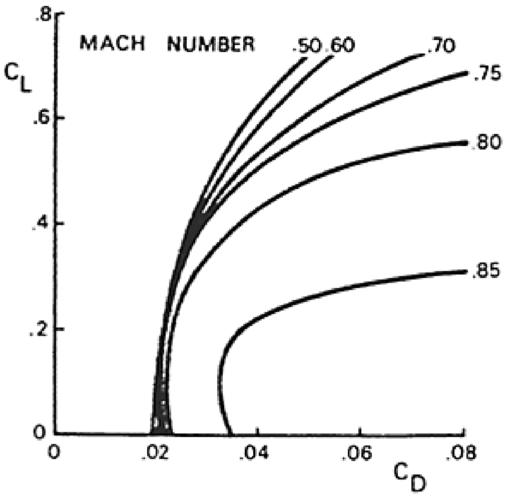


Figure 13-2: Effect of compressibility on the drag polar

13.5.3. Extra to wing drag:-

Extra to wing drag result from fuselage, u.c. if not retractable, nacelle, empennage…etc. the major method is exactly as explained in item (13-1) by considering each individual element separately and then associate areas drag together.

$$(C_{Do})_{extra}.S_w = (C_{Do}.S)_{fus} + (C_{Do}.S)_{u.c.} + (C_{Do}.S)_{nace} + (C_{Do}.S)_{tail} \ldots etc \qquad \ldots 13.20a$$

$$(C_{Do})_{extra} = \frac{(C_{Do})_{extra}.S_w}{S_w} \qquad \ldots 13.20b$$

13.6. Drag area, empirical method B :(optional for the student)

A simplified drag area approach based on statistical data is suggested by (E$gbert\ Torenbeek$). In the present method the zero-lift drag will be calculated according to the following basic equation.

$$C_{Do}.S_w = r_{Re}.r_{u.c.}\{r_t[(C_{Do}.S)_w + (C_{Do}.S)_f + (C_{Do}.S)_t + (C_{Do}.S)_n \ldots etc]\} \qquad \ldots 13.21$$

If there is an extra areas projected to the flow, it profile drag should also be added.

13.6.1. Wing:- Uncorrected drag area for smooth wings is:

$$(C_{Do}.S)_w = 0.0054.r_w\{1.+3.0(t/c)(\cos^2\Lambda)_{0.25}\}S_{gross} \qquad \ldots 13.22$$

$r_w = 1.0$ for cantilever wing.

$\quad = 1.1$ for braced wing.

t/c is mean thickness chord ratio.

$\Lambda_{0.25}$ is sweep angle at the quarter-chord line.

S_{gross} is gross plane-form area

13.6.2. Fuselage:- Uncorrected parasite drag for streamline shape:-

$$(C_{Do}.S)_f = 0.0031.r_f.l_f(b_f + h_f) \qquad \ldots 13.23$$

l_f : Fuselage length, including propeller spinner or jet engine out let.

$b_f; h_f$: Maximum width and length of the major cross section, including canopy.

r_f : Shape factor, the ratio of actual wetted area to that of fuselage with elliptical or circular cross section and cylindrical mid section.

---

r_f =1.30 for rectangular cross section.

=1.15 for one side of cross section rectangular other side is rounded off.

$=0.65 + 1.5\left(1/\lambda_f\right)$ for fully stream lined fuselage without cylindrical mid-section where $\lambda_f = l_f/d_f$ is the finesse ratio.

13.6.3. Tail:-_ (As for the wing)

$$(C_{Do}.S)_{v.t} = 0.0054.r_{v.t}\{1.+3.0(t/c)(\cos^2\Lambda)_{0.25}\}S_{v.t} \quad \text{... 13.24}$$

$$(C_{Do}.S)_{h.t} = 0.0054.r_{h.t}\{1.+3.0(t/c)(\cos^2\Lambda)_{0.25}\}S_{h.t} \quad \text{... 13.25}$$

$r_{v,t}$ & $r_{h,t} = 1.0$

13.6.4. Nacelle:-

- ❖ Turbojet engine:-

$$(C_D.S)_n = 1.72 r_n r_{th}\left(\frac{5+\lambda}{1+\lambda}\right)\frac{T_{to}}{\Psi_{to} p_{atm}} \quad \text{... 13.26}$$

r_n : =1.50 all engine podded.

=1.65 tow engines podded one is buried in fuselage tail.

=1.25 engines buried in nacelles, attached on the side of the fuselage.

=1.00 engines fully buried, with intake scoops on fuselage.

=0.30 engines fully buried, with wing root intakes.

r_{th} : =1.00 thrust reversers installed.

=0.82 no thrust reversers.

λ : by bass ratio.

Ψ_{to} : Specific thrust (jet thrust / air flow) at sea level condition.

T_{to} : Thrust at takeoff at sea level condition.

p_{atm} : Static atmospheric pressure at sea level condition.

- ❖ Turboprop engine:-

$$(C_D.S)_n = 1.10 r_n \frac{P_{to}}{\Phi_{to}} \quad \text{... 13.27}$$

r_n : =1.00 ring type inlets.

=1.60 scoop type inlets, increasing the frontal area.

Φ_{to} : power/engine frontal area at sea level.

P_{to} : Total (equivalent) takeoff horse power at sea level condition.

- ❖ Piston engine:-

$$(C_D.S)_n = 0.07\xi_n \frac{P_{to}}{\Phi_{to}} \quad \text{... 13.28}$$

$$\xi_n = \frac{nacelle\ frontal\ area}{cylinder\ volume}$$

$\xi_n = 0.012\ to\ 0.015\, ft^2/in^2$,($0.07\ to\ 0.09\ \ m^2/litter$) for engine power up to $500\ hp$.
$= 0.050\ to\ 0.070\ \ ft^2/in^2$ ($0.03\ to\ 0.04\ \ m^2/litter$). for engine power $2000\ to\ 4000\ hp$.

Φ_{to} : Specific thrust (jet thrust / air flow) at sea level condition.

P_{to} : Total (equivalent) takeoff horse power at sea level condition.

13.6.5. Corrections for (Re) and miscellaneous drag:-

The interference effect, surface irregularities, air scoops, slots, etc. generally affect the boundary layer more on small low speed a/c than they do on large high speed a/c, due to the difference in relative size. Figure (13-1) shows the relation between Re and correction factors r_{Re} for different aircrafts. To account for the effect of the Reynolds number on turbulent skin friction drag and miscellaneous drag items, the following formula is used:

$$r_{Re} = \frac{actual\ zero\ lift\ drag\ coefficient}{uncorrected\ drag\ coefficient} = 47\ Re_f^{-0.2} \qquad \ldots 13.29$$

$$Re_{fuselage} = \frac{\rho V_{cruise} l_{fuselage}}{\mu}$$

Figure (13-3) shows that the miscellaneous drag contributions are about ($25\ to\ 30\%$) for light a/c and about ($10\ to\ 15\%$) for large transport a/c.

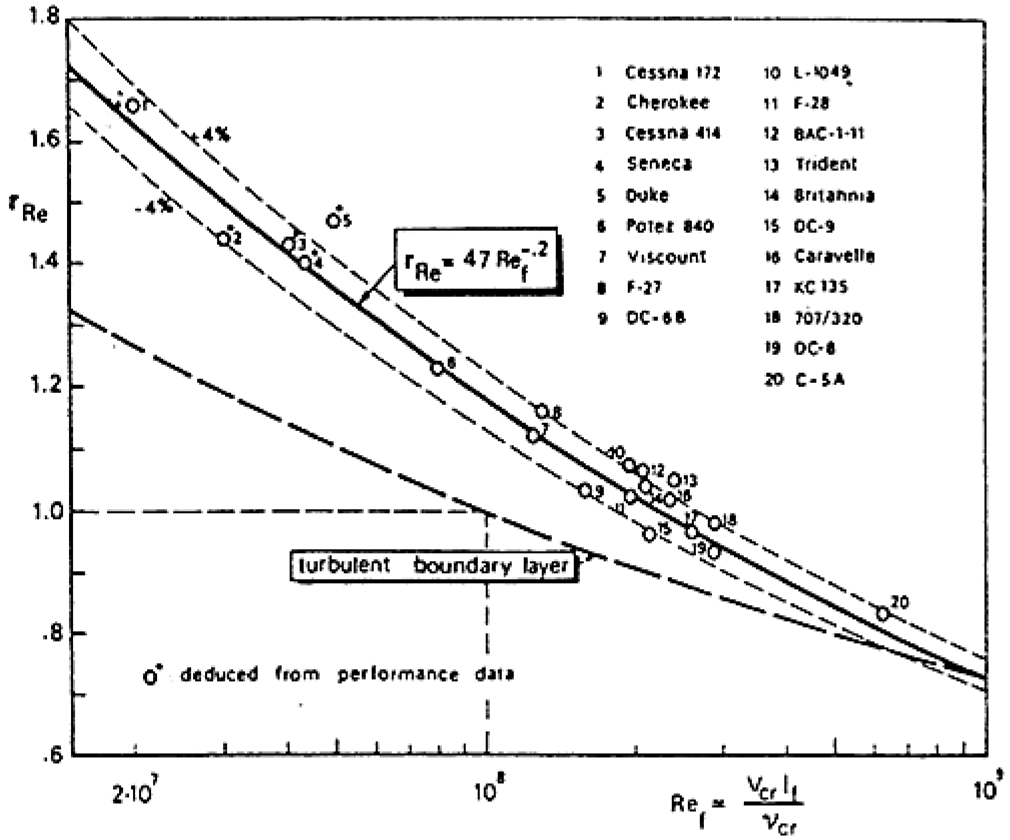


Figure 13.3: Correction factor for miscellaneous contributions as a function of $Re_{fuselage}$

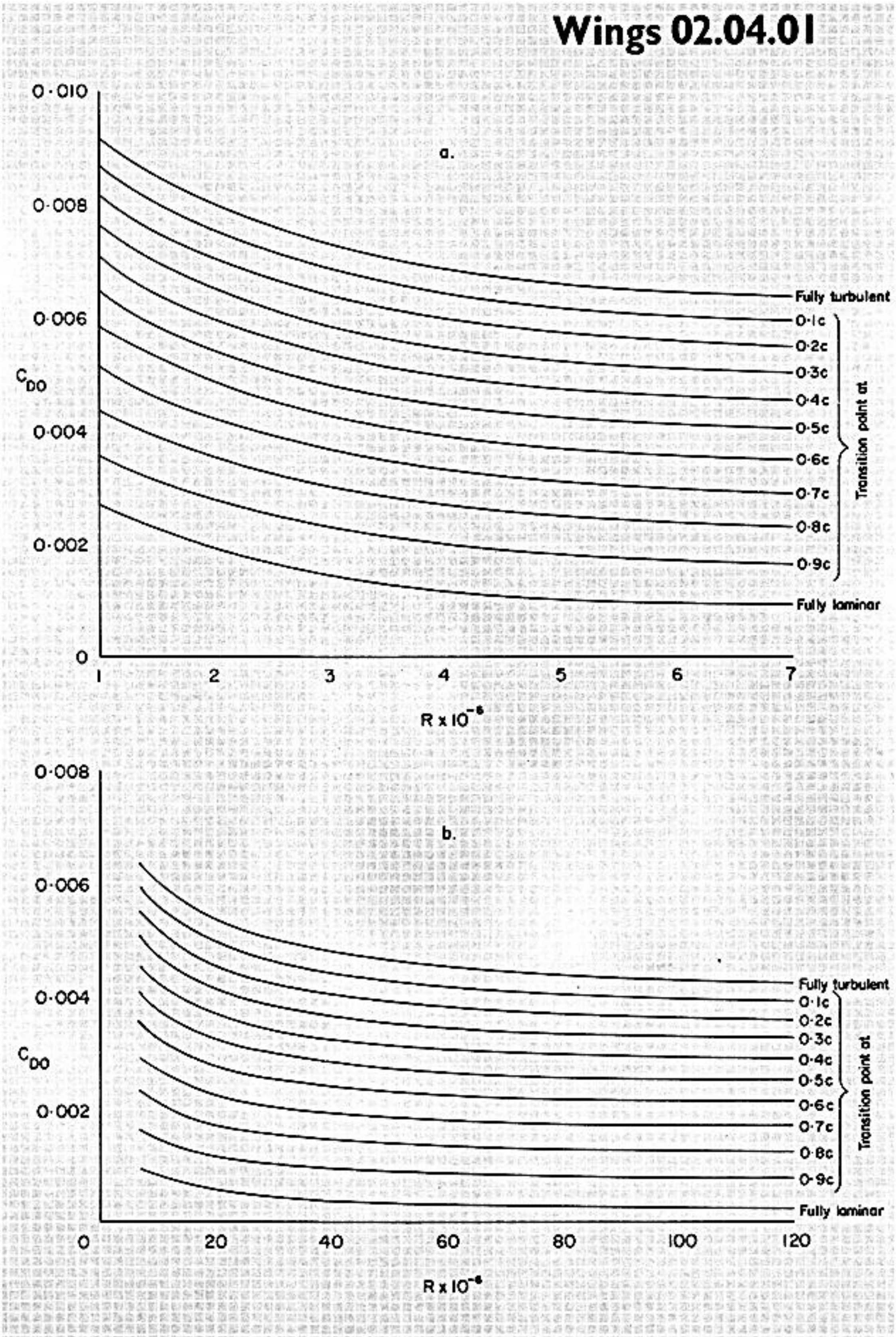


FIGURE 1. DRAG OF A SMOOTH FLAT PLATE AT ZERO INCIDENCE

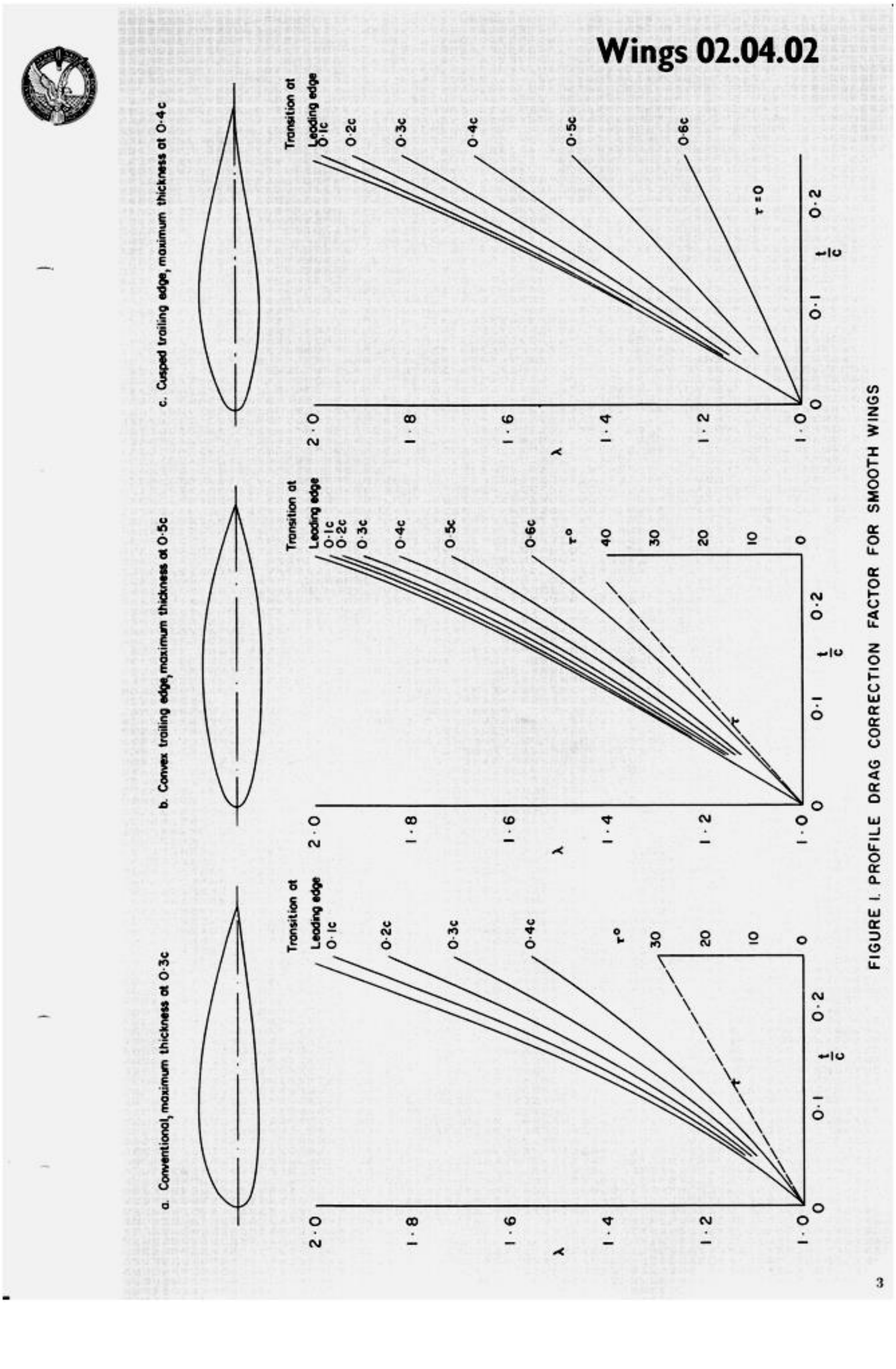
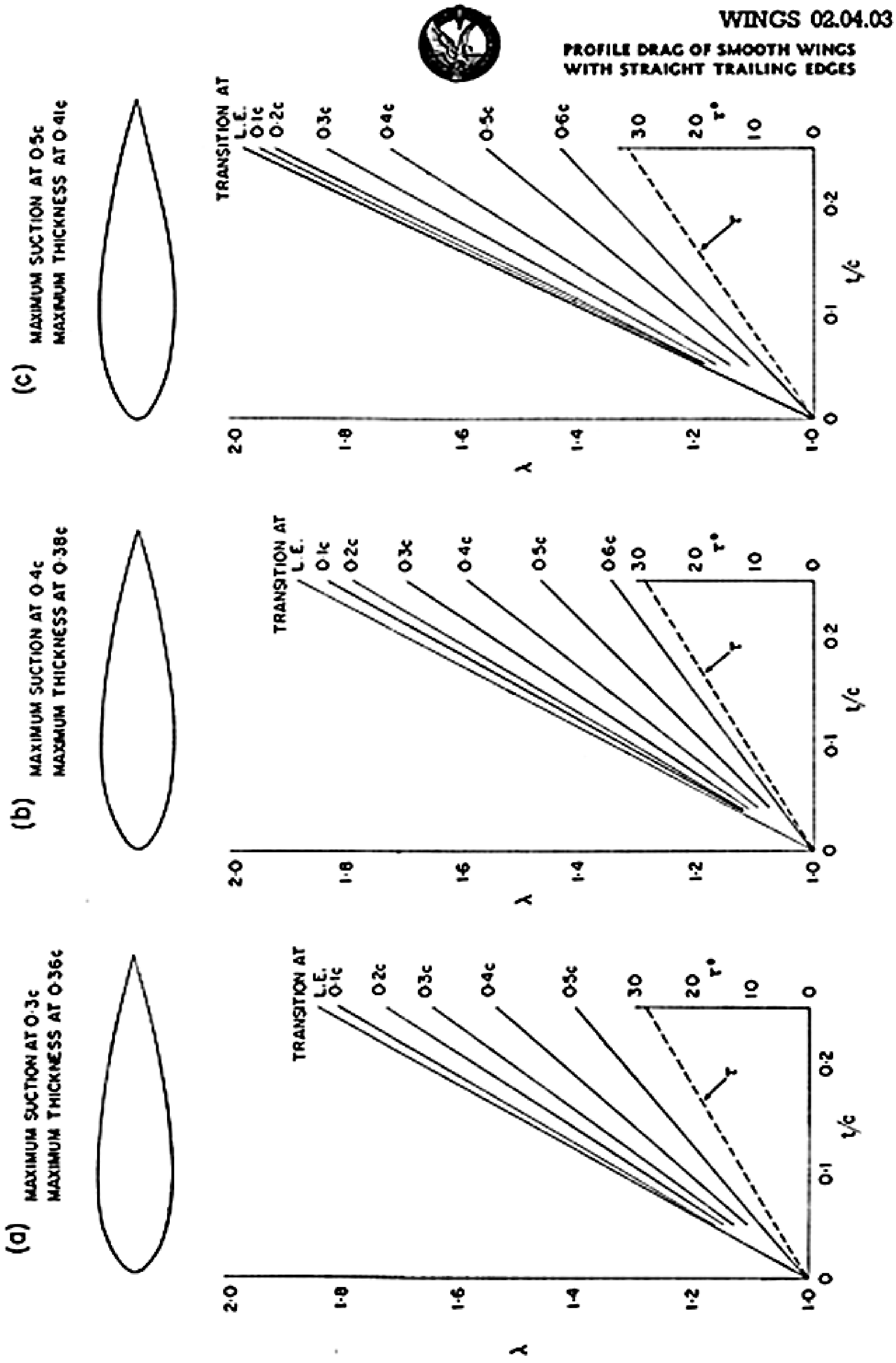
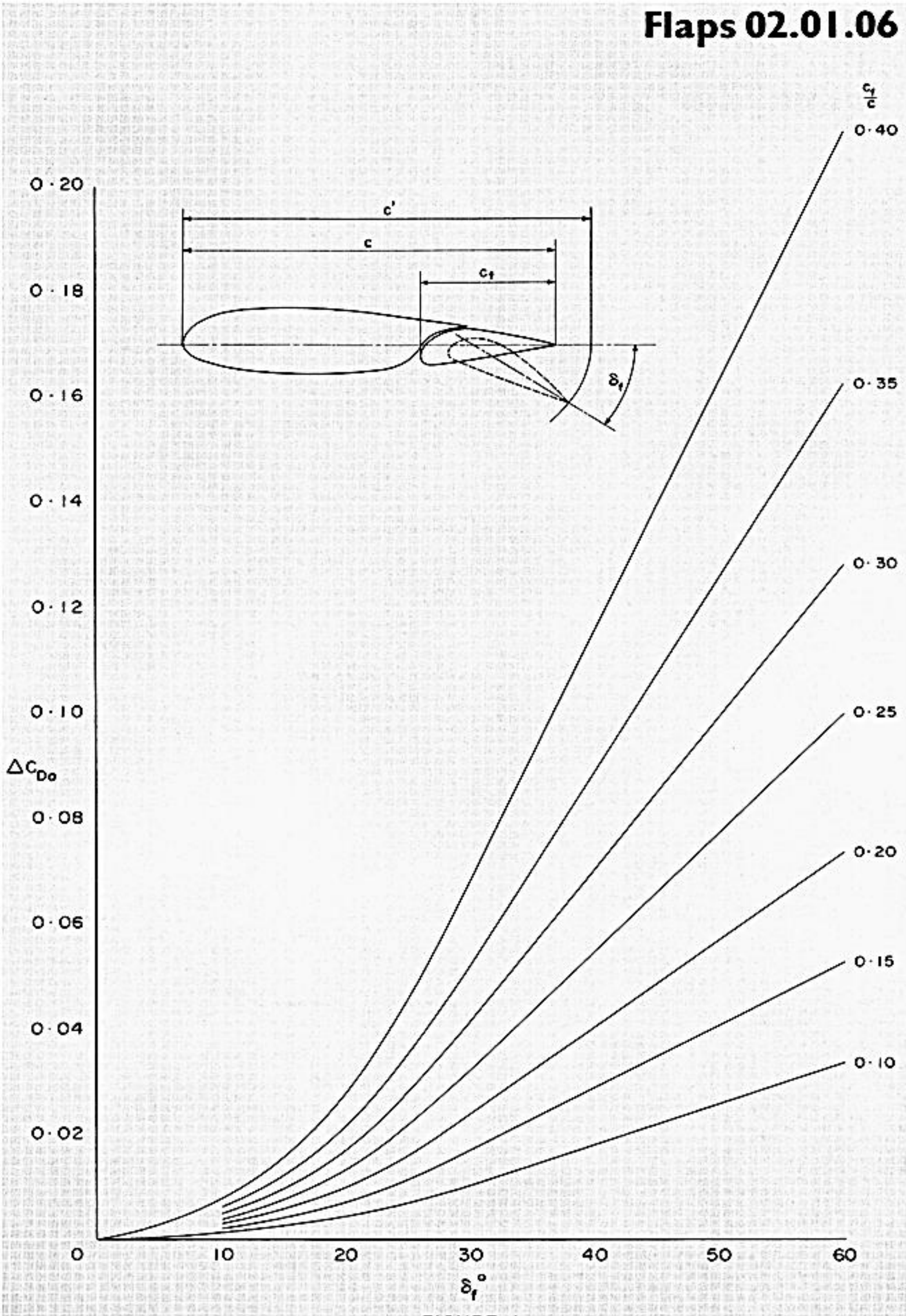


FIGURE I. PROFILE DRAG CORRECTION FACTOR FOR SMOOTH WINGS


WINGS 02.04.03
PROFILE DRAG OF SMOOTH WINGS
WITH STRAIGHT TRAILING EDGES
(a) MAXIMUM SUCTION AT 0·3c
MAXIMUM THICKNESS AT 0·36c
(b) MAXIMUM SUCTION AT 0·4c
MAXIMUM THICKNESS AT 0·38c
(c) MAXIMUM SUCTION AT 0·5c
MAXIMUM THICKNESS AT 0·41c
TRANSITION AT
L.E.
0·1c
0·2c
0·3c
0·4c
0·5c
0·6c
τ°
t/c
λ


Flaps 02.01.06
c'
c
c_f
δ_f
$\frac{c_f}{c}$
0·40
0·35
0·30
0·25
0·20
0·15
0·10
0·20
0·18
0·16
0·14
0·12
0·10
ΔC_{Do}
0·08
0·06
0·04
0·02
0
10
20
30
40
50
60
$\delta_f{}^\circ$

Flaps 02.01.07

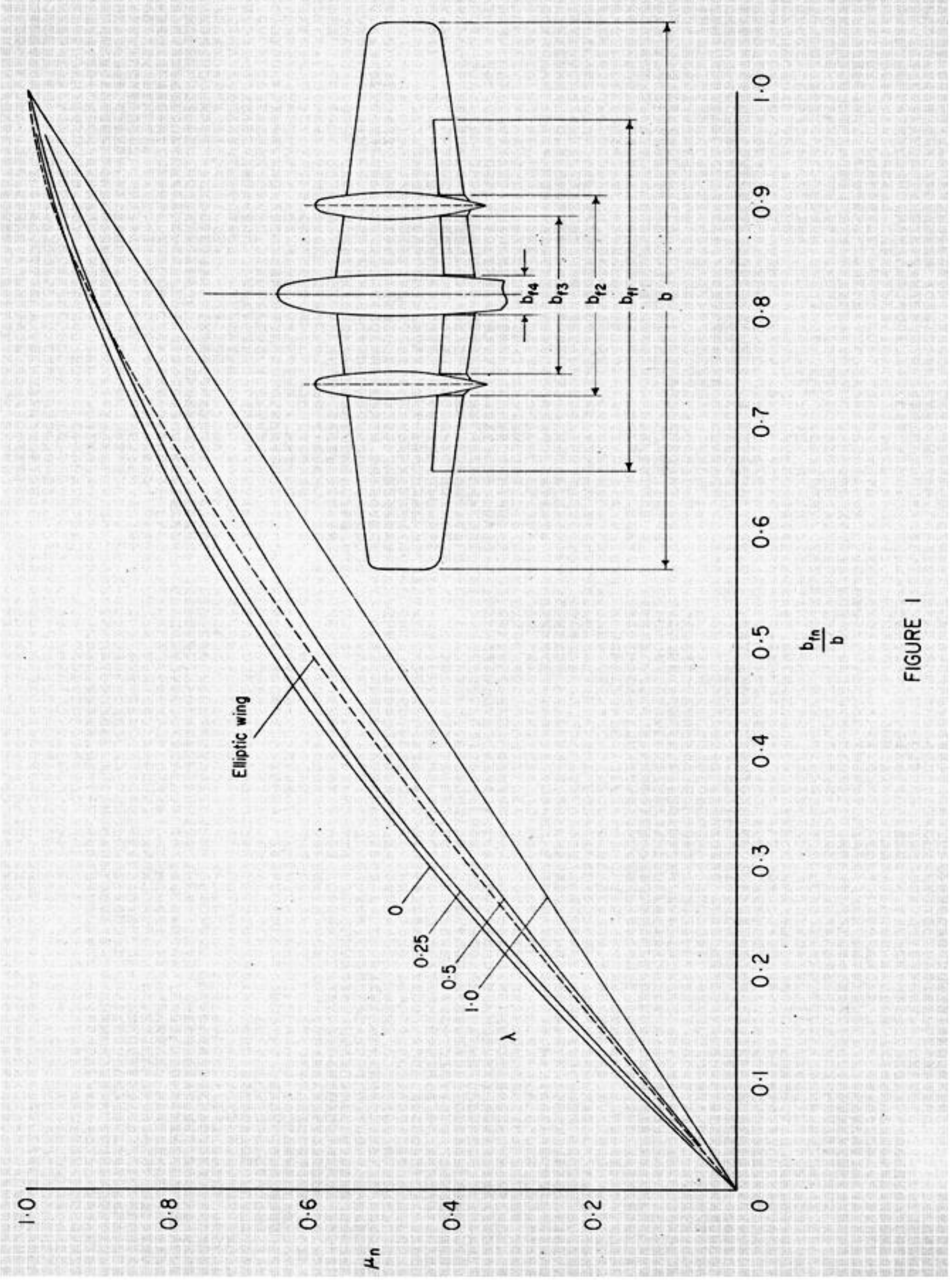
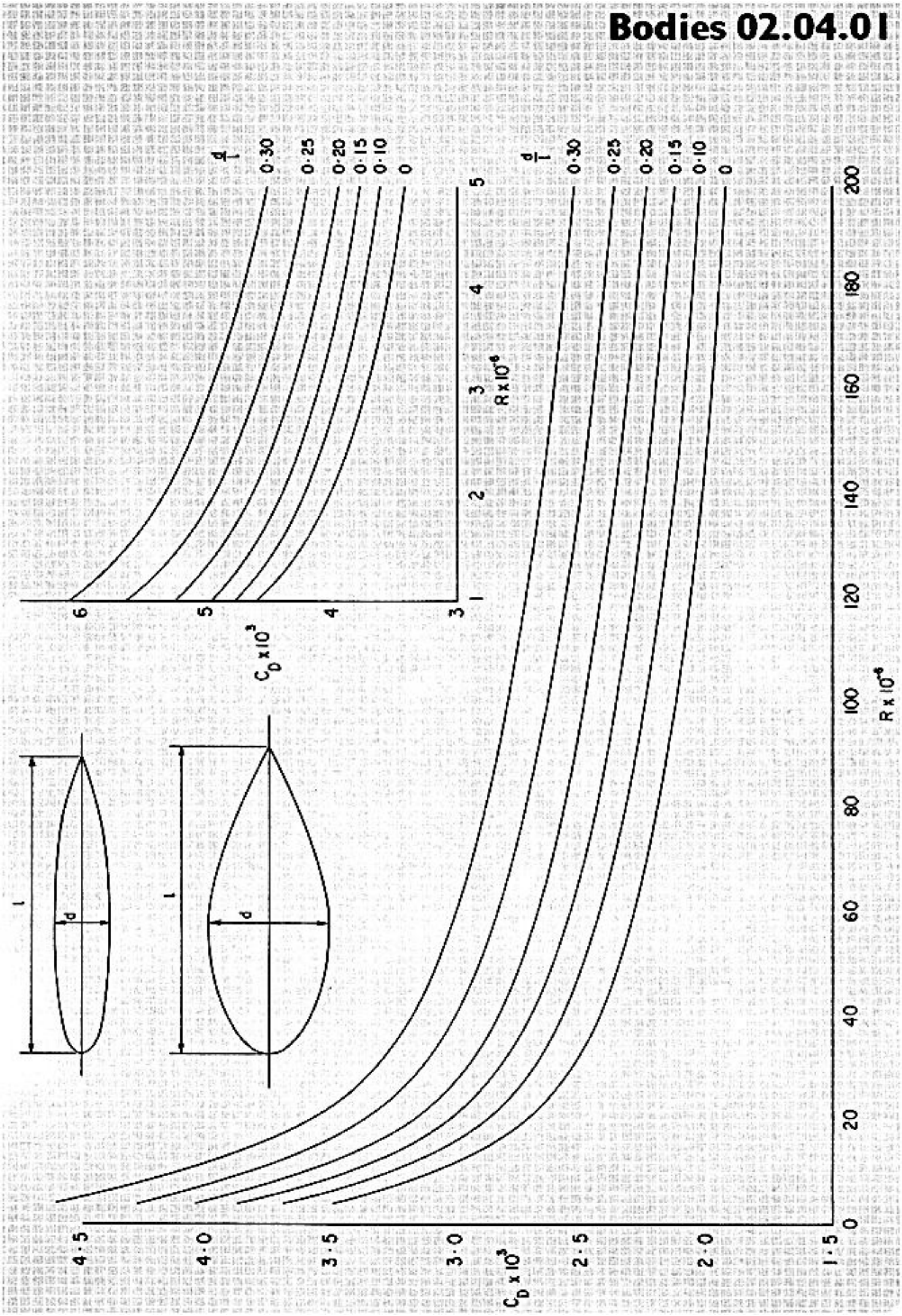
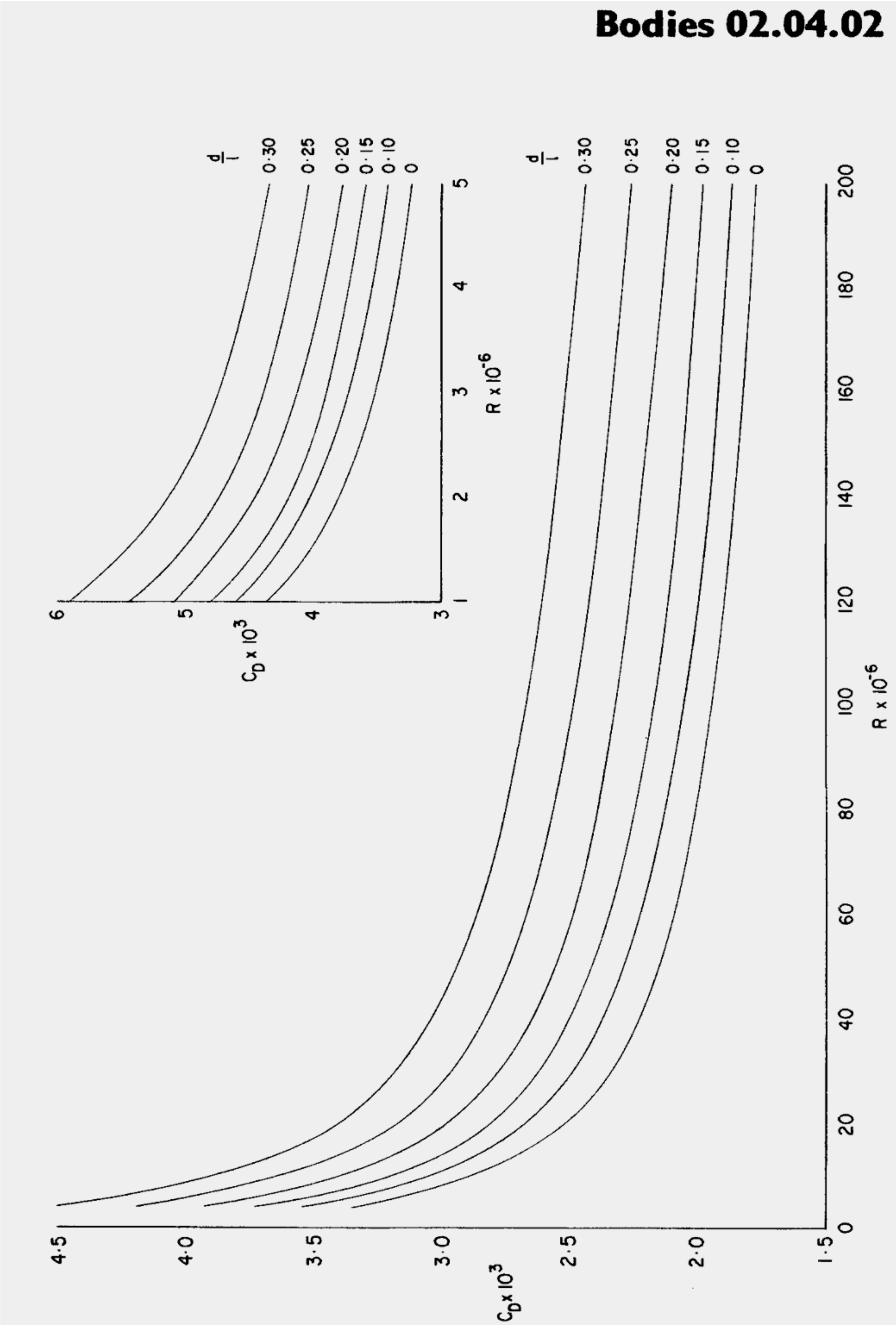
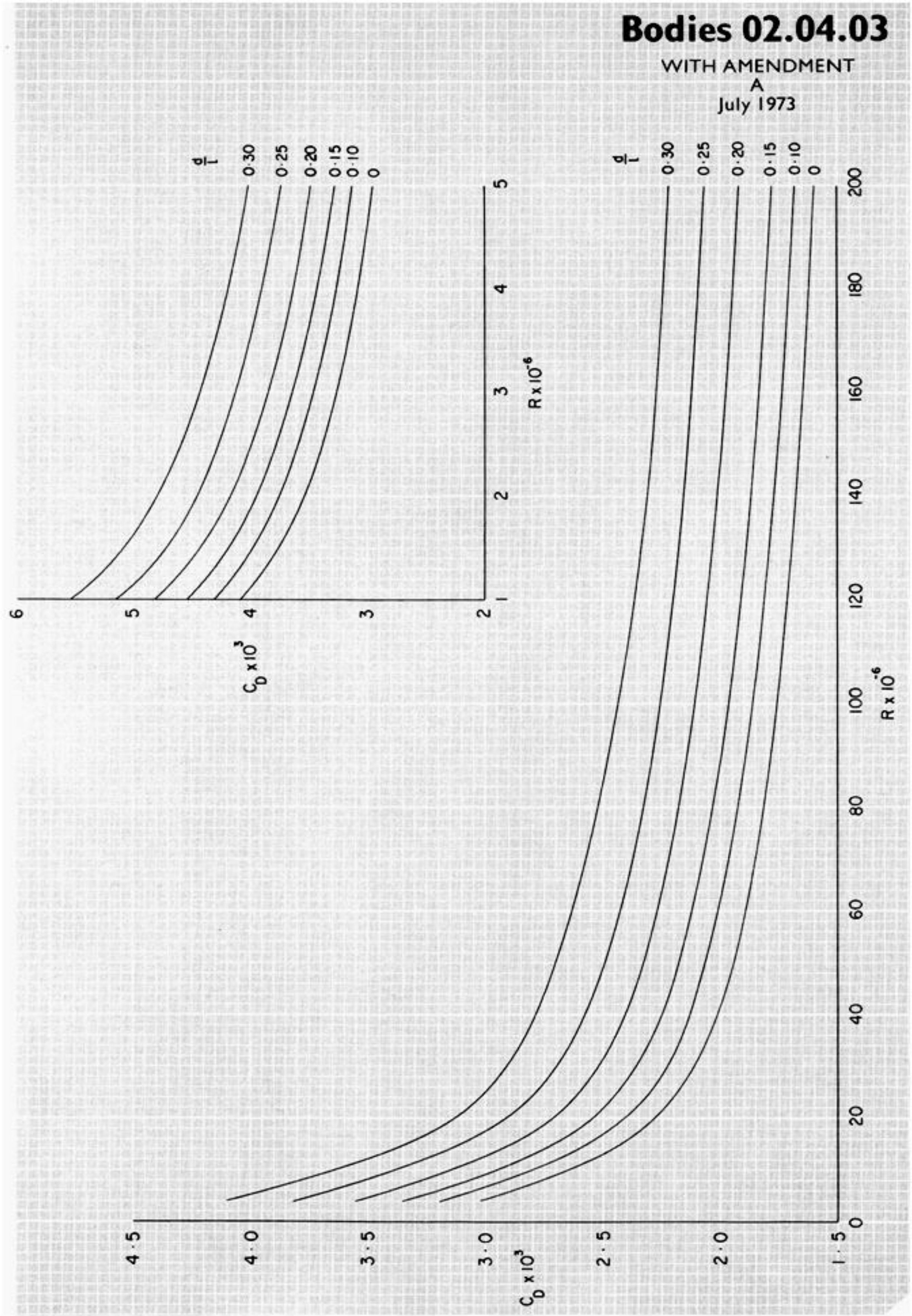


FIGURE 1


Bodies 02.04.01
d/l
0·30
0·25
0·20
0·15
0·10
0
$C_D \times 10^3$
$R \times 10^{-6}$
l
d


Bodies 02.04.02
d/l
0·30
0·25
0·20
0·15
0·10
0
$C_D \times 10^3$
$R \times 10^{-6}$


Bodies 02.04.03
WITH AMENDMENT
A
July 1973
d/l
0·30
0·25
0·20
0·15
0·10
0
R x 10^-6
C_D x 10^3